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Misspecification in Dynamic Models

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Within the class of ARMAX models we consider the effects omitted variables have on the dynamic shape and the exogeneity properties of economic relations. We provide necessary and sufficient conditions for dynamic shape, Granger causation, linear independence and structural invariance being preserved in the misspecified equation. We show that, barring particular cases, the misspecified equation does not preserve the properties of the "true" relation. In particular, the misspecified equation has a complex dynamic shape even though the true relation is static. These results apply to several misspecification problems, such as measurement errors, non-linearity, aggregation over agents and over time. We argue that misspecification must be regarded as a major source of dynamics for macroeconomic relations.

KEY WORDS: Stationary process, ARMAX model, Orthogonal projection, Dynamic shape, Exogeneity.

INRODUCTION

The consequences of the omission of a relevant variable in static models are well known. By contrast, little has been said on this subject within a dynamic framework. The issue is extremely interesting, since, as shown in Section 3, a wide class of misspecification problems can be thought of as omitted-variables problems. Examples are non-linearity, errors in variables, unobserved components, signal extraction, aggregation over agents, temporal aggregation.

The questions we deal with in this paper are well described by means of the following example. Assume that the variable Y_t satisfies the static relation

$$Y_t = aX_t + cZ_t + E_t, \tag{1}$$

where $aX_t + cZ_t$ is the best linear predictor of Y_t, given X_t, Z_t and all past values of Y_t, X_t and Z_t. Assume further that we omit the variable Z_t, e.g. because data are not available, and specify the relation as

$$Y_t = \beta X_t + R_t. \tag{2}$$

The first question is: does equation (2) provide the best linear predictor of Y_t given all the available information, i.e. X_t and *all of the past values of* X_t *and* Y_t? Put another way, is there any **dynamic** equation linking Y_t and X_t whose prediction-error variance is less than the variance of R_t? Consider, for instance, an equation much more general than (2), i.e. the ARMAX

$$\alpha(L)Y_t = \beta(L)X_t + \gamma(L)W_t, \tag{3}$$

where $\alpha(L)$, $\beta(L)$ and $\gamma(L)$ are polynomials in the lag operator L and $Y_t - W_t$ is the best linear predictor of Y_t given the available information. Is it generally true that equation (3) reduces to the static form (2)? If the answer is negative, what is the dynamic shape of the misspecified equation (3)? Can we state for instance that in general $\alpha(L) = \gamma(L)$, so that the misspecified equation is a ***rational distributed lag***?

The problem can be generalized by allowing for a more general specification of the "true" relation. Assume for instance that Y_t follows the rational distributed lag

$$Y_t = \frac{a(L)}{b(L)}X_t + \frac{c(L)}{d(L)}Z_t + E_t.$$

Can we state that the dynamic shape of the true relation is robust with respect to misspecification, i.e. also equation (3) is a rational distributed lag?

The questions above are concerned with the dynamic shape of equation (3). However, other important problems arise, concerning the exogeneity properties and the forecast performance of the misspecified equation.

First, it is easily seen that equation (3) cannot perform better than equation (1) in predicting Y_t. Indeed, it can be shown that $W_t = A_t + E_t$, where A_t is orthogonal to E_t; that is, the prediction error of the misspecified equation decomposes into two components: the prediction error of the true relation (1) and an additional error arising from misspecification, which can be termed ***misspecification error***. The question is: under what conditions does this additional error vanish, so that the misspecified equation retains all the information embedded into Z_t?

Second, suppose that in equation (1) E_t is orthogonal to all future values of the processes X_t and Z_t, i.e. Y_t does not Granger cause either X_t or Z_t, given the past of both the processes. Is there any feedback in relation (3)?

Third, assume that Y_t does not depend on X_t (given Z_t), i.e. in equation (1) $a = 0$ and E_t is orthogonal to the future of X_t. Under what conditions is Y_t ***independent of*** X_t in the misspecified equation?

Last, suppose that the parameters of the true relation are ***invariant*** with respect to some policy intervention, so that Lucas' (1976) critique does not apply to equation (1). Does the invariance property hold for the parameters of equation (3)?

In this paper we provide necessary and sufficient conditions for the properties listed above to be robust with respect to misspecification. The central result is a rather negative one. All of the dynamic properties of economic relations — i.e. prediction-error variance, dynamic shape, unidirectional causation, linear independence, parameter invariance — are destroyed upon misspecification, unless the joint covariance structure of the explanatory variables X_t and Z_t

satisfies restrictions which in most cases are unlikely to hold. The misspecified equation has a complex ARMAX shape, even though the true relation is static. The dependent variable Y_t Granger causes X_t, even though it does not Granger cause X_t conditionally on Z_t. The process X_t enters the misspecified equation, even though Y_t is independent of X_t in the true relation. Structural invariance is lost when the relation is misspecified, so that policy analysis may be seriously misleading, independently of Lucas' rational expectations argument.

These results suggest two observations. First, we must be very careful in specifying macroeconomic relations. In Section 3 we show that misspecification problems arising from aggregation may be mitigated by including "distributive" variables among the regressors. A similar point is made by Lau (1982), Jorgenson *et al.* (1982) and Stoker (1984, 1986), within a static, non-linear set-up.

Second, we must be very careful in interpreting estimated macro relations. Despite our conscientiousness, macroeconomic relations are likely to be affected by many misspecification problems, particularly aggregation over agents and over time, in addition to non-linearity and measurement errors. Therefore, the main dynamic properties of these relations may well be due to misspecification.

The dynamic shape of macroeconomic relations is usually explained as resulting from individual expectations, adjustment costs or agent's inertia (see e.g. Hendry *et al.* 1984, pp.1037-40). Equations with lagged dependent variables, for instance, may arise from search costs, transaction costs and optimization costs, or slow agents' reactions due to habits and lags in perceiving changes. Moreover, distributed lags may result from elimination of agents' expectations. While not denying the importance of these reasons, our results strongly suggest that linearization and aggregation both over agents and over time are major sources of the complex dynamic shape of macroeconomic relations.

Some results on the issues addressed here are already known, though they have never been explored in a systematic way. In particular, there is a large literature on Granger causation. Tiao and Wei (1976) show that unidirectional causation is not robust with respect to temporal aggregation. The same conclusion holds for unobserved components models (see e.g. Nerlove *et al.* 1979, pp.167-168; Sargent 1987, pp.346-348) and aggregation over agents (Lippi 1988a, 1988b). By contrast, little work has been done on the dynamic shape of misspecified equations. Weiss (1984) discusses the lag length of relations linking temporal aggregates of time series. Lippi (1988a, 1988b) and Lippi and Forni (1990) show that aggregation over agents completely modifies the dynamic shape of economic relations.

