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The Aggregate Escape Rate Analysed through the Queueing Model

by

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Abstract

Some empirical studies show a decreasing aggregate rate of escape and many theories of unemployment assume the heterogeneity of the unemployed in order to explain this decline. The purpose of this paper is to investigate the conditions under which a declining rate of escape occurs in a population of unemployed workers. The phenomenon of unemployment is represented by a queueing model: a container with an input given by individuals looking for a job and an output given by individuals who have found a job. The exit flow can be fixed at each simulation step, and, as in a hydraulic system, it can be controlled by a cut-off valve. Samples of completed spells are generated from homogeneous and heterogeneous populations, using a random selection method under different conditions. Assuming that durations are Weibull distributed, the maximum likelihood method is adopted to estimate the parameters of the distribution function relative to each sample and, hence, to easily obtain the hazard function. On the one hand, the results are those expected for a homogeneous population when the Weibull distribution fits the data generated well, but under certain conditions, the hazard or escape rate is increasing with duration. On the other hand, the results almost always show a declining escape rate in all the samples generated through random selection from a heterogeneous population. The author also examines the parameters of the distribution of completed spells observed in the real labour market.

1. Introduction[1]

One of the main issues in the literature on duration is whether the probability of leaving unemployment decreases, remains constant or increases as the duration of unemployment increases. Most search theories assume that the reservation wage is either constant or decreasing over time and, hence, the probability of reemployment is either constant or increasing

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with the duration of unemployment (positive duration dependence). This implies that the wage offer distribution degenerates because employers think that the skills of unemployed workers will depreciate over time, inducing them to accept any wage. This may lead to a reemployment probability which decreases over time (negative duration dependence). The use of the probability of finding a job over time, which is a conditional probability, also called hazard function or instantaneous escape rate, makes the modelling of unemployment spells easier[2].

Some empirical works show a declining aggregate escape rate (Salais, 1980; Lancaster and Nickell, 1980; Malinvaud, 1984) and the heterogeneity of the persons seeking employment is assumed as the cause of this decline. This assumption is not only plausible, but necessary for most theories to explain persistent unemployment (Salant, 1977).

The aim of this paper is to verify whether there is an increase in the aggregate escape rate given the assumption of heterogeneity between the unemployed workers. If this is not the case, i.e. the conditional probability of escaping from unemployment remains constant or increases, then these theories must reconsider some aspects of the process that have been left out.

The aggregate escape rate is analysed by using a simulation model which considers unemployed workers who are queueing for employment. Although a realistic model of unemployment is used, the results are open to criticism because it may argued that it is possible to obtain the desired outcomes by choosing suitable starting conditions for the simulation process. Nevertheless, the model represents a powerful tool to explore unknown situations or to verify some hypotheses, as we have attempted here. The durations of the spells observed in a real labour market are also analysed in order to provide some concrete examples of the aggregate escape rate.

The most important issue involved in this queueing model is how to select the individuals who will find jobs. This is a criticizable step in the model because perhaps it is not possible to identify the conditions under which people leave unemployment in the real world. In a simulation process, the selection criteria frequently used are LIFO (*Last In, First Out*), FIFO (*First In, First Out*), and a random one. The LIFO and FIFO criteria do not seem adequate to represent the exiting process of unemployed workers from the queue. So in our model, the individuals who find jobs are randomly selected using a uniform distribution. Although this is not the true selection mechanism working in the labour market, it is of interest because it shares some resemblance with the real one and represents the starting point for other distribution laws.

The simulation process is executed for homogeneous and heterogeneous queueing populations, under steady state and non-steady state conditions. Each individual's completed

[2] This approach was suggested by Nickell (1979a, b) and Lancaster (1979). Since then, a great number of works have attempted to model economic duration data on the basis of hazard function: Kiefer and Neumann (1979), Flinn and Heckman (1982), Solon (1985), Narendranathan, Nickell, and Stern (1985), Flinn (1986), Ham and Rea (1987), and Kiefer (1988), among others. Note that the method of hazard function is similar to the method of the life table used by Silcock (1954) and further developed by Salant (1977).

spell of unemployment is given by the time that has elapsed between entry into and exit from the queue. Samples of completed spells are obtained from the queueing model. Thus, the shape and location parameters of the relative distribution determining the trend of the aggregate escape rate can be estimated.

The heterogeneity of the unemployed workers is introduced by considering a queueing population made up of several groups, each characterized by a different probability of finding employment. Within each group, however, the probability can be the same[3]. Homogeneity is instead represented by a queue without groups and it is considered in order to make comparisons with the heterogeneity case. The steady state is obtained by fixing the exit rate of individuals leaving the queue and the non-steady state is obtained by varying it.

Only the features of the Weibull distribution are examined here. It is one of the distributions utilized to describe unemployment spell durations (Flinn, 1986; Kiefer, 1988) and is a very useful tool in the analysis of durations because it allows us to estimate its parameters easily and to verify immediately the trend of the escape rate.

The structure of this paper is the following. Section 2 deals with the structure of the completed spell generation model. Section 3 is dedicated to the plan of experiments. The hazard function, the Weibull distribution, and the estimation methods are described in Section 4. The estimates of the parameters of the distribution of the unemployment spells, generated from homogeneous and heterogeneous populations of unemployed workers under steady state and non-steady conditions are provided in Section 5. In addition, Section 5 includes the estimates of the parameters obtained from a sample of completed spells observed in the real labour market. Section 6 contains a brief discussion and conclusion.

2. The Simulation Model Used to Generate the Completed Spells

The labour market can be described by the following expression

$$LFJ + aD + bRs = NJ + cRt$$

where the left-hand side represents the unemployed persons: LFJ, D and Rs stand for individuals looking for their first jobs, individuals who have been dismissed or have resigned and are looking for new jobs, respectively. The right-hand side represents the vacancies: NJ and Rt stand for new jobs and retirements, respectively. The coefficients a, b, and c vary between 0 and 1. They indicate that only one part of the individuals who have been dismissed or have resigned are searching for another job, while some of their previous jobs or some jobs of retirements are included in the vacancies.

[3] Individuals belonging to groups with a high probability of employment will tend to leave the unemployment state quickly. For the selection made at random, only unlucky individuals will remain unemployed. This group is made up of people who are less available or have more difficulties in finding jobs. Thus, according to some theories, the tendency of the groups to leave the unemployment state as quickly as possible makes the aggregate escape rate decreasing as duration increases.

