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**Political exchange and the allocation of surplus:
a model of two-party competition**

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POLITICAL EXCHANGE AND THE ALLOCATION OF SURPLUS:
A MODEL OF TWO-PARTY COMPETITION

(M.Grillo - M.Polo)

Abstract

We discuss a simple model of political competition which is explicitly grounded on economic analysis of the political exchange. We argue that an act of exchange always gives rise to a common surplus on which both "parties" to the exchange are entitled to make a claim. We investigate exchange and competition in a political setting from this abstract perspective. A party's platform specifies the amount of surplus which will be distributed to society (thus stating, by subtraction, the surplus appropriated by parties) and the way in which it will be distributed among society's agents.

In the simplest case of two-party competition with linear platforms, a unique solution for the game is found which is the only subgame perfect Nash equilibrium that strictly Pareto-dominates every other equilibrium. In this equilibrium we obtain a result which is weaker than the standard median voter's: parties' platforms are not undifferentiated to voters' eyes - they are ideologically identifiable -, though they appear indifferent to the median voter. Moreover a positive share of surplus will in equilibrium be appropriated by parties. Finally, both the degree of ideological characterization of parties and the share of the surplus distributed to society are increasing in the slope of the function that maps shares of votes into shares of "power". When this slope goes to infinity the surplus is entirely distributed to society, but parties - though not necessarily voters - are indifferent among all platforms.

POLITICAL EXCHANGE AND THE ALLOCATION OF SURPLUS:

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"Cuanto más que saliendo yo desnudo,
como salgo, no es menester otra señal
para dar a entender que he gobernado
como un ángel. ¹
M. de Cervantes, Don Quijote, part II,
cap.LIIII.

1. Introduction and summary

In this paper we present and discuss a simple model of political competition which is explicitly grounded on economic analysis of the political exchange (i.e., the exchange between voters and representatives, or parties). We argue that an act of exchange always gives rise to a common surplus on which both "parties" to the exchange (are entitled to) make a claim. Thus there is a "price" - inherent in every act of exchange - to which a specific distribution of the common surplus is associated.

We investigate exchange and competition in a political setting from this abstract perspective. Therefore we depart from the standard Hotelling-Downs' tradition where "prices" are ignored and restore a two-stage approach (in the first stage, parties choose a "location"; in the second stage, they define a "price") which is more in line with the original paper by

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Hotelling.

In our model voters choose parties on the basis of their platforms. A platform specifies the amount of surplus which will be distributed to society (thus stating, by subtraction, the surplus appropriated by parties) and the way in which it will be distributed among society's agents.

The analysis is confined to the simplest case of two-party competition with linear platforms. We find a unique solution for the game which is the only subgame perfect Nash equilibrium that strictly Pareto-dominates every other equilibrium. In this equilibrium we obtain a result which is weaker than the standard median voter's: parties' platforms are not undifferentiated to voters' eyes - they are ideologically identifiable -, though they appear indifferent to the median voter. Moreover a positive share of surplus will in equilibrium be appropriated by parties. Finally, both the degree of ideological characterization of parties and the share of the surplus distributed to society are increasing in the slope of the function that maps shares of votes into shares of "power". When this slope goes to infinity the surplus is entirely distributed to society, but parties - though not necessarily voters - are indifferent among all platforms.

The paper is organized as follows. Sections 2 and 3 discuss the basic concepts of exchange and competition, and confront their significance in an economic market and in a political setting. The general discussion derives inspiration from an early

contribution by G.Stigler (1972). The formal model is presented in section 4 and results are analytically derived in section 5. Section 6 concludes with some comments.

2. Political competition and economic competition

The concept of party competition has been developed in the literature of public choice by building on the basic model of spatial competition due to H.Hotelling (1929). In Hotelling's model two firms compete for customers by choosing an appropriate location on a road along which customers are distributed. By elaborating on an intuition put forward by the same Hotelling, A.Downs (1957) modelled political competition as a movement of (two) parties' platforms toward voters which, in analogy with Hotelling's model, are supposed to be distributed along a scale of preferences.