The outline of the paper is as follows. Section 1 provides assumptions and definitions. The main results are presented in Section 2. Section 3 collects examples and applications to measurement errors, non-linear models, aggregation over agents, unobserved components and temporal aggregation. In this Section it is shown that the main results stated in the previous literature can be easily derived from the proposition proved in Section 2. In Section 4 some concluding remarks are provided.

1. ASSUMPTIONS AND DEFINITIONS

1.1 The true relation

The time series Y_t, X_t, and Z_t are zero-mean, jointly covariance stationary, purely non deterministic processes with non-singular, rational spectral-density matrix. The process Y_t follows the relation

$$Y_t = \frac{a(L)}{b(L)} X_t + \frac{c(L)}{d(L)} Z_t + E_t, \tag{4}$$

where the functions in the lag operator L are polynomials. Relation (4) is the *true relation*. In equation (4) the roots of $b(L)$ and $d(L)$ are of modulus greater than one, $b(0) = d(0) = 1$, $a(L)$ and $b(L)$ as well as $c(L)$ and $d(L)$ have no common roots, E_t is a white-noise disturbance orthogonal to Y_{t-h}, $h > 0$ and X_{t-k}, Z_{t-k}, all k. These conditions ensure that $[a(L)/b(L)]X_t + [c(L)/d(L)]Z_t$ is the best linear predictor of Y_t within the Hilbert space spanned by the past of Y_t and the past, present ***and future*** of X_t and Z_t, while E_t is the prediction error.

1.2 The explanatory variables

Since the vector process $(\, Y_t \quad Z_t \quad X_t\,)$ has a rational spectral-density matrix (i.e. possesses an ARMA representation), the vector process $(\, Z_t \quad X_t\,)$ has a rational spectral-density matrix (Lütkepol 1984), and admits the representation:

$$e(L)Z_t = f(L)X_t + g(L)U_{Zt} \tag{5}$$
$$k(L)X_t = h(L)Z_t + g(L)U_{Xt}, \tag{6}$$

where the polynomials in L satisfy the following conditions:

(*i*) $g(L)$ has no factors common to all other polynomials;
(*ii*) $e(0) = k(0) = g(0) = 1$, $h(0) = 0$;
(*iii*) the roots of $g(L)$ are of modulus greater or equal to one;
(*iv*) $e(L)k(L) - f(L)h(L)$ vanishes only outside the closed unit circle, except for zeroes of $g(L)$. Moreover,
(*v*) U_{Zt} is orthogonal to X_{t-k}, Z_{t-h}, $k \geq 0$, $h > 0$, while U_{Xt} is orthogonal to X_{t-h}, Z_{t-h}, $h > 0$.[1]

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[1] The variables Z_t and X_t have a rational joint Wold representation, i.e.

$$\begin{pmatrix} Z_t \\ X_t \end{pmatrix} = A(L) \begin{pmatrix} V_{Zt} \\ V_{Xt} \end{pmatrix}, \tag{7}$$

where $A(L)$ is a matrix of rational functions in L with no poles of modulus less or equal to one, $A(0) = I$, $\det[A(L)]$ has no roots of modulus less than one and $(\, V_{Zt} \quad V_{Xt}\,)$ is a vector white noise orthogonal to Z_{t-h}, X_{t-h}, $h > 0$. Representation (5)-(6) is obtained from the Wold representation by premultiplying both sides of (7) by

$$A^*(L) \begin{pmatrix} 1 & -\frac{\operatorname{cov}(V_{Zt}, V_{Xt})}{\operatorname{var}(V_{Xt})} \\ 0 & 1 \end{pmatrix},$$

where $A^*(L)$ is the adjoint of $A(L)$, and by eliminating denominators and common factors.

By construction, U_{Zt} and U_{Xt} are white noises orthogonal at all leads and lags. U_{Zt} is the residual of the orthogonal projection of Z_t on the space spanned by X_t and the lagged variables X_{t-k}, Z_{t-h}, $k \geq 0$, $h > 0$, while U_{X_t} is the residual of the orthogonal projection of X_t on the space spanned by X_{t-h}, Z_{t-h}, $h > 0$. Therefore, $Z_t - U_{Zt}$ is the best linear predictor of Z_t within the first space, while $X_t - U_{Xt}$ is the best linear predictor of X_t within the second one. We will refer to equation (5) as the ***model of Z_t conditional to X_t*** or simply the ***relation*** linking Z_t and X_t. Equation (6) is the ***marginal model*** of X_t.

It will prove useful to define the set of admissible parameters of equations (5) and (6). A particular set of values $\mathcal{F}$ for the parameters of equations (5) and (6), i.e. $e(L)$, $f(L)$, $g(L)$, $k(L)$, $h(L)$, $\mathrm{var}(U_{Zt})$ and $\mathrm{var}(U_{Xt})$, is ***admissible*** if it satisfies the conditions (i) through (iv) listed above along with $\mathrm{var}(U_{Zt}) \geq 0$ and $\mathrm{var}(U_{Xt}) \geq 0$. The class of all admissible $\mathcal{F}$ is denoted by Φ. It can be proved that, since the spectral-density matrix of $(Z_t \quad X_t)$ is non-singular almost everywhere on the interval $[-\pi, \pi]$, i.e. neither U_{Zt} nor U_{Xt} are zero, representation (5) is unique; that is, given the joint covariance structure of the processes Z_t and X_t, the parameters $\mathcal{F}$ listed above are univocally determined by conditions (i) through (v) (see Hannan 1970, chs. 2,3 or Rozanov 1967, chs. 1,2).

The conditions imposed on E_t in equation (4) ensure that the residuals U_{Zt} and U_{Xt} in equation (5) are orthogonal not only to the past of the processes X_t and Z_t but also to the past of the process Y_t. In fact, U_{Zt} and U_{Xt} are orthogonal to the whole process E_t, while the lagged variables Y_{t-h}, $h > 0$, are linear combinations of the past of X_t, Z_t and E_t. Therefore Y_t does not Granger cause either X_t or Z_t, ***given the past of both the processes.***

1.3 The misspecified equation

The ***misspecified model*** is

$$\alpha(L)Y_t = \beta(L)X_t + \gamma(L)W_t \tag{8}$$

$$\delta(L)X_t = \theta(L)Y_t + \gamma(L)U_t. \tag{9}$$

The ***misspecified relation*** (8) is the model of Y_t conditional to X_t: the polynomials in (8) and the process W_t satisfy the restrictions imposed on the corresponding polynomials and on the process U_{Zt} in equation (5). Therefore, $Y_t - W_t$ is the best linear predictor of Y_t given X_t and all past values of X_t and Y_t. Equation (9) is the corresponding marginal model; $X_t - U_t$ is the best linear predictor of X_t given all past values of X_t and Y_t.