In order to ascertain whether the aggregate escape rate decreases over time the relevant variable is the time an unemployed worker spends looking for a job regardless of which component of the labour force market he/she comes from. Therefore, the previous scheme may be represented by two containers communicating with each other as in a hydraulic system: one for the unemployed and another for the employed workers (see Figure 1). It is possible, without any loss of generality, to consider only the container for the unemployed, with an entry for the individuals who are looking for a job and an exit for the individuals who have found a job. The group of people in search of jobs constitute a queueing population, $\mathcal{P}_u$. At every moment t, l_t individuals "leave" the queue after finding a job, and e_t individuals "enter" because they have either lost their jobs or they are joining the "job-search area" for the first time. The exit component may be controlled by fixing the number of people as in a hydraulic system, in which the flow is regulated by a cut-off valve.

The model is used to study both homogeneous and heterogeneous populations of queueing individuals. A population of unemployed workers is homogeneous when all the subjects have the same probability of finding a job (independently of skill, sex, education, attitude toward risk, etc.). A population of unemployed workers is heterogeneous when the probability of finding employment depends on some distinctive features such as personal characteristics, family composition, local labour demand, and income variables. Therefore, the heterogeneous population, $\mathcal{P}_u$, may be considered as a group made of g groups. The i-th group is composed of s_i individuals looking for a job; its members, as a whole, have a probability p_i of leaving the queue. The probabilities of exiting from the queue are different between the groups, whereas within the groups, the probabilities are equal, without any loss of generality. Let j be any individual belonging to the i-th group. The probability of him/her exiting from the queue (or finding a job) is:

$$\frac{1}{s_{ij}} = \frac{1}{s_i} \qquad j = 1, 2, \dots, s_i \quad i = 1, 2, \dots, g \tag{1}$$

and the following equalities hold:

$$\begin{gathered} \sum_{i=1}^{g} p_i = 1 \\ \sum_{i=1}^{g} p_i \sum_{j=1}^{s_i} \frac{1}{s_{ij}} = \sum_{i=1}^{g} \sum_{j=1}^{s_i} p_i \frac{1}{s_i} = 1. \end{gathered} \tag{2}$$

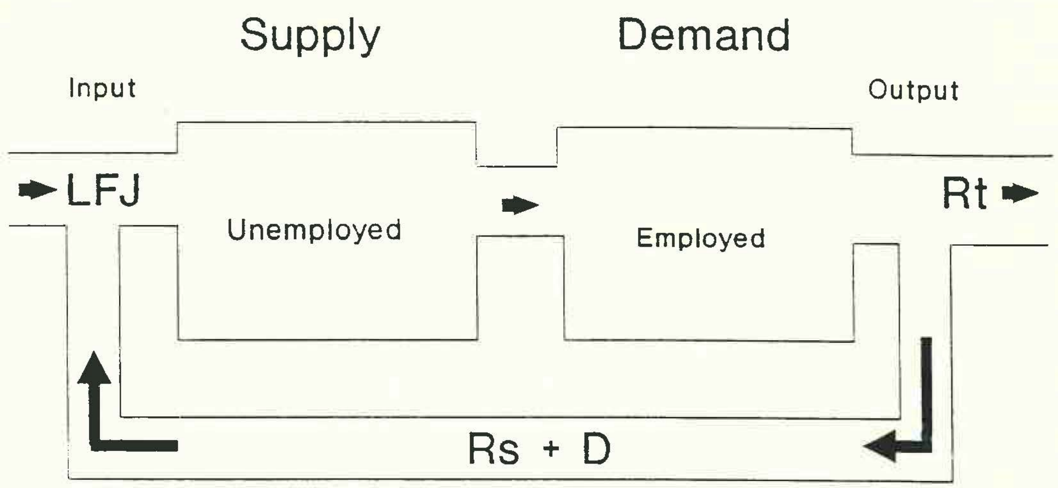
Note that the homogeneous population is a particular case of the heterogeneous population, i.e., when the latter is made up of only one group: $g = 1$. In fact, the assumption in equation (1) implies a homogeneity within the groups (each member has the same probability of finding a job). Each unemployed worker in the queue thus has a probability of finding a job equal to p_i/s_i.

The flow of output can be fixed during the simulation process. At each moment, t, l_t individuals exit from the queue, so it is possible to define the exit rate as:

$$r = \frac{l_t}{\mathcal{P}_u} \tag{3}$$

Figure 1

Scheme representing the labour force market



which only specifies the individuals who leave the unemployment state. Moreover, it can be varied in the model in order to study the distribution of the spell durations. The exit rate differs from the escape rate; the latter is an instantaneous rate and a conditional probability which may depend on time and the estimation procedure.

From the moment the simulation process starts, all the unemployed workers begin to queue with a spell equal to zero. As time goes on, they start to exit from and enter into the queue. At the beginning, the observed lengths of the spells are similar. As the process goes on, they begin to differ. This situation, known as the "transient", has to change in order to generate unbiased spell durations. The transient may be considered to be eliminated when all the subjects in the initial population have exited from the queue or when the queue is entirely renewed. This condition is necessary, but not restrictive. The time, t_s, at which the lengths of the spells being observed can be expressed as a multiple of the inverse of the exit rate:

$$t_s = m\frac{P_u}{l_t} = \frac{m}{r}. \tag{4}$$

In fact, the inverse of the exit rate denotes the number of simulation steps necessary to extract from the queue a number of individuals equal to the population size. With the FIFO (*First In, First Out*) method, these steps imply the complete renewal of the original population, whereas with other extraction criteria, an appropriate multiple of the inverse exit rate is sufficient to eliminate the transient. Therefore, it is necessary to search for the value of m which guarantees this renewal.

When the simulation experiments are carried out with a very large queueing population, the number of individuals who leave the queue may be greater than the size of the sample used to estimate the parameters of the distribution of the unemployment spells. Therefore, the model provides the possibility of specifying through a proportion the number of individuals to select from the exiting ones and to be included in the sample. This proportion it is called the sampling rate, s_{rl}, and it allows us to study the changes in the distribution of the parameters as it varies.

The input-output flow relationship may affect the distribution of the spell durations. In a steady state, the number of people exiting from the queue is equal to the number of entrants over time, that is, $l_t = e_t$. In a non-steady state, on the one hand, it may happen that l_t increases and e_t remains constant or decreases. On the other hand, it may happen that l_t remains constant or decreases and e_t increases. It is possible to obtain these situations by simply varying the exit rate during the simulation steps. In fact, the increase in the stock of unemployed, together with the constancy or the decrease in l_t, implies a decrease in the exit rate; *viceversa*, the decrease in the stock of unemployed, together with the constancy or the increase in l_t, implies an increase in the exit rate. Therefore, we have always assumed that $l_t = e_t$.