Provided that parties seek to maximize votes per se (in a sense which will be made clearer), the focal result in the public choice literature on political competition is the so-called "median voter" theorem: both parties will be located "in the middle of the road", more precisely, the equilibrium platforms of both parties will be the same and coincide with the platform most preferred by the median voter.

G.Stigler (1972) has challenged the view that convergence to the median voter's position can in any sense be interpreted as a result of "competition". He argues that it will pay even "a

rational single party (a tyrant) which seeks to maximize the emoluments of office [not to] defy the majority wish" so that a median voter result should be expected to obtain even in a non competitive context. This is because "if the single party does not seek the most popular policy...[it] reduce[s] the amount of [its] return ([because of] more self-defensive costs)".

The essence of Stigler's critique is that the economic theory of democracy has not in fact made full use of the concept of competition as it has been developed in economic analysis. In the market for a good, the role of competition is to eliminate "unnecessary returns" to producers. In the same way, Stigler suggests, in a political setting the role of competition is to eliminate "unnecessary returns" to parties.

We argue that the failure to examine political competition within the proper role it has in the economic market originates from the partial use that the Downsian tradition makes of Hotelling's model of spatial competition. In Hotelling's firms compete not only by choosing a location, but also by fixing a "price" for the good they sell. However there is no formal role for this latter variable in Downs' model.

To stress this lack of concern for a "price" is not a case for pedantry. What goes lost is indeed a thorough comprehension of the abstract concept of "exchange", i.e. the basic concept that the economic analysis of politics admittedly borrows from economics.

An "exchange" always has a twofold dimension:

- it is an instance of cooperation for mutual advantage of agents who exploit the benefits of social division of labour;

- it involves an inherent conflict of interest among parties on how to split the social surplus generated by the division of labour. By assuming a specific value within the interval between the reservation utility of the buyer and the cost to the seller (where the length of the interval gives the total amount of surplus) the price at which a good is bought and sold in an economic exchange always represents a specific solution to this inherent conflict of interest.

In what follows we shall argue: (i) that a "political" exchange (i.e., the exchange between a voter and a representative, or a party), as well generates a social surplus; (ii) that, more to the point, every political exchange has a price which represents a specific solution to a distributive problem. Without a "price", in fact, not only is a genuine conflict of interest missed in the Hotelling-Downs' tradition, but moreover the model lacks a precise perspective from where to evaluate the common "surplus" that springs out from the act of political exchange.

In the following sections we discuss a stylized model of two-party competition, where the generation of a social surplus from the act of political exchange is explicitly described and in which the role of competition is evaluated in terms of the division of the surplus (in Stigler's words, the conditions for the elimination of "unnecessary" appropriation of the surplus by

parties are investigated).

3. Exchange and competition in a political market.

This section gives an informal presentation of the model, which will formally be analyzed in section 4. Let us assume a society where a public good problem (e.g., defense) is to be solved: a surplus can be generated if a collective decision (on how to produce the public good and how to redistribute the surplus obtained) is taken.

However, suppose that to summon every society's agent to a "communal" assembly intended to solve the public good problem requires a waste of resources greater than the surplus which can be obtained. Since transaction costs are too high, the good will not be produced. Now let us suppose that some agents in society "specialize" in political activity. They will organize in parties ² and seek delegation by other agents in order to represent them and their interests in a "Parliament". In an abstract perspective, a Parliament, being far smaller than the "communal" assembly, involves lower transaction costs. Hence a surplus (equal to the difference between the value of the public good and the costs of the representative mechanism) will be obtained, as a result of the social division of labour.

Delegation is conferred through votes. Each competing party will propose a "platform" that specifies the way in which the surplus ³ will be distributed among society's agents. A platform

is characterized by two elements, representing the fact that distribution of the surplus involves two different aspects which deserve attention.

First, we assume that parties, because of their being one of the "parties" to an act of exchange, will make a claim on the surplus. In an abstract perspective there is no difference between a baker (a society's agent delegated to produce bread on the account of other agents, according to the principle of social division of labour) and a political party (delegated to "produce" a collective decision from which a surplus originates). As the baker tries to sell its bread at the highest price, so will a political party.