A particular choice of the parameters in equations (8) and (9), i.e. $\alpha(L)$, $\beta(L)$, $\gamma(L)$, $\delta(L)$, $\theta(L)$, $\mathrm{var}(W_t)$ and $\mathrm{var}(U_t)$, is denoted by $\mathcal{O}$, while Ω indicates the set of all admissible $\mathcal{O}$. The spectral-density matrix of the vector $(Y_t \quad X_t)$ is denoted by

$$S = \begin{pmatrix} S_{YY} & S_{YX} \\ S_{XY} & S_{XX} \end{pmatrix}.$$

The assumptions in 1.1 ensure that S is a matrix of rational functions in $e^{-i\lambda}$, $-\pi \leq \lambda < \pi$. Moreover, S is non-singular almost everywhere. Therefore representation (8)-(9) always exists and is unique.

1.4 The dynamic shape

We say that the misspecified equation (8) is

(a) ***rational distributed lag*** (RDL), if and only if $\alpha(L) = \gamma(L)$ and the roots of $\gamma(L)$ are of modulus greater than one;

(b) ***unrestricted ARMAX***, if and only if it is not a rational distributed lag. Moreover, if the misspecified equation is RDL, it is

(a′) ***finite distributed lag*** (FDL), if and only if $\gamma(L) = 1$;

(a″) ***static***, if and only if it is FDL and $\beta(L) = \beta$.

Similar definitions hold for equation (5). By definition, the true relation (4) cannot take the shape (b).

1.5 Causation, independence and invariance

There is a one-way causation in the misspecified model, that is Y_t ***does not Granger cause*** X_t, if and only if $\theta(L) = 0$ in equation (9), i.e. U_t is the residual of the orthogonal projection of X_t on its own past. There is a one-way causation in model (5)-(6) if and only if $h(L) = 0$. There is always a one-way causation in the true model, since by construction Y_t does not Granger cause either X_t or Z_t conditionally on both the processes.

Y_t is ***independent of*** X_t if, and only if, $\beta(L) = 0$ in equation (8) and Y_t does not Granger cause X_t (i.e. Y_t and X_t are orthogonal at all leads and lags). Y_t ***is independent of*** X_t ***conditionally on*** Z_t if and only if $a(L) = 0$ in relation (4).

The misspecified relation (8) is ***invariant with respect to*** Φ if and only if the parameters of (8), i.e. $\alpha(L)$, $\beta(L)$, $\gamma(L)$ and var(W_t), do not depend on the parameters in Φ — that is, they do not depend on the joint covariance structure of Z_t and X_t. A similar definition holds for the true relation.

1.4 The misspecification error

Consider the projection of $[c(L)/d(L)]Z_t$ on X_t and the past values of X_t and Y_t. Call this projection P_t and the residual A_t. Then

$$Y_t = \frac{a(L)}{b(L)}X_t + P_t + A_t + E_t.$$

It turns out that $A_t + E_t = W_t$, where W_t is the residual of the misspecified relation. In fact, both A_t and E_t are orthogonal to X_t and the past values of X_t and Y_t, while $[a(L)/b(L)]X_t + P_t$ belongs to the Hilbert space spanned by the same variables. Moreover, E_t is orthogonal to P_t and Z_{t-k}, $k \geq 0$; since $A_t = [c(L)/d(L)]Z_t - P_t$, E_t is orthogonal to A_t, so that var$(W_t) =$ var$(A_t) +$ var(E_t).

Hence, the prediction error W_t decomposes into two orthogonal components, E_t, that is the prediction error of the true relation, and the additional error

A_t. The variance of A_t measures the information we lost when predicting Y_t using the misspecified relation instead of the true relation. The process A_t is the *misspecification error*; i.e. the component of the prediction error arising from misspecification.

2. RESULTS

2.1 Prediction

Proposition 1. *The misspecification error A_t is zero if and only if $c(L) = 0$.*

Proof. The sufficiency part is obvious. To prove necessity, assume $W_t = E_t$. From (4) and (8) it is obtained

$$\alpha(L)\frac{c(L)}{d(L)}Z_t = \left[\beta(L) - \frac{a(L)}{b(L)}\alpha(L)\right] X_t + [\gamma(L) - \alpha(L)]E_t. \tag{10}$$

The last term on the right-hand side belongs to the space spanned by E_{t-h}, $h \geq 0$. However, it belongs also to the space spanned by the present and past of the processes Z_t and X_t. Since the former space is orthogonal to the latter, the last term of (10) is orthogonal to itself and therefore is zero. If $c(L) \neq 0$, equation (10) implies that the processes Z_t and X_t span the same space and have a singular spectral-density matrix, contrary to the assumptions in 1.1.

2.2 Invariance

Lemma 1. *Define Ω' the set of all parameter choices $\mathcal{O}$ of the misspecified relation such that*

$$\mathrm{var}(E_t) \leq S_{YY} - \frac{S_{XY}S_{YX}}{S_{XX}} \tag{11}$$

on the interval $[-\pi, \pi)$. If $c(L) \neq 0$, whatever $\mathcal{O}$ in Ω' can be obtained for the misspecified equation by a suitable choice of $\mathcal{F}$ in Φ, i.e. by a suitable choice of the parameters of equations (5) and (6).

Proof. Take some $\mathcal{O}^*$ in Ω'. This determines univocally $S = S^*$. Define

$$R^* = S^* - \begin{pmatrix} \mathrm{var}(E_t) & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$G^* = \begin{pmatrix} \frac{d(z)}{c(z)} & -\frac{a(z)d(z)}{b(z)c(z)} \\ 0 & 1 \end{pmatrix} R^* \begin{pmatrix} \frac{d(z^{-1})}{c(z^{-1})} & 0 \\ -\frac{a(z^{-1})d(z^{-1})}{b(z^{-1})c(z^{-1})} & 1 \end{pmatrix}, \tag{12}$$

where $z = e^{-i\lambda}$, $-\pi \leq \lambda < \pi$. Condition (11) along with $c(z) \neq 0$ ensure that matrices R^* and G^* are well-defined rational spectral-density matrices. Therefore there is always one (and only one) $\mathcal{F}^*$ in Φ such that the spectral-density matrix of $(Z_t \quad X_t)$ is equal to G^*. If we choice $\mathcal{F}^*$ for the parameters of equations (5) and (6) we obtain $S = S^*$ by equation (12). This determines univocally $\mathcal{O} = \mathcal{O}^*$ for the parameters of the misspecified model.