The process works as follows. We must fix some characteristics of the model at the beginning, such as:

1. the size of the queueing population of unemployed, or stock, P_u;
2. the number of groups, g, constituting the stock P_u;
3. the probabilities of the groups, p_i, of being extracted, as a whole;

4. the number of members, s_i, in the groups;
5. the exit rate, r, defining the number of individuals exiting from the queue at each moment, t;
6. the starting time to begin recording the lengths of the spells, t_s;
7. the sampling rate for the individuals leaving the queue, s_{rl}, which defines the number of individuals to be surveyed at each moment t;
8. the sampling size, n, defining the total number of individuals to be surveyed at each simulation step or experiment;
9. the number of repetitions of the experiments, N_e, defining the end of the process.

Each experiment begins at a time equal to 0. The members of the exiting group are **selected at random** from the queueing population, on the basis of their probabilities of changing states, that is, of finding employment. At any moment t, a number of individuals, equal to the exit rate of the queueing population $l_t = rP_u$, leave the queue and are all replaced by other workers. Between $t = 1$ and $t = t_s - 1$, the spells of those leaving are not recorded. The recording process begins when $t = t_s$, terminates when the fixed sampling size has been reached, and refers only to those individuals who are selected from the exiting group on the basis of the sampling rate. The parameters of the Weibull distribution on the basis of the lengths of the spells surveyed are then estimated. In addition, two tests of goodness of fit of the Weibull distribution are performed: the Kolmogorov-Smirnov test, λ_{KS}, and the *chi-squared* test, χ^2, with nine degrees of freedom. These estimates are recorded and a new experiment is begun at time equal to 0. When all the N_e experiments have been executed, the means and the standard deviations of the estimates, the goodness of fit tests, and the likelihood function are calculated.

By changing some basic characteristics of the simulation process, the trend of the aggregate escape rate may be explored under different exiting conditions.

3. The Plan of the Experiments

The simulation process is strongly influenced by the values of P_u, g, p_i, s_i, r, t_s, s_{rl}, n, N_e. Among these characteristics, we can fix P_u, t_s, s_{rl}, and N_e definitively, after a sensitivity analysis of the changes in the distribution of the lengths of the spells. For example, if the stock of unemployed does not affect the shape of the distribution, then its size may be set equal to a number which does not involve high computer costs (CPU) for data processing and remains constant in all the experiments. The values of the determinants of the process used to study the modifications in the aggregate escape rate were set as follows.

1. The stock of unemployed, P_u, was fixed at 5,000 individuals because experiments with up to 30,000 individuals have shown the same results.
2. The number of groups, g, in the stock P_u was:
   — 1 for a homogeneous population,
   — 2 and 5 for a heterogeneous population.
3. The probability, p_i, of selecting each group was:

— obviously $p_i = p_1 = 1$ for a homogeneous population (in such a case, one could also have many groups ($g > 1$) with $p_i = 1/g$, for any i);

— equal to the following combinations of probability for a heterogeneous population:

- $g = 2$ and $p_1 = 0.7, p_2 = 0.3$;
- $g = 5$ and $p_1 = 0.40, p_2 = 0.30, p_3 = 0.15, p_4 = 0.10, p_5 = 0.05$;

4. The number of subjects composing the groups was equal to $s_i = s = P_u/g$, in any case. Then

   — for an individual, the probability of leaving the selected group was equal to $1/s$;

   — for any individuals in i-th group, the probability of leaving the queue was p_i/s;

   — the equality $P_u = g \cdot s$ held.

5. The exit rate, r, was equal to 0.5%, 1%, 2%, 5%, 10%, 20%, 30% at each stage.
6. The starting time for recording spell lengths depends on the value of m, as in equation (4). The value of m was fixed at 6 because values greater than 6 have not shown any changes in the shape of the distribution of the spell lengths, then $t_s = 6/r$.
7. The sampling rate of the individuals who exit from the queue, s_{rl}, was fixed at 5%, but it does not affect the aggregate escape rate.
8. The sampling size, n, was equal to 200, 400, 800, and 1200.
9. The number of repetitions of the experiments, N_e, was fixed at 300, but sufficiently accurate estimates were obtained only after 100 experiments.

4. Statistical methods

The conditional probability of leaving the unemployment state, that is, of leaving the queue, for an individual, may be expressed by the equation

$$\frac{F'(t)}{F(t)} = \frac{f(t)}{F(t)} = -\lambda(t) \tag{5}$$

where $f(t)$ is the probability density, $F(t)$ is the distribution function denoting the probability of becoming employed between 0 and t, $\lambda(t)$ is the hazard function denoting the instantaneous rate of exiting from the unemployment state, given that the individual has not left the state as of time t (Cox & Oakes, 1984; Gnedenko, 1969). Hereafter, $\lambda(t)$, the employment rate, will be referred to as the "escape rate"[4]. The general solution of the previous equation is

$$F(t) = e^{-\int_0^t \lambda(u)du} = e^{-\Lambda(t)}, \tag{6}$$

where $\Lambda(t)$ denotes the integrated hazard which does not represent a probability. It is useful in practice, but there is no convenient interpretation of it.

[4] In the theory of reliability, $\lambda(t)$ is the failure rate and in survival data analysis, $\lambda(t)$ is the hazard function or mortality intensity.

4.1. The Weibull distribution

The Weibull distribution depends on a ***shape parameter*** ($\alpha > 0$) and a ***scale parameter*** ($\lambda > 0$), which is sometimes referred to as the ***characteristic life***. It is obtained by setting $\lambda(t) = \alpha\lambda(\lambda t)^{\alpha-1}$ in equation (6). The process is described by the following expressions:

$$\begin{aligned} F(t) &= 1 - e^{-(\lambda t)^{\alpha}} \\ f(t) &= \alpha\lambda(\lambda t)^{\alpha-1}e^{-(\lambda t)^{\alpha}} \\ \lambda(t) &= \alpha\lambda(\lambda t)^{\alpha-1} \\ \Lambda(t) &= (\lambda t)^{\alpha-1}. \end{aligned} \tag{7}$$

The mean and the variance of the Weibull distribution are $\mu = (\lambda\alpha)^{-1}\Gamma(1/\alpha)$ and $\sigma^2 = (\lambda\alpha)^{-2}[2\alpha\Gamma(2/\alpha) - \Gamma^2(1/\alpha)]$ respectively.