In political science literature a party's claim on the surplus is usually tackled by having recourse to ethical or legal considerations. Most circumstances in which appropriation of the surplus is sought by political representatives are described as "corruption", a case which, if it can judicially be identified and detected, is prosecuted by law. However, in an economist's perspective, competition has historically been introduced and analyzed as a social mechanism intended to reduce the "demand for morality", by producing the same results which are expected to obtain from the application of moral norms ⁴. This is the point of Stigler's critique.

Thus we will assume that the first element of a party's platform specifies a division of the total surplus according to a share y , $0 \leq y \leq 1$, to be distributed among society's agents, and a

share $(1-y)$ to be appropriated by the party. Therefore, what we ask of a theory of political competition is whether and under what conditions the share y converges to 1.

Secondly, we assume society's agents to be heterogeneous so that they can be ordered according to an index t . The second element of a party's platform specifies how y is distributed among agents according to a function $f(t)$ which gives the amount of surplus assigned to every agent of type t . In the model we also assume that every voter is assigned a non-negative amount of surplus by every platform. This can be justified on the ground that the existence of a Parliament as a place where to take collective decisions is the consequence of a "constitutional" rule, and that constitutional rules require unanimity.

Voters behaviour is easily described. Each voter will vote for the party whose platform assigns to him a greater amount of surplus, while all the votes coming from voters to whom parties' platforms assign the same amount of surplus will be equally split between the parties.

Now, the basic assumption of the model is concerned with the question: what do parties maximize? We argue that parties do not maximize votes per se, but that votes are only a means to get power, and power is needed to get command over a share of the social surplus. Loosely interpreted this view is undoubtedly implicit in most public choice literature. If interpreted in a narrow sense it can instead be contested by arguing that it reduces the richness of the general model, since "immaterial"

benefits also accrue to a party in power, which are psychologically relevant and may not strictly depend on the amount of the social surplus appropriated. Our view is that, while psychological considerations can be introduced in a party's objective function, this model is specifically intended to start an explicit investigation on the genuine conflict of interest which is inherent in the act of political exchange. Therefore, we will restrict to this aspect, by assuming that the amount of surplus appropriated is in fact all that really matters to a party, that is the maximand of its objective function.

However, a second question arises: how will the surplus be appropriated by a party? In tackling this question this paper again derives inspiration from G. Stigler (1972, p.98) where it is argued that "it is not useful to characterize the outcome of a political rivalry as failure (-1) or success (+1) for a party: in an important sense, political outcomes range continuously from failure to success". Hence, it will be assumed that the surplus appropriated by a party i will be the total amount of surplus $(1-y_i)$ claimed in its platform only if it obtains 100% of votes. In every other circumstance the surplus claimed in the winner's platform will be shared between the parties according to some rule which depends on the share of votes obtained by each party. In other words we are assuming that even the party who loses the election is entitled to a share of the surplus, though the total amount from which this share is taken is determined by the winner's platform. This assumption intends to catch the idea that

minority has a role in the political process, even after the majority has won the election, and deserves some comments.

Some of the arguments that can be put forward in support of this hypothesis are empirical in nature. G. Stigler (1972, p.99) argues: "...all political systems contain some element of division of power so a minority will hold a share of minor offices which responds to its relative size" and gives examples. However the fact that such institutional arrangements are observed still lacks a strong theoretical explanation.

A possible direction of analysis is to connect the role of minority to the so-called "Millian" view ⁵ according to which elections are seen not only as a means to choose a government, but primarily as an instrument to signal general preferences and opinions. Even after an election has been won, the winner's political platform still gives only a vague idea of which decisions are to be taken in specific situations, and how they are to be implemented. In the abstract, there will be a large set of specific solutions to the collective decision problem, all consistent with the general principles stated in the winner's platform.

Minority will in general not be indifferent among those solutions. Moreover it usually has the power, within the limits set by the constitutional rules, to impose costs upon the majority in enforcing its policies, and these costs can be particularly high against policies that the minority abhors ⁶. Now the point is simply that in a world of uncertainty, majority

has the best information on the preferences of its constituencies since it is able to monitor them continuously. The same cannot be said as far as the preferences of minority's constituencies are concerned. However, majority has an interest in getting informed on these latter preferences in order to be able to maximize (minimize the costs of) political consensus even from minority positions. This leaves room for an after-elections bargain.