Proposition 2. *Assume that the true relation is invariant with respect to Φ. Then, the misspecified equation is invariant with respect to Φ if and only if $c(L) = 0$.*

Proof. If $c(L) = 0$ the misspecified equation and the true equation are the same. If $c(L) \neq 0$, take two choices $\mathcal{O}^*$ and $\mathcal{O}^{**}$ in Ω' such that at least one of the equalities $\alpha^*(L) = \alpha^{**}(L)$, $\beta^*(L) = \beta^{**}(L)$, $\gamma^*(L) = \gamma^{**}(L)$, $\text{var}(W_t)^* = \text{var}(W_t)^{**}$ does not hold. By Lemma 1, both $\mathcal{O}^*$ and $\mathcal{O}^{**}$ can be obtained for the misspecified equation by varying $\mathcal{F}$ in Φ.

2.3 Granger causation

Lemma 2. *If $c(L)$ does not vanish within the unit circle, the model of the omitted term $S_t = [c(L)/d(L)]Z_t$ conditional on X_t is*

$$e(L)d(L)S_t = f(L)c(L)X_t + \frac{c(L)g(L)}{c(0)}[c(0)U_{Zt}], \tag{13}$$

while the corresponding marginal model is

$$\frac{k(L)c(L)}{c(0)}X_t = \frac{h(L)d(L)}{c(0)}S_t + \frac{c(L)g(L)}{c(0)}U_{Xt} \qquad (14).$$

Proof. Equations (13) and (14) are obtained from equations (5) and (6). Since the variables S_{t-h}, $h > 0$, belong to the space spanned by Z_{t-h}, $h > 0$, it follows that $c(0)U_{Zt}$ is orthogonal to S_{t-h}, X_{t-k}, $h > 0$, $k \geq 0$, while U_{Xt} is orthogonal to S_{t-h}, X_{t-h}, $h > 0$. Moreover, $c(L)g(L)$ does not vanish within the unit circle because of the assumptions on $c(L)$. The other conditions on the polynomials in L are easily verified.

Proposition 3. *Y_t does not Granger cause X_t if and only if the omitted term $S_t = [c(L)/d(L)]Z_t$ does not Granger cause X_t.*

Proof. Call Q_t the orthogonal projection of Y_t on the space $\mathcal{H}_X$ spanned by X_{t-k}, all k, and H_t the residual. Hence, $S_t = \Lambda_t + K_t$, where $\Lambda_t = Q_t - [a(L)/b(L)]X_t$ and $K_t = H_t - E_t$. Since both H_t and E_t are orthogonal to $\mathcal{H}_X$, K_t is orthogonal to $\mathcal{H}_X$. On the other hand, Λ_t belongs to $\mathcal{H}_X$; therefore, Λ_t is the projection of S_t on $\mathcal{H}_X$. It is clear from the definition of Λ_t that Λ_t belongs to the subspace of $\mathcal{H}_X$ spanned by the present and past of X_t if, and only if, Q_t belongs to this subspace.

Proposition 4. *(a) If Z_t does not Granger cause X_t, then Y_t does not Granger cause X_t. (b) If $c(L)$ does not vanish within the unit circle, Y_t does not Granger cause X_t if and only if Z_t does not Granger cause X_t.*

Proof. (a) The projection of S_t on the whole process X_t is $\Lambda_t = [c(L)/d(L)]\Pi_t$, where Π_t is the projection of Z_t on the same space. If Π_t belongs to the subspace spanned by X_{t-k}, $k \geq 0$, then also Λ_t belongs to this subspace, so that S_t does not Granger cause X_t and by Proposition 3 Y_t does not Granger cause X_t. (b) It is clear from Lemma 2, equation (14), that if $c(L)$ does not vanish within the unit circle, then S_t Granger causes X_t if and only if Z_t Granger causes X_t. The result follows from Proposition 3.

2.4 Independence

Proposition 5. *If Y_t is independent of X_t conditionally on Z_t, Y_t is independent of X_t if and only if (i) $c(L) = 0$, or (ii) Z_t is independent of X_t.*

Proof. If $a(L) = 0$, then $S_{YX} = S_{ZX}c(z)/d(z)$, where $z = e^{-i\lambda}$, $-\pi \leq \lambda < \pi$. The right-hand side vanishes if and only if either (i) or (ii) hold.

2.5 Dynamic shape

Proposition 6. *If $c(L) \neq 0$, an arbitrary dynamic shape can be obtained for the misspecified equation and the marginal model (9) whatever the dynamic shape of the true relation (4), by suitably setting $\mathcal{F}$ and* var(E_t).

Proof. Take some $\mathcal{O}^*$ in Ω and set var(E_t) such that (11) is satisfied. The result follows from Lemma 1.

Remark. Assume that the true equation is static. If var$(E_t) = 0$ then $\Omega = \Omega'$, where Ω' is as in Lemma 1. By Proposition 6, the misspecified equation can take whatever dynamic shape, depending on the parameters in (5) and (6). On the contrary, if var$(E_t) > 0$ there are some parameters $\mathcal{O}$ and some associated spectra S which do not satisfy (11), i.e. $\Omega' \subset \Omega$. The misspecified models characterized by such parameters cannot be obtained from a static equation. The greater var(E_t), the smaller Ω' and the larger is the set of these models.