If $\alpha > 1$, the escape rate is monotone increasing with duration, starting from $t = 0$.

If $\alpha < 1$, the escape rate is monotone decreasing with duration and it is undefined at $t = 0$.

If $\alpha = 1$, the Weibull distribution becomes exponential, which implies a constant escape rate or hazard. Thus, the past is uninfluential on the future and the probability of finding a job within the time interval depends only on the length of the time interval. The exponential distribution is widely used because of its simplicity, but its dependence on a single parameter limits its flexibility as it does not allow for a "separate adjustment" of the mean and the variance. On the other hand, the Weibull distribution, which depends on a supplementary parameter α, represents a generalization of the exponential distribution.

Therefore, the value of the shape parameter α allows us to verify immediately whether the aggregate escape rate declines as duration increases. However, there are no explicit theoretical arguments indicating that a Weibull distribution should be used (Johnson & Kotz, 1970) in almost all practical cases. It is just a power transformation of a random variable exponentially distributed, representing a convenient way of introducing some flexibility into the model. Only in the case where $\alpha = 2$ is it possible to create a mechanism generating the Weibull as a limit distribution; the inherent conditions of the process can be found in Gnedenko, Béliaev, & Soloviev (1972, pag. 107).

4.2. The Method of Estimation

The estimates of the parameters are obtained by maximizing the likelihood function. Given a sample of n independent spell durations, $t_1, ..., t_n$, the logarithm of the likelihood of the Weibull is:

$$L = n\alpha log\lambda + nlog\alpha + \sum_{i=1}^{n}[(\alpha - 1)logt_i - (\lambda t_i)^{\alpha}]. \tag{8}$$

The estimates, as is well-known, are obtained by solving a system of partial differential first order equations with respect to λ and α.

$$\frac{\partial L}{\partial \lambda} = n - \sum_{i=1}^{n} (\lambda t_i)^{\alpha} = 0$$
$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} [1 - (\lambda t_i)^{\alpha}] log(\lambda t_i) = 0. \tag{9}$$

There are various methods of solution (Goldfield & Quandt, 1972; Aitkin & Clayton, 1980). In this case, after obtaining λ from the first equation and replacing it in the second one, we obtain an expression which depends only on α:

$$\frac{n}{\alpha} + \sum_{i=1}^{n} [1 - (\frac{n}{\sum_{i=1}^{n} t_i^{\alpha}})\ t_i^{\alpha}] log(t_i) = 0. \tag{10}$$

This equation is solved by using the sign method, checking the Hessian to verify whether the point is a maximum. This procedure is a modified version of the bisection method (Ralston, 1971, p. 461).

Variance of the estimates is given by the expected value of the diagonal elements of the inverse of the Fisher information matrix. It is defined by the second partial derivatives with signs changed. The expected value may be determined consistently by calculating the second partial derivatives at the point defined by the maximum likelihood estimates (Rao, 1973), as in the following:

$$\frac{\partial^2 L}{\partial \lambda^2} = -\frac{1}{\lambda}\frac{\partial L}{\partial \lambda} - \frac{\alpha^2}{\lambda^2} \sum_{i=1}^{n} (\lambda t_i)^{\alpha}$$
$$\frac{\partial^2 L}{\partial \lambda \partial \alpha} = \frac{1}{\alpha}\frac{\partial L}{\partial \lambda} - \frac{\alpha}{\lambda} \sum_{i=1}^{n} (\lambda t_i)^{\alpha} log(\lambda t_i) \tag{11}$$
$$\frac{\partial^2 L}{\partial \alpha^2} = -\frac{n}{\alpha^2} - \sum_{i=1}^{n} (\lambda t_i)^{\alpha} log^2(\lambda t_i)$$

5. The Results of the Experiments

We have included some tables in Appendix A, to provide a picture of the results of the simulation according to different escape rates (r), various sampling sizes (n), and several sets of probabilities of selecting groups (p_i), in the cases of homogeneous and heterogeneous populations. The parameters of the Weibull (the location and the shape), the Kolmogorov-Smirnov test, the *chi-squared* test[5], and the logarithm of the likelihood function are reported for each set of experiments in these tables.

[5] The *chi-squared* test depends on how the data have been grouped and, at times, the outcome may vary accordingly. Ten classes of equal probability have been used implying nine degrees of freedom.

As non-steady state conditions did not lead to any particular change in steady-state results, a subsection is devoted to a brief description of the outcomes in this case.

5.1. The homogeneous population

According to our scheme, for the homogeneous stock of unemployed, the probability of escaping is $1/P_u$. This implies that the durations of the spells must be exponentially distributed and the Weibull distribution must have a shape parameter equal to one. The results, shown in Table 1, may be summarized as follows.

1. An increase in the exit rate determines an increase in the location parameter or a decrease in the mean of the spells lengths, a slight increase in the variance of the mean, and a slight increase in the shape parameter. When the tests of goodness of fit do not lead to the rejection of the Weibull, the shape parameter does not differ from one, that is, the spell durations are exponentially distributed.
2. An increase in the sampling size determines a decrease in the value of the exit rate below which the tests of goodness of fit do not lead to the rejection of the exponential distribution. This dependence, although not completely unexpected, seems relevant because the conclusions reached from a survey may change as the sampling size changes. In fact, the badness of fit and, therefore, the inadequacy of the distribution, were revealed only in large samples.
3. The upper bound of the interval of exit rate, within which the Weibull fits the data well, is never greater than 10% and it diminishes with the sampling size. On the contrary, the lower bound does not show any restriction. Experiments have been made with exit rates up to 0.01%, which is already an unrealistic value.

Thus, it may be concluded that the spell lengths observed in homogeneous populations are exponentially distributed only when the exit rates are lower than the values which depend on the sampling size. An increase in the replacement rate of the stock of the unemployed determines a modification in the distribution.

5.2. The Heterogeneous Population

According to our scheme, for the heterogeneous stock of unemployed the probability of escaping is $p_i/(P_u/g)$ and the values of p_i are different for every group in the stock. The results —some of which are reported in Tables 2 and 3— seem to confirm the decreasing escape rate for the heterogeneous populations, as foreseen by numerous theories on job-search. The relations between the exit rate r, the sampling size n, and the parameters of the distribution are almost the same as the ones reported for the homogeneous population.