We argue that, once the election has been won, the minority party has basically an informative role which is institutionally performed in Parliament's debates. This informative role can give a theoretical explanation for minority's claim on part of the producer's surplus. Moreover, since the value of information depends on the size of minority, it seems natural to connect the amount of the surplus claimed by minority to its size.

By stressing the informative role of minority we have been looking for an explanation which is in a sense exogenous to the logic of the model. However, there might also be "endogenous" reasons on which this model itself may throw some light: as will be shown, by accepting a sharing rule ex-post, parties can ex-ante maximize the expected amount of surplus. More on this will be said in the concluding section.

Therefore we will assume in the paper that the surplus, which according to the winner's platform is not to be distributed to society's agents, will be shared among all parties according to a proportion which depends on the distribution of votes obtained.

Before entering into analytical details we sketch the general

structure of the model. We decided to keep the flavor of the original Hotelling's paper by assuming that the two elements in a party's platform have different temporal dimensions. The parameters of the function $f(t)$ that describe the distribution of the surplus among society's agents reflect a sort of ideological commitment and are long-run in character. They play a role similar to that of the location variable in standard spatial competition models. On the other hand, the share y of the surplus to be "globally" distributed to society is similar to a "price" variable. This is to our view the missed variable in a theory of political competition based on the economic analysis of political exchange. As with the "price" in Hotelling's we assume that y is more short-run in character.

Thus the model we study is two-stage. In the first stage parties compete on "ideology" and in the second stage they compete on "prices", by treating ideology as not modifiable in the short run. As usually we look for Selten subgame-perfect Nash equilibria of the semi-extensive form of the game. It means that equilibrium values of the ideological variables are chosen in the first stage by correctly anticipating optimal (i.e. Nash equilibrium) strategies in the second stage.

4. The Model

In this section we set up a very simple model coherent with the assumptions discussed above: since most of the features of

our analysis have been extensively discussed in the previous section, the following pages should be intended as a formal rephrasing of the previous informal discussion.

We consider the provision of a public good which generates a surplus (net of production costs) equal to 1. There is a continuum of voters uniformly distributed according to an index $t \in [0,1]$ which represents the individual type; it can be thought of as referred to income, geographical location or any other variable. Voters' types are observable.

Each party i competes by designing a platform $P_i(y_i, f_i(t))$ where y_i , $0 \leq y_i \leq 1$, is the share of surplus to be distributed to society, and $f_i(t)$ is a "density" function that specifies the amount $f_i(t)dt$ of surplus which will be assigned to each subset dt of voters.

We consider a class of simple political platforms, with $f_i(t)$ linear in t : a typical electoral program is

$$(1) \quad P_i(y_i, f_i(t)) = a_i + b_i t \quad t \in [0,1]$$

such that

$$(2) \quad \int_0^1 (a_i + b_i t) dt = y_i \quad \text{with } y_i \in [0,1]$$

and

$$(3) \quad P_i(\cdot) \geq 0 \quad \forall t \in [0,1]$$

Solving (2) and substituting back in (1) we obtain

$$(4) \quad P_1(y_1, b_1, t) = y_1 - b_1/2 + b_1 t$$

The constraint (3) implies that, as discussed in the previous section, all the feasible platforms distribute non negative surplus to any voter. Hence we must have that $y_1 \geq b_1/2$ and $y_1 \geq -b_1/2$, where the former corresponds to $P_1(t=0) \geq 0$ and the latter to $P_1(t=1) \geq 0$. Which of the two constraints is relevant depends upon the sign of b_1 : if $b_1 > 0$, $P_1(t=0) \geq 0$ can eventually bind, while if $b_1 < 0$, $P_1(t=1) \geq 0$ is to be considered. In summary, the constraint (3) can be written as

$$(5) \quad y_1 \geq |b_1/2|$$

Hence, (5) sets a lower bound to the amount of surplus distributed to the voters, given the chosen b_1 .

We consider an electoral competition with two parties.