Proposition 7. *The misspecified equation is a rational distributed lag if and only if the model of the omitted term $S_t = [c(L)/d(L)]Z_t$ conditional on X_t is a rational distributed lag.*

Proof. Assume that in the misspecified equation $\alpha(L) = \gamma(L)$ and $\gamma(L)$ vanishes only outside the closed unit circle. Equation (4) implies

$$S_t = [\beta(L)/\gamma(L) - a(L)/b(L)]X_t + (W_t - E_t). \tag{15}$$

W_t belongs to the space spanned by E_t, S_t and X_{t-k}, $k \geq 0$, so that E_{t-h} is orthogonal to W_t for $h > 0$. Since W_t is orthogonal to Y_{t-h}, X_{t-k}, $h > 0$, $k \geq 0$, it is also orthogonal to $S_{t-h} = Y_{t-h} - [a(L)/b(L)]X_{t-h} - E_{t-h}$, $h > 0$. Hence $W_t - E_t$ is orthogonal to S_{t-h}, X_{t-k}, $k \geq 0$, $h > 0$. The model of S_t conditional on X_t is then obtained by (15) eliminating denominators and common factors, so that it is RDL. Conversely, let us assume that the model of S_t conditional on X_t is

$$S_t = [f'(L)/g'(L)]X_t + U_{St}, \tag{16}$$

Then

$$Y_t = [a(L)/b(L) + f'(L)/g'(L)]X_t + (E_t + U_{St}). \tag{17}$$

E_t and U_{St} are orthogonal at all leads and lags, since E_t is orthogonal to the processes S_t and X_t. Moreover, U_{St} is orthogonal by definition to S_{t-h}, X_{t-k}, $h > 0$, $k \geq 0$. Therefore U_{St} is orthogonal to $Y_{t-h} = [a(L)/b(L)]X_{t-h} + S_{t-h} + E_{t-h}$, $h > 0$. Hence, $E_t + U_{St}$ is orthogonal to X_{t-k}, Y_{t-h}, $k \geq 0$, $h > 0$. Thus, the misspecified equation is obtained from (17) eliminating denominators and common factors, so that it is RDL.

Proposition 8. *If $c(L)$ does not vanish within the unit circle, the misspecified equation is a rational distributed lag if and only if*[2] $e(L)d(L) = c(L)g(L)/c(0)$. *In this case,* $\alpha(L) = \gamma(L) = b(L)g(L)$, $\beta(L) = a(L)g(L) + c(0)f(L)b(L)$ *and* $W_t = c(0)U_{Zt} + E_t$.

Proof. By Lemma 2, if $c(L)$ does not vanish within the unit circle, equation (13) is the model of S_t conditional on X_t. This model is RDL if and only if $e(L)d(L) = c(L)g(L)/c(0)$, so that by Proposition 7 the last condition is equivalent to the misspecified equation being RDL. The expressions for $\alpha(L)$, $\beta(L)$, $\gamma(L)$ and W_t follow from equations (13), (16) and (17).

Proposition 9. *If the true relation is static and $c(0) \neq 0$, the misspecified equation is (i) RDL, (ii) FDL, (iii) static, if and only if the relation linking Z_t and X_t is respectively (i) RDL, (ii) FDL, (iii) static.*

Proof. By Proposition 8, if $c(L) = c(0) \neq 0$, $a(L) = a(0)$ and $d(L) = b(L) = 1$, the misspecified equation is RDL if and only if $e(L) = g(L)$, i.e. the model of Z_t conditional on X_t is RDL. Moreover, in this case $\alpha(L) = \gamma(L) = e(L) = g(L)$ and $\beta(L) = a(0)e(L) + c(0)f(L)$. Therefore $\alpha(L) = \gamma(L) = 1$, i.e. the misspecified equation is FDL, if and only if $e(L) = g(L) = 1$, i.e. the model of Z_t conditional on X_t is FDL. In this case, $\beta(L) = a(0) - c(0)f(L)$. It follows that $\beta(L) = \beta(0)$ (the misspecified equation is static) if and only if $f(L) = f(0)$ (the model of Z_t conditional on X_t is static).

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[2] This condition implies that (a) if the true relation is static in Z_t, i.e. $c(L) = c(0)$ and $d(L) = 1$, the misspecified equation is RDL if and only if the model of Z_t conditional on X_t is RDL; (b) if $c(L) = 0$ and the model of Z_t conditional on X_t is RDL, then the misspecified relation is RDL if and only if the true relation is static in Z_t.

Proposition 10. *If Z_t does not Granger cause X_t, then in equations (8) and (9)*

$$\begin{aligned} \alpha(L) &= b(L)d(L)e(L) \\ \beta(L) &= a(L)d(L)e(L) + c(L)f(L)b(L) \\ \gamma(L)/\delta(L) &= k(L)/g(L) \end{aligned} \tag{18}$$

and $\gamma(L)$, $\mathrm{var}(W_t)$ *are such that*

$$|\gamma(z)|^2 \mathrm{var}(W_t) = |b(z)|^2 \left(|c(z)g(z)|^2 \mathrm{var}(U_{Zt}) + |d(z)e(z)|^2 \mathrm{var}(E_t)\right), \tag{19}$$

where $z = e^{-i\lambda}$, $-\pi < \lambda \leq \pi$.

Proof. From equations (4), (5) and (6) we obtain

$$\begin{aligned} S_{XX} &= \frac{|g(z)|^2}{|k(z)|^2} \mathrm{var}(U_{Xt}) \\ S_{YX} &= \frac{|g(z)|^2}{|k(z)|^2} \mathrm{var}(U_{Xt}) \left(\frac{a(z)}{b(z)} - \frac{c(z)f(z)}{d(z)e(z)} \right) \\ S_{YY} &= \frac{|g(z)|^2}{|k(z)|^2} \mathrm{var}(U_{Xt}) \left| \frac{a(z)}{b(z)} - \frac{c(z)f(z)}{d(z)e(z)} \right|^2 + \\ &\qquad \frac{|c(z)g(z)|^2}{|d(z)e(z)|^2} \mathrm{var}(U_{Zt}) + \mathrm{var}(E_t). \end{aligned}$$

Imposing the equalities (18) and (19) the same spectra are obtained from (8) and (9).

3. EXAMPLES AND APPLICATIONS

3.1 Errors in variables

Assume that the variables $\hat{Y}_t$ and Z_t are linked by the relation

$$\hat{Y}_t = \frac{c(L)}{d(L)} Z_t + R_t,$$

where $c(L)$ and $d(L)$ do not vanish within the unit circle and R_t is orthogonal to $\hat{Y}_{t-h}$, Z_{t-k}, $h > 0$, all k. Assume further that $\hat{Y}_t$ and Z_t are subject to a measurement error. The observed values are $Y_t = \hat{Y}_t + O_t$ and $X_t = Z_t + U_t$, where the errors O_t and U_t are joint white noises with diagonal variance-covariance matrix, orthogonal to $\hat{Y}_{t-k}$ and Z_{t-k} for all k. Hence

$$Y_t = \frac{c(L)}{d(L)} Z_t + E_t, \tag{20}$$

where $E_t = O_t + R_t$. It is easily verified that E_t is orthogonal to Y_{t-h}, X_{t-k}, Z_{t-k}, $h > 0$, all k, so that (20) has the same properties as equation (4).