1. An increase in the exit rate determines, once again, an increase in the location parameter or a decrease in the mean of the spell lengths, a slight increase in the variance of the mean, and a slight increase in the shape parameter. When the Weibull fits the data well, the shape parameter is almost always lower than one and it implies a decreasing hazard.
2. An increase in the sampling size determines, once again, a decrease in the value of the exit rate, below which the tests of goodness of fit do not lead to a rejection of the Weibull with a shape parameter lower than one. This dependence is relevant to empirical studies because the size of sample may affect the conclusions concerning the inadequacy of the

distribution adopted.

3. The upper bound of the interval of the exit rate, within which the Weibull is not rejected by the goodness of fit tests, is never greater than 10% and it diminishes with the sampling size. On the other hand, the lower bound, once again, shows no restriction. Experiments have been made with exit rates up to 0.01%.
4. When the difference between the escape probabilities of the various groups increases (at least in the $g = 2$), a decrease in the parameter α is observed.

5.3. The non-steady state conditions

The results already described refer to a constant exit rate relative to a steady state. The model generating the unemployment spells allows us to examine how their distribution changes under non-steady state conditions. These conditions are introduced in the experiments by varying the exit rate at random, in opposite directions and in a more complicated way. For example, a decreasing exit rate arises when the number of individuals who exit from the queue (l_t) decreases as the queueing population (P_u) remains constant; see equation (3). This case may represent different situations in the labour market: a reduction of new jobs and/or dismissals and/or resignations and/or retirements. Likewise, an increasing exit rate arises when l_t increases and this may happen when there are additional new jobs and/or dismissals and/or retirements, and so on.

It is appropriate at least to distinguish the sporadic changes in the exit rate from the frequent changes over time. The former require a time interval between the changes large enough to allow the queue to return to a steady state. In such a case, it is possible to analyse the changes one at a time because we have two steady states separated by a transient.

When the exit rate varies continually it generates a sort of continuous transient. If the values of the exit rates are lower than the upper bound found in the previous subsections, the distribution of the spell lengths remains a Weibull and its scale parameter increases or decreases as the exit rate increases or decreases. In fact, an increase in the scale parameter implies a decrease in the mean of the spell lengths, as well as an increase in the exit rate. However, the variations of the exit rate may occur in a given interval with an upper bound lower or almost equal to the one previously found. The results reported in Table 4 show that the parameters of the distribution of the unemployment spells, generated under "continuous" transient conditions, are similar to the parameters of the distribution of the unemployment spells generated under steady-state conditions and a fixed exit rate equal to the mean of exit rates of the transient.

5.4. The spell durations observed in the real labour market

As an application, let us consider a sample of completed spells for close to 12,500 individuals from 4,677 households in the Italian region of Emilia-Romagna. The sample was drawn according to the same technique used by the Istituto Centrale di Statistica (ISTAT), the most important national data collection center in Italy, with individual interviews conducted in January 1984. We surveyed personal characteristics, family composition, and local labour demand, starting from January 1983 and reconstructed the changes in states over 52 weeks in 1983. Hence, it is possible to obtain the completed spells of employed and unemployed workers, for which the beginning and the end of the spells are given by retrospective data.

We have distinguished the first-job seekers from those who had been dismissed and those who had resigned and were looking for new jobs. Table 5 reports the estimated parameters of the distribution of the observed spells, together with the results obtained by Flinn (1986) for the same period in Italy. The shape and the scale parameters of the first-job seekers are different from those relative to the group made up of dismissed individuals and those who resigned. The former have a shape parameter slightly greater than one, which implies an exponential distribution in any case, whereas the latter have a shape parameter statistically lower than one. The individuals looking for their first jobs have a scale parameter lower than the dismissed individuals and those who resigned. Hence, the former have a higher mean of spell duration than the latter. These results show the difficulties in finding new jobs. In fact, the means of the spells are higher than those reported in the literature for other countries (Salant, 1977; Solon, 1985; etc.).

The data reveal a marked difference between males and females. In fact, 59 percent of females were looking for their first jobs and 65 percent of females were looking for new jobs. Moreover, the means of the spells for females are higher than those for males. These figures indicate that the females experience many disadvantages in the labour market.

6. Discussion and Conclusion

In a recent work (Lalla, 1989), the distribution of unemployment spell durations has been analysed by means of a model focusing only on the generation mechanism, thus with the exit rate depending on several characteristics of the simulation process. This is not only unrealistic, but it complicates the results. On the contrary, in the model presented in this paper, the exit rate is independent of other determinants of the process and can be controlled during the generation of spell lengths. The practical advantage is that by varying the exit rate, we can study both steady state and non-steady state conditions and what happens at a given exit rate. This model makes it easier to examine large stocks of the unemployed too. However, the results reported here are slightly different from those obtained with the other model and they seem to confirm hypotheses and distribution patterns of some interest to theoretical and empirical research.

The main aspects of the results obtained may be summarized as follows.

1. Homogeneous unemployed workers experience spell lengths that are exponentially distributed.
2. Heterogeneous unemployed workers experience spell lengths distributed as a Weibull, with a shape parameter lower than one or a decreasing hazard.
3. The Weibull fits the data well only when the exit rates are lower than an upper bound, which is never greater than 10%, but this upper bound decreases when the sampling size increases.
4. Whenever the exit rate varies at levels lower or close to the upper bound, the distribution of the spell lengths, generated under non-steady state conditions, does not reveal changes

in the goodness of fit.

The first and the second conclusions correspond to the patterns expected for the distribution of unemployment spells in the cases of homogeneity and heterogeneity, respectively. Therefore, they provide further evidence of the validity of the assumption of heterogeneity of unemployed workers to explain the declining aggregate escape rate. On the other hand, the third and the fourth conclusions are the more interesting findings of this study and provide some information about the limits of the validity of the expected results. Thus, when the exit rate is greater than 10%, we may have some doubts about the exponential and Weibull laws of duration. This suggests that when there is a large group of individuals who frequently enter and exit from the queue amongst the queueing population, the Weibull distribution may be inadequate. The dependence of the goodness of fit on the sampling size is still confirmed, but it lends some uncertainty to empirical results, which must change as the sample size changes. The steadiness of the distribution law of durations can be easily assumed when the exit rate decreases and/or increases, but it remains prevalently below the upper bound mentioned previously.

Although the results reported above present interesting characteristics and a robust trend, applicable also to large queueing populations, the job market may be studied further by slightly modifying the model.