The utility of a voter of type t is assumed to ultimately depend upon the amount of surplus that he will in fact receive when the public good is supplied according to the results of the election. Two elements determine how the surplus is actually distributed after the election: which party has the majority of votes and whether the policy which will actually be implemented reflects in some sense the results of the election and is therefore a sort of weighted combination of the two original platforms.

However, the key point is that, for any expectation on the election outcome and any rule that determines ex-post policies, it is optimal for a voter of type t to select from among the original platforms that one which gives him the higher surplus. Such a behaviour shares some feature of a (trembling hand) perfect equilibrium. Consider the case of a finite number of voters: each voter has a strictly positive probability of being the casting voter for at least some deviations from the equilibrium behaviour of the remaining voters; hence, a perfect equilibrium requires that each voter selects the platform which gives him the higher payment. Moreover, if the policy actually implemented is a weighted combination of the electoral platforms, it is convenient for the voter to increase the weight of the party which gives him the higher payment. When each voter is of measure zero, as in our case, the suggested choice remains optimal, although in a weak sense.

Turning to the outcome of the electoral competition, define $v_i \in [0,1]$ as a percentage of votes, so that v_i corresponds to the electoral result of party i . Let us label the parties so that $b_2 \geq b_1$. If $b_2 = b_1$ a platform designed according to (4) implies that

$$(6) \quad v_1 = \begin{cases} 1 & \text{if } y_1 > y_2 \\ 1/2 & \text{if } y_1 = y_2 \\ 0 & \text{if } y_1 < y_2 \end{cases}$$

If $b_2 > b_1$ there exists a voter \hat{t} which is indifferent between

the two platforms i.e.

$$(7) \quad \hat{t} : y_1 - b_1/2 + b_1\hat{t} = y_2 - b_2/2 + b_2\hat{t}$$

Solving explicitly

$$(8) \quad \hat{t} = 1/2 + (y_1 - y_2)/(b_2 - b_1)$$

An internal solution for \hat{t} ($\hat{t} \in [0,1]$) implies that

$$(9) \quad \begin{aligned} y_1 - b_1/2 &> y_2 - b_2/2 \\ y_1 + b_1/2 &> y_2 + b_2/2 \end{aligned}$$

In this case

$$(10) \quad v_1 = \hat{t} \quad ; \quad v_2 = 1 - \hat{t}$$

Now we turn to the payoff to the political parties: each party is assumed to maximize the share of surplus that it receives. This amount of surplus depends upon two elements: which is the winning platform and which are the rules that determine how the surplus appropriated according to the winner's platform is shared between the two parties given the election's results. We can imagine many different institutional rules, from the "winner takes all" to a distribution of the surplus claimed by the parties in proportion of the votes received by each. To represent this variety of rules we define a class of functions $S(v) \in \Sigma : [0,1] \rightarrow [0,1]$ with the following properties:

(A1) $S(0)=0$; $S(1/2)=1/2$; $S(1)=1$; $S(v) \in (0,1/2)$ if $v \in (0,1/2)$;
 $S(v) \in (1/2,1)$ if $v \in (1/2,1)$; $|S(v) - v| = |S(1-v) - (1-v)|$

(A2) S' and S'' are defined everywhere for $v \in [0,1]$

(A3) $S' > 0$ for $v \in (0,1)$; $S'' \geq 0$ if $v < 1/2$; $S'' \leq 0$ if $v \geq 1/2$

We define Smooth Majority Premium (SMP) any institutional rule which satisfies A1-A3. Intuitively a SMP does not bias the "focal" results $v = \{0, 1/2, 1\}$ while it may overweight the votes of the winner and symmetrically underweight the votes of the loser. However, such distortions are smooth and no discontinuity occurs. In figure 1 some examples are represented.

It should be noticed that the crucial feature of the class of functions $S(v) \in \Sigma$ is their shape at $v=1/2$, i.e. $S'(1/2)$: for $S'(1/2)=1$ we have $S(v)=v$, a system in which each party is weighted according to the votes received, while for higher values of $S'(1/2)$ the winner receives an increasingly higher (smooth) premium. In the limit for $S'(1/2) \rightarrow \infty$ we approach the "winner takes all" case.