Assume that the univariate Wold representation of Z_t is $Z_t = [\mu(L)/\nu(L)]V_{Zt}$, where $\mu(L)$ has no unit-modulus roots. It follows that

$$\begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \begin{pmatrix} \mu(L)/\nu(L) & 0 \\ \mu(L)/\nu(L) & 1 \end{pmatrix} \begin{pmatrix} V_{Zt} \\ U_t \end{pmatrix}. \qquad (21)$$

The joint Wold representation of Z_t and X_t is obtained from (21) by inserting the factor

$$I = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

between the matrix and the vector on the right-hand side. Following the steps indicated in Subsection 1.2, footnote 1, we get

$$\begin{pmatrix} (1-p)\nu + p\mu & -p\mu \\ \nu - \mu & \mu \end{pmatrix} \begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \mu \begin{pmatrix} U_{Zt} \\ U_{Xt} \end{pmatrix}, \qquad (22)$$

where $p = \text{var}(V_{Zt})/[\text{var}(V_{Zt}) + \text{var}(U_t)]$, $U_{Zt} = (1-p)V_{Zt} - pU_t$ and $U_{Xt} = U_t + V_{Zt}$. The first line of (22) is the model of Z_t conditional on X_t, while the second line is the marginal model of X_t.

Consider now the misspecified model, that is the model of Y_t conditional on X_t and the corresponding marginal model. By Propositions 1, the misspecification error is zero if and only if $c(L) = 0$, i.e. Y_t is a white-noise process orthogonal to Z_t at all leads and lags. This condition is also equivalent to the misspecified relation being invariant with respect to the autocovariance structure of Z_t. Since $c(L)$ does not vanish within the unit circle, by Proposition 4 Y_t does not Granger cause X_t if and only if $\mu(L) = \nu(L)$, i.e. Z_t is white noise. Lastly, by Proposition 8 the misspecified relation is RDL if and only if

$$d(L)[(1-p)\nu(L) + p\mu(L)] = c(L)\mu(L)/c(0).$$

It follows that if the true relation is static, either $\nu(L) = \mu(L)$, i.e. Z_t is white noise, so that the misspecified equation is static, or Z_t is not white noise, in which case the misspecified relation is an unrestricted ARMAX.

3.2 Non-linearity

In this Subsection we assume that Y_t and X_t are strictly stationary and are linked by the static quadratic relation

$$Y_t = aX_t + c[X_t^2 - \text{var}(X_t)] + E_t,$$

where E_t is independent of Y_{t-h}, X_{t-k}, $h > 0$, all k. The problem is to find the *linear* model of Y_t and X_t.

To simplify matters, assume that the explanatory variable X_t is AR(1), i.e. $X_t = \phi X_{t-1} + U_t$, where U_t is independent of X_{t-h}, $h > 0$. It follows that $Z_t = X_t^2 - \text{var}(X_t)$ satisfies

$$Z_t = \phi^2 Z_{t-1} + R_t,$$

where $R_t = U_t^2 - \text{var}(U_t) + 2\phi U_t X_{t-1}$. It is easily seen that R_t is a zero-mean white noise independent of X_{t-h} and Z_{t-h} for $h > 0$. Therefore Z_t and X_t are jointly covariance stationary and their Wold representation is

$$\begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \begin{pmatrix} 1/(1-\phi^2 L) & 0 \\ 0 & 1/(1-\phi L) \end{pmatrix} \begin{pmatrix} R_t \\ U_t \end{pmatrix}.$$

The model of Z_t and X_t is

$$\begin{pmatrix} 1-\phi^2 L & -p(1-\phi L) \\ 0 & 1-\phi L \end{pmatrix} \begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \begin{pmatrix} U_{Zt} \\ U_t \end{pmatrix}, \tag{23}$$

where $p = E(U_t^3)/E(U_t^2)$ and $U_{Zt} = R_t - pU_t$.

Since Z_t does not Granger cause X_t, by Proposition 4 Y_t does not Granger cause X_t and, by Proposition 10, the misspecified equation is

$$(1-\phi^2 L)Y_t = [a(1-\phi^2 L) + cp(1-\phi L)]X_t + \gamma(L)W_t, \tag{24}$$

where $\text{var}(W_t) = c\,\text{var}(R_t) + \text{var}(E_t)$. The polynomial $\gamma(L)$ is identified by the condition on the roots and by the equality

$$|\gamma(z)|^2 = \frac{c\,\text{var}(R_t) + \text{var}(E_t)|1-\phi^2 L|^2}{c\,\text{var}(R_t) + \text{var}(E_t)},$$

where $z = e^{-i\lambda}$, $-\pi < \lambda \leq \pi$. It is apparent from (24) that a change in the model generating X_t modifies the parameters of the misspecified relation. Moreover, if X_t is white noise, that is $\phi = 0$, the misspecified equation is static; conversely, if $\phi \neq 0$, the misspecified equation is a general ARMAX.

3.3 Aggregation over agents

In this Subsection we analyze a simplified version of Lippi's (1989) aggregation problem. There are two groups of consumers. Within the first group each consumer follows the static behavioral rule $y_{it} = a_1 x_{it} + e_{it}$, where y_{it} and x_{it} are respectively consumption and income of agent i. Within the second group the behavioural rule is $y_{jt} = a_2 x_{jt} + e_{jt}$. Summing over individuals we get, with an obvious notation,

$$Y_{1t} = a_1 X_{1t} + E_t \tag{25}$$

for the first group and

$$Y_{2t} = a_2 X_{2t} + E_t \tag{26}$$

for the second one. We assume that both E_{1t} and E_{2t} are orthogonal to $Y_{1(t-h)}$, $Y_{2(t-h)}$, $X_{1(t-k)}$, $X_{2(t-k)}$, $h > 0$, all k. Our aim is to study the model of the aggregate consumption $Y_t = Y_{1t} + Y_{2t}$ conditional on the aggregate income $X_t = X_{1t} + X_{2t}$.

Summing equations (25) and (26) it is obtained

$$Y_t = Z_t + E_t, \tag{27}$$

where $Z_t = a_1 X_{1t} + a_2 X_{2t}$ and $E_t = E_{1t} + E_{2t}$. It is easily verified that equation (27) satisfies the properties of equation (4), so that in our terminology equation (27) is the true relation. Aggregate consumption depends on the joint distribution (among individuals) of income and propensity to consume. Indeed, the variable Z_t is the covariance of this distribution. If Z_t is not observable, and we substitute the aggregate income X_t for Z_t, the resulting equation is misspecified. As we will see in the following Subsection, the misspecified aggregate relation is in general an unrestricted ARMAX.