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APPENDIX A

Table 1 – *Estimates of Weibull Parameters, Tests of Goodness of Fit (λ_{KS}, χ^2), Likelihood Function (L), and Standard Deviation for Completed Spells Produced by Random Selection from a* **Homogeneous Population** *at different values of the exit rate and sampling size.*

n and r values	λ S D	α S D	λ_{KS} S D	$\chi^2_{(9)}$ S D	L S D
$n = 200$					
$r = 0.5\%$	0.0051	1.005	0.586	6.22	−1257.30
	0.0004	0.056	0.143	3.54	13.85
$r = 1\%$	0.0100	1.028	0.599	6.15	−1119.78
	0.0007	0.055	0.146	3.33	13.78
$r = 2\%$	0.0198	1.040	0.644	6.77	−981.32
	0.0015	0.056	0.149	3.75	14.41
$r = 5\%$	0.0491	1.075	0.833	9.09	−796.46
	0.0034	0.057	0.168	4.86	12.90
$r = 10\%$	0.0956	1.136	1.084	15.95	−657.57
	0.0070	0.055	0.190	8.19	15.06
$r = 20\%$	0.1873	1.241	1.669	69.40	−512.79
	0.0111	0.056	0.202	19.86	12.94
$r = 30\%$	0.2739	1.350	2.232	176.85	−426.06
	0.0156	0.065	0.224	27.79	14.03
$n = 400$					
$r = 0.5\%$	0.0050	1.017	0.592	6.08	−2516.29
	0.0003	0.040	0.136	3.18	18.57
$r = 1\%$	0.0100	1.023	0.640	6.62	−2239.04
	0.0005	0.037	0.154	3.41	20.65
$r = 2\%$	0.0199	1.035	0.713	7.26	−1961.15
	0.0011	0.038	0.146	3.72	20.90
$r = 5\%$	0.0487	1.077	0.956	11.73	−1595.64
	0.0023	0.038	0.166	6.17	18.74
$r = 10\%$	0.0960	1.133	1.385	27.16	−1313.65
	0.0045	0.041	0.187	10.65	19.46
$r = 20\%$	0.1864	1.237	2.230	133.23	−1027.86
	0.0078	0.042	0.204	33.82	18.00
$r = 30\%$	0.2737	1.345	3.114	352.80	−852.69
	0.0104	0.048	0.223	39.96	18.69

(continued)

(continued)

n and r values	λ S D	α S D	λ_{KS} S D	$\chi^2_{(9)}$ S D	L S D
$n = 800$					
$r = 0.5\%$	0.0050	1.013	0.629	6.75	−5035.05
	0.0002	0.024	0.143	3.93	25.64
$r = 1\%$	0.0099	1.022	0.681	7.46	−4483.28
	0.0003	0.028	0.148	3.97	25.73
$r = 2\%$	0.0198	1.036	0.814	9.15	−3926.32
	0.0007	0.026	0.164	4.77	27.04
$r = 5\%$	0.0486	1.074	1.187	17.70	−3194.25
	0.0016	0.026	0.185	9.03	25.89
$r = 10\%$	0.0953	1.127	1.805	46.62	−2634.27
	0.0029	0.028	0.193	16.48	24.31
$r = 20\%$	0.1864	1.231	3.071	272.85	−2056.97
	0.0059	0.032	0.215	62.38	29.97
$r = 30\%$	0.2735	1.337	4.322	707.28	−1708.69
	0.0075	0.032	0.237	59.18	25.86
$n = 1200$					
$r = 0.5\%$	0.0050	1.012	0.651	6.74	−7555.71
	0.0002	0.022	0.155	3.62	35.74
$r = 1\%$	0.0100	1.022	0.699	7.95	−6719.62
	0.0003	0.022	0.141	4.09	36.74
$r = 2\%$	0.0198	1.032	1.000	9.49	−5890.71
	0.0006	0.022	0.161	5.29	33.96
$r = 5\%$	0.0488	1.070	1.384	24.33	−4787.82
	0.0013	0.022	0.179	12.44	32.91
$r = 10\%$	0.0958	1.127	2.152	67.10	−3944.65
	0.0025	0.019	0.189	20.11	31.43
$r = 20\%$	0.1862	1.234	3.708	410.61	−3085.22
	0.0049	0.024	0.239	94.46	33.90
$r = 30\%$	0.2727	1.341	5.214	1042.83	−2564.22
	0.0061	0.026	0.251	65.09	32.56

Table 2 – *Estimates of Weibull Parameters, Tests of Goodness of Fit (λ_{KS}, χ^2), Likelihood Function (L), and Standard Deviation for Completed Spells Produced by Random Selection from a* **Heterogeneous Population** *at different values of the exit rate and sampling size with the following probability of selecting two subgroups:* $p_1 = 0.7, p_2 = 0.3$.

n and r values	λ S D	α S D	λ_{KS} S D	$\chi^2_{(9)}$ S D	L S D
$n = 200$					
$r = 0.5\%$	0.0052	0.941	0.671	7.10	−1256.50
	0.0004	0.049	0.159	3.82	16.75
$r = 1\%$	0.0104	0.951	0.674	6.65	−1118.52
	0.0009	0.050	0.162	3.37	16.60
$r = 2\%$	0.0206	0.962	0.740	6.97	−980.62
	0.0017	0.042	0.201	4.46	16.71
$r = 5\%$	0.0512	0.990	0.898	11.46	−795.58
	0.0041	0.047	0.154	7.13	15.29
$r = 10\%$	0.1005	1.044	1.183	18.93	−655.89
	0.0072	0.045	0.181	16.29	15.23
$r = 20\%$	0.1943	1.139	1.805	91.77	−514.68
	0.0133	0.056	0.216	20.25	15.68
$r = 30\%$	0.2864	1.237	2.423	220.17	−426.93
	0.0165	0.065	0.245	33.98	14.59
$n = 400$					
$r = 0.5\%$	0.0052	0.939	0.713	7.09	−2511.96
	0.0003	0.035	0.177	4.14	20.06
$r = 1\%$	0.0104	0.945	0.746	7.63	−2235.50
	0.0006	0.036	0.181	4.00	21.57
$r = 2\%$	0.0208	0.954	0.849	9.00	−1957.15
	0.0011	0.033	0.185	4.74	22.71
$r = 5\%$	0.0511	0.990	1.100	17.52	−1592.12
	0.0028	0.033	0.190	10.62	23.06
$r = 10\%$	0.1000	1.041	1.526	26.96	−1313.83
	0.0051	0.035	0.192	23.75	22.29
$r = 20\%$	0.1936	1.133	2.416	182.33	−1031.21
	0.0083	0.040	0.234	25.96	18.95
$r = 30\%$	0.2839	1.231	3.316	432.91	−858.04
	0.0115	0.049	0.241	51.81	20.43

