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Trend, Cycle and 'Fortuitous Cancelations'
A Note on a Paper by Nelson and Plosser

di

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TREND, CYCLE AND "FORTUITOUS CANCELATIONS"
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1. Some economists have recently discussed the traditional decomposition of economic time series into a deterministic trend and a stationary cycle. A partial list of papers dealing with this issue includes Beveridge and Nelson (1981), Nelson and Plosser (1982), Harvey (1985), Watson (1986), Campbell and Mankiw (1987), Cochrane (1988), Quah (1988), Rappoport and Richelin (1989), Lippi and Richelin (1989). Nelson and Plosser argue that the trend component of US GNP is better characterized as a stochastic difference-stationary process than as a deterministic function of time. Moreover, they claim that the variance of the trend component is large compared with the variance of the cyclical component, so that a major part of GNP fluctuations is due to permanent shocks.

This conclusion relies heavily on the following argument. The authors find that the first difference of US GNP (in logs) is a first-order moving average process. From this empirical evidence, Nelson and Plosser infer that, barring "fortuitous cancelations", both trend and cycle must be MA(1). Taking this for granted and noting that US GNP exhibits a positive autocorrelation at lag one, it is not difficult to show that the variance of the trend component cannot be small in comparison with that of the cycle.

The claim made by Nelson and Plosser has been criticized by several authors (see for instance Harvey 1985, Rappoport and Reichlin 1989, Lippi and Reichlin 1989), but the argument stated before has never been questioned. In the present note I maintain that this line of reasoning is wrong. The trend component may have an arbitrarily small variance, so that it can closely approach a deterministic function of time, even though output (in differences) is MA(1) with a positive first-order autocorrelation.

2. Let us define the problem in some more detail. The output y_t is an integrated process of order one. It is the sum of a trend T_t, which is also integrated of order one, and a stationary cycle c_t. The cycle follows the model $c_t = \psi(L)u_t$, while the first difference of the trend follows the model $(1 - L)T_t = \theta(L)v_t$. The vector process $(\, u_t \quad v_t \,)$ is a zero-mean white noise; $\psi(L)$ and $\theta(L)$ are (not necessarily finite) polynomials in the lag operator

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L. Output therefore satisfies the relation

$$(1 - L)y_t = \theta(L)v_t + (1 - L)\psi(L)u_t.$$

Moreover, Δy_t is MA(1), and $\text{cov}(\Delta y_t, \Delta y_{t-1}) > 0$. Starting from these assumptions, Nelson and Plosser assert that the order of $\theta(L)$ must be one and the order of $\psi(L)$ must be zero. Given that $\text{cov}(\Delta y_t, \Delta y_{t-1}) > 0$, this statement implies $\text{var}(v_t) \geq \text{var}(u_t)$, i.e. the trend innovation is bigger than the cycle innovation.

My argument is the following. Cycle and trend (in differences) are not in general MA(1), even though output is MA(1). If a process, say x_t, is the sum of ξ_t and z_t and we know that both ξ_t and z_t are MA(q), we can conclude that in general x_t is MA(q). However the converse is not true: if we know that x_t is MA(q), we cannot conclude that both ξ_t and z_t are MA(q).

Consider the following example. For the sake of simplicity, let us suppose that cycle and trend (in differences) are MA processes of order not higher than two, so that $\theta(L) = 1 + \theta_1 L + \theta_2 L^2$ and $\psi(L) = 1 + \psi L$. Consider then the following restriction:

$$\theta_2 \text{var}(v_t) - \psi \text{var}(u_t) + (\theta_2 - \psi)\text{cov}(v_t, u_t) = 0. \tag{1}$$

If equation (1) holds, the autocorrelation of output at lag two is zero and the order of Δy_t is not higher than one. By contrast, if (1) does not hold, Δy_t is MA(2). The values of the parameters θ_1, θ_2, ψ, $\text{var}(u_t)$, $\text{var}(v_t)$, $\text{cov}(v_t, u_t)$ which satisfy relation (1) form a zero-measure set in R^6. If no *a priori* information about these values is available, equation (1) holds with probability zero. Hence, unless certain values of the above mentioned parameters are economically more likely than others, the case of output being MA(1) (given that both cycle and trend are MA of order not greater than two) must be regarded as fortuitous.