A similar problem arises if the parameters of the micro equations are all equal, but the micro equations either are non-linear or include unobserved explanatory variables. Consider for instance the micro equations $y_{it} = ax_{it} + c[x_{it}^2 - \text{var}(x_{it})] + e_{it}$. Summing over individuals yields

$$Y_t = aX_t + cZ_t + E_t,$$

where $Z_t = \sum_i x_{it}^2 - \sum_i \text{var}(x_{it})$. Also in this case aggregate consumption does not depend on aggregate income only, but on the variable Z_t, which is the variance of the distribution of income minus its mathematical expectation. Therefore Z_t should be included among the explanatory variables in the aggregate relation. However, if data on Z_t are not available, the best we can do is to specify a model linking Y_t and X_t. As stated in Proposition 2, the properties of this model depend on the joint covariogram of Z_t and X_t.

3.4 Unobserved components

In this Subsection we discuss the unobserved component model of Nerlove *et al.* (1979, pp.167-68). Since this model is formally identical to the two-groups aggregation model discussed above, the conclusions of this Subsection apply to the aggregation problem as well.

Assume that there are two economic series, Y_t and X_t, each one having two components, seasonal and non-seasonal:

$$Y_t = Y_{Nt} + Y_{St}; \qquad X_t = X_{Nt} + X_{St}.$$

The non-seasonal components are linked each other by the relation

$$Y_{Nt} = a_N X_{Nt} + E_{Nt}, \tag{28}$$

while the seasonal components follow the relation

$$Y_{St} = a_S X_{St} + E_{St}. \tag{29}$$

The residuals E_{Nt} and E_{St} are orthogonal to $Y_{N(t-h)}$, $Y_{S(t-h)}$, $X_{N(t-k)}$, $X_{S(t-k)}$, $h > 0$, all k. The extension to the case of (28) and (29) being RDL is straightforward.

As usual, we are interested in the model of Y_t conditional to X_t. Summing (28) and (29) gives

$$Y_t = Z_t + E_t,$$

where $Z_t = a_{Nt}X_{Nt} + a_{St}X_{St}$ and $E_t = E_{Nt} + E_{St}$. Clearly, if $a_N = a_S = a$, then $Z_t = aX_t$ and no misspecification problems arise. Conversely, if $a_N \neq a_S$ the spectral-density matrix of $(\, Z_t \quad X_t \,)$ is non-singular. Since $c(L) = 1$, by Proposition 1 the misspecification error is non zero and by Proposition 2 the model of Y_t conditional to X_t is not invariant with respect to the joint model of Z_t and X_t. In order to obtain this model, we must explore the covariance structure of the explanatory variables X_{Nt} and X_{St}.

To simplify calculations, we retain Nerlove's assumption that X_{Nt} and X_{St} are orthogonal at all leads and lags. It should be noted, however, that this restriction can be easily dropped in our framework, whereas it is essential in Nerlove's one. The joint Wold representation of X_{Nt} and X_{St} is therefore

$$\begin{pmatrix} X_{St} \\ X_{Nt} \end{pmatrix} = \begin{pmatrix} \mu(L)/\nu(L) & 0 \\ 0 & \phi(L)/\psi(L) \end{pmatrix} \begin{pmatrix} V_{St} \\ V_{Nt} \end{pmatrix}.$$

It follows that

$$\begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \begin{pmatrix} a_S\mu/\nu & a_N\phi/\psi \\ \mu/\nu & \phi/\psi \end{pmatrix} \begin{pmatrix} V_{St} \\ V_{Nt} \end{pmatrix}.$$

Inserting the factor

$$\frac{1}{a_S - a_N} \begin{pmatrix} 1 & -a_N \\ -1 & a_S \end{pmatrix} \begin{pmatrix} a_S & a_N \\ 1 & 1 \end{pmatrix}$$

between the matrix and the vector on the right-hand side, we obtain the joint Wold representation of Z_t and X_t, i.e.

$$\begin{pmatrix} Z_t \\ X_t \end{pmatrix} = \frac{1}{a_S - a_N} \begin{pmatrix} a_S\mu/\nu - a_N\phi/\psi & a_S a_N(\phi/\psi - \mu/\nu) \\ \mu/\nu - \phi/\psi & a_S\phi/\psi - a_N\mu/\nu \end{pmatrix} \begin{pmatrix} V_{Zt} \\ V_{Xt} \end{pmatrix},$$

where $V_{Zt} = a_S V_{St} + a_N V_{Nt}$ and $V_{Xt} = V_{St} + V_{Nt}$. The model of Z_t conditional on X_t and the corresponding marginal model are

$$\begin{pmatrix} \dfrac{\nu\phi(a_S - p) + \mu\psi(p - a_N)}{a_S - a_N} & \dfrac{-\nu\phi a_N(a_S - p) - \mu\psi a_S(p - a_N)}{a_S - a_N} \\ \dfrac{\nu\phi - \mu\psi}{a_S - a_N} & \dfrac{\nu\phi a_S - \mu\psi a_N}{a_S - a_N} \end{pmatrix} \begin{pmatrix} Z_t \\ X_t \end{pmatrix}$$
$$= \mu\phi \begin{pmatrix} U_{Zt} \\ U_{Xt} \end{pmatrix}, \tag{30}$$

where $p = \text{cov}(V_{Zt}, V_{Xt})/\text{var}(V_{Xt})$, $U_{Zt} = V_{Zt} - pV_{Xt}$ and $U_{Xt} = V_{Xt}$.

It is easily seen from (30) that Y_t does not Granger cause X_t if and only if $\nu(L)\phi(L) = \mu(L)\psi(L)$, i.e. the autocorrelation structures of the seasonal and non-seasonal components are the same (Proposition 3). This is Nerlove's result. Moreover, by Proposition 9, the model of Y_t conditional on X_t is static if and only if

$$\begin{aligned}\nu\phi a_N(a_S - p)/p + \mu\psi a_S(a_N - p)/p \\ = \nu\phi(a_S - p) + \mu\psi(a_N - p) = \mu\psi(a_S - a_N)\end{aligned}$$

From the first inequality we get $\nu(L)\phi(L) = \mu(L)\psi(L)$; substituting into the second yelds $\psi(L) = \phi(L)$ and $\mu(L) = \nu(L)$. Therefore the relation linking Y_t and X_t is static if and only if the unobserved components X_{Nt} and X_{St} are white noises.