(continued)

(continued)

n and r values	λ SD	α SD	λ_{KS} SD	$\chi^2_{(9)}$ SD	L SD
$n = 800$					
$r = 0.5\%$	0.0052	0.930	0.852	9.41	−5027.04
	0.0002	0.025	0.210	5.41	32.25
$r = 1\%$	0.0104	0.941	0.888	9.62	−4476.55
	0.0004	0.026	0.185	4.65	33.18
$r = 2\%$	0.0207	0.950	1.025	6.15	−3918.87
	0.0008	0.026	0.173	6.15	29.74
$r = 5\%$	0.0509	0.983	1.394	32.34	−3188.20
	0.0020	0.024	0.211	19.34	31.69
$r = 10\%$	0.0999	1.037	2.025	44.44	−2629.34
	0.0035	0.025	0.207	32.22	30.35
$r = 20\%$	0.1936	1.129	3.298	366.25	−2063.48
	0.0061	0.027	0.211	27.79	29.85
$r = 30\%$	0.2850	1.230	4.584	865.63	−1712.61
	0.0080	0.033	0.238	72.98	29.18
$n = 1200$					
$r = 0.5\%$	0.0052	0.931	0.933	11.16	−7542.44
	0.0002	0.018	0.191	4.93	41.77
$r = 1\%$	0.0103	0.937	0.997	11.18	−6718.70
	0.0003	0.019	0.184	5.17	36.94
$r = 2\%$	0.0207	0.948	1.142	14.66	−5881.90
	0.0007	0.020	0.221	7.30	39.46
$r = 5\%$	0.0510	0.981	1.634	52.40	−4782.97
	0.0016	0.019	0.209	29.56	37.76
$r = 10\%$	0.0997	1.033	2.417	58.79	−3949.26
	0.0032	0.019	0.204	32.60	41.37
$r = 20\%$	0.1935	1.126	3.973	549.74	−3097.49
	0.0056	0.023	0.234	37.41	40.99
$r = 30\%$	0.2849	1.224	5.656	1304.48	−2572.76
	0.0072	0.029	0.252	107.27	38.79

Table 3 – *Estimates of Weibull Parameters, Tests of Goodness of Fit (λ_{KS}, χ^2), Likelihood Function (L), and Standard Deviation for Completed Spells Produced by Random Selection from a* **Heterogeneous Population** *at different values of the exit rate and sampling size with the following probability of selecting five groups:* $p_1 = 0.40, p_2 = 0.30, p_3 = 0.15, p_4 = 0.10, p_5 = 0.05$.

n and r values	λ S D	α S D	λ_{KS} S D	$\chi^2_{(9)}$ S D	L S D
$n = 200$					
$r = 0.5\%$	0.0058	0.866	0.813	8.72	−1243.29
	0.0005	0.043	0.194	4.37	17.05
$r = 1\%$	0.0115	0.873	0.854	8.80	−1106.08
	0.0011	0.043	0.229	5.11	18.61
$r = 2\%$	0.0228	0.872	0.897	10.33	−968.19
	0.0018	0.043	0.193	5.23	16.41
$r = 5\%$	0.0564	0.913	1.090	13.80	−784.05
	0.0050	0.045	0.187	6.52	18.03
$r = 10\%$	0.1100	0.954	1.398	50.39	−646.69
	0.0092	0.045	0.209	16.84	18.11
$r = 20\%$	0.2121	1.046	2.042	130.90	−506.06
	0.0157	0.051	0.221	26.69	17.57
$r = 30\%$	0.3088	1.139	2.712	293.25	−421.03
	0.0192	0.063	0.234	39.72	16.62
$n = 400$					
$r = 0.5\%$	0.0058	0.854	0.987	11.93	−2488.84
	0.0004	0.029	0.209	5.59	24.63
$r = 1\%$	0.0113	0.862	1.019	11.62	−2217.84
	0.0007	0.030	0.206	5.51	25.29
$r = 2\%$	0.0228	0.870	1.148	15.49	−1937.23
	0.0014	0.029	0.227	7.13	25.36
$r = 5\%$	0.0560	0.906	1.352	20.95	−1571.12
	0.0033	0.033	0.208	9.61	24.26
$r = 10\%$	0.1083	0.951	1.807	108.88	−1299.23
	0.0064	0.033	0.221	26.32	25.59
$r = 20\%$	0.2098	1.033	2.799	253.60	−1018.44
	0.0107	0.039	0.219	30.08	24.88
$r = 30\%$	0.3102	1.126	3.764	601.28	−842.31
	0.0136	0.048	0.245	54.36	23.75

(continued)

(continued)

n and r values	λ SD	α SD	λ_{KS} SD	$\chi^2_{(9)}$ SD	L SD
$n = 800$					
$r = 0.5\%$	0.0058	0.850	1.293	19.64	−4977.75
	0.0003	0.020	0.251	8.29	34.72
$r = 1\%$	0.0114	0.853	1.351	20.67	−4429.51
	0.0005	0.022	0.248	8.11	34.46
$r = 2\%$	0.0228	0.864	1.447	23.92	−3874.97
	0.0011	0.021	0.227	9.48	36.87
$r = 5\%$	0.0556	0.897	1.835	38.11	−3151.04
	0.0024	0.023	0.233	13.50	37.13
$r = 10\%$	0.1087	0.940	2.490	212.90	−2599.57
	0.0047	0.023	0.216	28.45	38.11
$r = 20\%$	0.2109	1.032	3.807	501.23	−2032.67
	0.0079	0.028	0.227	47.46	35.78
$r = 30\%$	0.3096	1.113	5.265	1205.82	−1691.03
	0.0113	0.032	0.243	90.96	37.62
$n = 1200$					
$r = 0.5\%$	0.0057	0.843	1.492	26.93	−7478.37
	0.0002	0.016	0.243	9.13	42.27
$r = 1\%$	0.0115	0.847	1.583	28.55	−6645.45
	0.0004	0.018	0.241	9.72	40.71
$r = 2\%$	0.0227	0.860	1.720	33.33	−5819.05
	0.0008	0.018	0.234	12.57	40.52
$r = 5\%$	0.0555	0.890	2.188	52.91	−4732.26
	0.0021	0.018	0.220	14.50	46.95
$r = 10\%$	0.1086	0.935	2.977	313.61	−3903.32
	0.0034	0.020	0.224	37.86	42.02
$r = 20\%$	0.2106	1.021	4.618	753.68	−3056.93
	0.0062	0.020	0.229	57.17	40.65
$r = 30\%$	0.3078	1.108	6.387	1799.28	−2545.93
	0.0092	0.028	0.269	107.77	47.24