Nevertheless, the problem facing Nelson and Plosser is rather different. The process $(1 - L)y_t$ is known to be MA(1). Therefore, equation (1) must hold and the aforementioned "fortuitous cancelations" must occur. A particular case of restriction (1) is

$$\theta_2 = 0; \qquad \psi = 0. \tag{2}$$

In this case, both cycle and trend are MA(1). But notice that if, for instance,

$$\text{var}(v_t) = \text{var}(u_t; \qquad \theta_2 = \psi, \tag{3}$$

equation (1) still holds, even though conditions (2) are not satisfied. Unless more information is available, we cannot regard (3) as more unlikely than (2).

As a matter of fact, restrictions (2) (as well as (3)) are unlikely to hold. The values of θ_1, θ_2, ψ, $\text{var}(u_t)$, $\text{var}(v_t)$, $\text{cov}(v_t, u_t)$ which satisfy (2) belong to a zero-measure set in R^5, whereas the set of values satisfying (1) have a non-zero measure in the same space. Therefore, the probability of the former set is zero. In other words, equation (1) characterizes the general case, whereas the case of both cycle and trend being of order one is fortuitous.

3. Dropping the hypothesis that both ΔT_t and Δc_t are MA of order not greater than two, the possibilities for cycle and trend not being MA(1) are increased. Let us consider the case where cycle and trend are respectively ARMA(1,2) and ARMA(1,1):

$$(1 - L)y_t = \frac{1 + \theta L}{1 - \alpha_1 L} v_t + \frac{(1 - L)(1 + \psi L)}{1 - \alpha_2 L} u_t. \tag{4}$$

To make things simple, let us suppose that v_t and u_t are orthogonal at all leads and lags. Then the process Δy_t is MA(1) if and only if

$$\alpha_1 = \alpha_2 = \alpha$$
$$\text{var}(v_t)\alpha(\alpha + \theta)(1 + \alpha\theta) = \text{var}(u_t)(1 - \alpha)^2(\alpha + \psi)(1 + \alpha\psi). \tag{5}$$

Although restrictions (5) describe a particular case within model (4), Nelson and Plosser's case is even more peculiar, since it is obtained by imposing (5) as well as the further restriction $\alpha = 0$. Indeed, setting $\alpha = 0$, the second equation in (5) reduces to $\psi = 0$, so that both trend and cycle are MA(1).

Given that ΔT_t and Δc_t are not MA(1), the conclusion of Nelson and Plosser is no longer valid. The trend innovation may be smaller than the cycle innovation, even though output is MA(1) with a large positive autocorrelation. Indeed, it can be shown that if we allow for ΔT_t and Δc_t being AR (or ARMA) then the variance of v_t can be arbitrarily close to zero, whatever the autocorrelation structure of Δy_t (see Quah 1988). This is to say that, in spite of Δy_t being I(1), the trend component may be arbitrarily close to a deterministic function of time. Unless we introduce further identifying assumptions into the model, we cannot rule out decompositions of output with a small-variance trend.

Consider the following example. Setting $\text{var}(u_t) = 0.8$, $\text{var}(v_t) = 0.04$, $\alpha = 0.8$, $\theta = \psi = 0.8$ in model (4), the Wold representation of output is

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$(1 - L)y_t = (1 + 0.8L)\epsilon_t$. Thus, although the variance of u_t is twenty times the variance of v_t, Δy_t is MA(1) with a strong first-order autocorrelation (0.8). Similar examples are easily obtained by imposing, in addition to (5), the relation

$$\frac{\text{var}(v_t)}{\text{var}(u_t)} = \frac{(1 - \alpha)^2}{\alpha}. \tag{6}$$

Equation (6), together with (5), implies $(1 - L)y_t = (1 + \theta L)\epsilon_t$. Hence, provided that θ is positive, there are values of α (those satisfying (6)) such that the autocorrelation of Δy_t at lag one is positive, whatever the ratio $\text{var}(v_t)/\text{var}(u_t)$.

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