3.5 Temporal aggregation

Let us consider a two-period version of the temporal-aggregation model discussed by Tiao and Wei (1976). Assume for simplicity that the true relation linking y_t and x_t is the FDL

$$y_t = p(L)x_t + e_t, \tag{31}$$

where e_t is orthogonal to y_{t-h}, x_{t-k}, $h > 0$, all k and $p(L) = p_0 + p_1L + \cdots + p_{(2r)}L^r$. However, data are available only for $Y_T = y_{(2t)} + y_{(2t-1)}$ and $X_T = x_{(2t)} + x_{(2t-1)}$. We are interested in the relation linking Y_T and X_T.

Summing y_t and y_{t-1} gives

$$Y_t = p(L)X_t + E_t, \tag{32}$$

where $Y_t = y_t + y_{t-1}$, $X_t = x_t + x_{t-1}$ and $E_t = e_t + e_{t-1}$. From (32) we get

$$Y_T = a(B)X_T + c(B)Z_T + E_T, \tag{33}$$

where $B = L^2$, $a(B) = p_0 + p_2B + p_4B + \cdots + p_{2r}B^r$, $c(B) = p_1 + p_3B + \cdots + p_{(2r-1)}B^r$, $E_T = E_{(2t)}$ and $Z_T = X_{T-1}$. Note that in equation (33) E_T is orthogonal to Y_{T-2h}, X_{T-2k}, Z_{T-2k}, $h > 0$, all k. Equation (33) transforms our temporal aggregation problem into an omitted-variable problem. Indeed, the misspecified relation is obtained from the true relation (33) by omitting Z_T.

If $p(L) = p_0$, then $c(L) = 0$ and the aggregate equation is $Y_T = p_0X_T + E_T$. Therefore, if the true relation is static the aggregate relation is static. Conversely, if $p(L)$ is not a constant and the odd coefficients of $p(L)$ are not all zero, then $c(B) \neq 0$, so that by Proposition 1 a misspecification error arises and by Proposition 2 the aggregate relation is not invariant with respect to the model of Z_T and X_T.

Let us assume that the Wold representation of x_t is

$$x_t = [1/b(L)]v_t$$

, where $b(L) = 1 + b_1 L + \cdots + b_{(2s)} L^{2s}$.[2] It follows that

$$b(L)X_t = v_t + v_{t-1}$$

and

$$\begin{pmatrix} \nu(B) & \mu(B) \\ \mu(B) & \nu(B)B \end{pmatrix} \begin{pmatrix} Z_T \\ X_T \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ B & 1 \end{pmatrix} \begin{pmatrix} v_T \\ V_T \end{pmatrix},$$

where $\mu(B) = 1 + b_2 B + \cdots + b_{(2s)} B^s$, $\nu(B) = b_1 + b_3 B + \cdots + b_{2s-1} B^{s-1}$, $v_T = v_{(2t)}$, $V_T = v_{(2t-1)}$. Note that the residuals v_T and V_T are orthogonal to X_{T-2h} and Z_{T-2h} for $h > 0$.

Representation (5)-(6) for Z_T and X_T is

$$\begin{pmatrix} \frac{2-b_1}{q}\mu - \frac{1-b_1+B}{q}\nu & -\frac{1-b_1+B}{q}\mu + \frac{2-b_1}{q}B\nu \\ [1-(1-b_1)B]\nu - b_1\mu & [1-(1-b_1)B]\mu - b_1 B\nu \end{pmatrix} \begin{pmatrix} Z_T \\ X_T \end{pmatrix} = (1-B) \begin{pmatrix} U_{ZT} \\ U_{XT} \end{pmatrix}, \tag{34}$$

where $q = 1 + (1-b_1)^2$ and $U_{ZT} = -V_T/q - (1-b_1)v_T/q$.

As Tiao and Wei (1976) point out, the causation structure of model (31) is destroyed upon temporal aggregation. From (34) and Proposition 4 it follows that if $c(L)$ does not vanish within the unit circle then Y_T does not Granger cause X_T if and only if $[1-(1-b_1)B]\nu(B) = b_1\mu(B)$. This condition imposes restrictions devoid of economic meaning on the parameters $b_1, \ldots, b_{2s}$. The only interesting case is $b_h = 0$ for $h > 0$, i.e. x_t is white noise. In this case Y_T does not Granger cause X_T.

The dynamic structure of equation (31) is completely modified. The aggregate equation is not in general a FDL. Indeed, by Proposition 8 the relation linking Y_T and X_T is an unrestricted ARMAX unless

$$\frac{2-b_1}{q}\mu(B) - \frac{1-b_1+B}{q}\nu(B) = \frac{(1-B)c(B)}{p_1}. \tag{35}$$

Note that a necessary condition for (35) is $\mu(1) = \nu(1)$, i.e. $1 + b_2 + \cdots + b_{(2s)} = b_1 + b_3 + \cdots + b_{(2s-1)}$. If x_t is white noise the latter condition is not satisfied.

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[2] The case of a general ARMA can be treated in a similar way.

CONCLUDING REMARKS

When a relevant explanatory process is omitted, the resulting misspecified equation does not preserve in general the dynamic shape and the exogeneity properties of the original relation. The parameters of the misspecified relation depend on the parameters of the model linking the omitted variable Z_t and the explanatory variable X_t, so that the misspecified relation is not invariant with respect to policy interventions affecting the latter model. Unidirectional causation is lost, unless Z_t does not Granger cause X_t. If the dependent variable Y_t does not depend on X_t in the "true" relation, it depends on X_t in the misspecified relation, unless Z_t and X_t are orthogonal at all leads and lags. Lags of the dependent variable occur in general in the misspecified relation even though such lags do not occur in the true relation. If the true relation is static, the misspecified equation is dynamic, unless the model of Z_t conditional on X_t is static.

These results apply to misspecification arising from measurement errors, non-linearities, unobserved components, aggregation over agents, systematic sampling and temporal aggregation. Such kinds of misspecification destroy exogeneity and produce relations with a complex dynamic shape. Since estimated macroeonomic relations are likely to be affected by measurement errors, non-linearity, imperfect aggregation over agents and temporal aggregation, their dynamic properties are unlikely to reflect the underlying economic behaviour. Moreover, misspecification must be regarded as a major source of dynamics for macroeconomic relations.

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