Table 4 – *Estimates of Weibull Parameters, Tests of Goodness of Fit (λ_{KS}, χ^2), Likelihood Function (L), and Standard Deviation for Completed Spells Produced by Random Selection from* **a Homogeneous and a Heterogeneous Populations** *varying the exit rate over time for sampling size equal to 400.*

n and r values	λ SD	α SD	λ_{KS} SD	$\chi^2_{(9)}$ SD	L SD
r Uniformly distributed in the interval specified					
Homogeneous population					
$r \in [1\%, 3\%]$	0.0198	1.036	0.704	7.30	−1963.63
	0.0011	0.039	0.151	4.18	21.83
$r \in [3\%, 7\%]$	0.0490	1.073	0.961	10.68	−1594.38
	0.0028	0.044	0.163	5.25	22.85
$r \in [6\%, 14\%]$	0.0958	1.130	1.391	27.11	−1315.21
	0.0058	0.045	0.198	9.60	24.76
$r \in [15\%, 25\%]$	0.1862	1.232	2.218	134.35	−1029.19
	0.0103	0.047	0.225	36.30	24.16
Heterogeneous population (p_1 =0.4, p_2 =0.3, p_3 =0.15, p_4 =0.1, p_5 =0.05)					
$r \in [1\%, 3\%]$	0.0225	0.841	1.194	17.86	−1948.50
	0.0014	0.041	0.235	8.34	26.00
$r \in [3\%, 7\%]$	0.0551	0.874	1.424	25.08	−1585.07
	0.0040	0.038	0.209	10.26	31.10
$r \in [6\%, 14\%]$	0.1089	0.919	1.814	98.68	−1304.01
	0.0081	0.038	0.246	22.38	31.48
$r \in [15\%, 25\%]$	0.2101	1.001	2.724	259.41	−1023.48
	0.0145	0.048	0.281	47.70	31.96
r Lognormally distributed $\mathbf{L}(m, \sigma^2)$					
Homogeneous population					
L(2%, 1/4%)	0.0197	1.032	0.708	7.23	−1965.51
	0.0009	0.037	0.158	4.08	18.76
L(5%, 4%)	0.0487	1.078	0.958	10.89	−1594.94
	0.0023	0.038	0.182	4.96	18.14
L(10%, 16%)	0.0957	1.133	1.380	26.23	−1314.79
	0.0050	0.037	0.181	11.18	21.94
L(20%, 25%)	0.1862	1.229	2.229	136.65	−1029.54
	0.0079	0.038	0.202	33.91	18.07

(continued)

(continued)

n and r values	λ SD	α SD	λ_{KS} SD	$\chi^2_{(9)}$ SD	L SD
Heterogeneous population (p_1 =0.4, p_2 =0.3, p_3 =0.15, p_4 =0.1, p_5 =0.05)					
L(2%, 1/4%)	0.0224	0.838	1.216	18.49	−1949.71
	0.0015	0.032	0.233	8.42	28.17
L(5%, 4%)	0.0551	0.873	1.399	24.75	−1584.41
	0.0030	0.039	0.211	11.48	23.99
L(10%, 16%)	0.1072	0.914	1.800	103.77	−1311.19
	0.0065	0.041	0.242	20.35	27.61
L(20%, 25%)	0.2092	1.003	2.726	254.87	−1026.01
	0.0119	0.046	0.243	38.30	27.53
r increases linearly in the interval specified					
Homogeneous population					
$r \in [1\%, 3\%]$	0.0269	1.015	0.779	8.14	−1844.40
	0.0015	0.039	0.164	4.35	23.00
$r \in [3\%, 7\%]$	0.0625	1.060	1.126	18.58	−1498.62
	0.0029	0.040	0.177	9.54	20.84
$r \in [6\%, 14\%]$	0.1230	1.131	1.636	34.49	−1214.38
	0.0060	0.040	0.193	33.27	20.92
$r \in [15\%, 25\%]$	0.2168	1.245	2.554	202.80	−965.20
	0.0087	0.048	0.231	27.52	19.75
Heterogeneous population (p_1 =0.4, p_2 =0.3, p_3 =0.15, p_4 =0.1, p_5 =0.05)					
$r \in [1\%, 3\%]$	0.0305	0.815	1.342	24.26	−1831.47
	0.0019	0.033	0.242	10.90	27.58
$r \in [3\%, 7\%]$	0.0710	0.858	1.627	52.67	−1486.84
	0.0046	0.034	0.232	35.46	28.12
$r \in [6\%, 14\%]$	0.1385	0.915	2.175	99.09	−1208.88
	0.0084	0.042	0.221	13.52	28.93
$r \in [15\%, 25\%]$	0.2467	1.013	3.116	397.62	−957.82
	0.0139	0.049	0.235	44.43	29.44

Table 5 – *Estimates of Weibull Parameters, Tests of Goodness of Fit (λ_{KS}, χ^2), Likelihood Function (L), and Standard Deviation for Completed Spells (measured in weeks) observed in a sample of individuals in the Italian region of Emilia-Romagna, and the estimates obtained by Flinn for Italy.*

Group	λ S D	α S D	λ_{KS} S D	$\chi^2_{(9)}$ S D	L S D
Individuals looking for their first jobs					
Males ($n = 48$)	0.0195	0.898	0.540	4.13	−239.01
	0.0033	0.100			
Females ($n = 68$)	0.0112	1.280	0.854	6.24	−364.90
	0.0011	0.116			
M + F ($n = 116$)	0.0137	1.072	0.538	7.32	−609.68
	0.0013	0.076			
Individuals looking for new jobs					
Males ($n = 115$)	0.0373	0.988	0.653	7.21	−494.01
	0.0037	0.068			
Females ($n = 217$)	0.0334	0.883	1.042	23.01	−966.92
	0.0027	0.043			
M + F ($n = 332$)	0.0347	0.910	1.027	11.35	−1462.64
	0.0022	0.036			
Flinn (IT, $n = 1035$)	0.0850	1.021			−3559.08
without heterogeneity	0.0070	0.052			
Flinn (IT, λ_{1S})	0.0270	1.868			−3545.25
with heterogeneity	0.0030	0.198			
(λ_{2S})	0.0820				
	0.0060				

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