[figure 1 about here]

If the institutional rules are consistent with a SMP the payoff functions to the two parties are

$$\begin{aligned}
 (11) \quad & \pi_1 = (1-y_1)S(v_1) & ; & \quad \pi_2 = (1-y_1)(S(v_2)) & \quad \text{if } v_1 > v_2 \\
 & \pi_1 = \max(1-y_1, 1-y_2)/2 & ; & \quad \pi_2 = \max(1-y_1, 1-y_2)/2 & \quad \text{if } v_1 = v_2 \\
 & \pi_1 = (1-y_2)S(v_1) & ; & \quad \pi_2 = (1-y_2)(S(v_2)) & \quad \text{if } v_1 < v_2
 \end{aligned}$$

Hence, the parties share the amount of surplus claimed by the winner's platform according to the institutional rule $S(\cdot)$. It must be noticed that the function π_1 in (11) is defined for $y_1 \in [|b_1/2|, 1]$, since any electoral platform cannot distribute negative surplus to any voter. Now we can analyze the equilibrium in the electoral game, that will be discussed in the next section.

5. The equilibrium in the electoral game

We first consider the equilibrium platforms in an electoral game in which a SMP holds; other institutional rules different from a SMP will be considered later. Parties are assumed to decide at first the way in which they distribute the surplus among voters, i.e. the parameter b_1 , and secondly the share of surplus that is distributed to the voters, y_1 . This multistage sequence has been discussed in section 3 relying the two variables to long term - i.e. ideology - and short run - i.e. "price" - political competition.

We begin by solving the last stage game, in which y_1 is to be set, considering the two cases $b_1=b_2$ and $b_1 < b_2$.

i) Case $b_1=b_2$

From (6), the distribution of votes is discontinuous, with a discrete increase when $y_1 > y_2$. The following Lemma establishes the features of the equilibrium.

PROPOSITION 1: If $b_1 = b_2$ there exists a unique symmetric Nash equilibrium $y_1 = y_2 = 1$ for any $S(v) \in \Sigma$.

The proof of Proposition 1 is trivial once noticed that, given (6) and (11), for any $S(v) \in \Sigma$ we have a Bertrand game: since it is always convenient to set $y_1 = y_2 + \epsilon$ for $y_2 \in [0, 1)$ and to set $y_1 = y_2$ if $y_2 = 1$, the result follows. In the perspective of a median voter theorem, the above Proposition makes explicit an important implication of the "median voter" result: when the two parties offer the same electoral platform, they distribute completely the surplus obtained through the production of the public good. The complementary case of $b_1 < b_2$ offers a very different picture of short run political competition.

ii) Case $b_1 < b_2$

When $b_1 < b_2$ the payoff of the political parties is obtained by substituting (10) in (11): notice that $v_1 = v_2 \iff t = 1/2 \iff y_1 = y_2$. Define the best reply function of party i as

$$y_i = R_i(y_j) \quad i, j = 1, 2 \quad i \neq j$$

and consider the function:

$$(12) \quad \hat{y}_i(y_j) = \arg \max (1-y_i)S(v_i) \quad i,j=1,2 \quad i \neq j$$

In (12) it is defined the maximizer of the function $(1-y_i)S(v_i)$, which corresponds to party i 's payoff only if $y_i \geq y_j$. Hence, the maximizer in (12) is the maximizer of the party's payoff if and only if $\hat{y}_i(y_j) > y_j$. The following Lemma allows to characterize the best reply functions of the parties in this game.

LEMMA 1: $R_i(y_j) = \max \{ |b_i/2|, \hat{y}_i(y_j), y_j \}$

PROOF: Party i 's strategy space is given by $y_i \in [|b_i/2|, 1]$, since the platform cannot distribute negative surplus to any voter, as discussed in (5). The payoff, given A1-A3, (5), (10) and (11), is always single peaked; for sufficiently high values of y_j it is increasing in y_i for $y_i < y_j$ and strictly concave for $y_i \geq y_j$. For a given b_i define y_j^p such that $\hat{y}_i(y_j^p) = b_i/2$ and y_j' such that $\hat{y}_i(y_j') = y_j'$. For $y_j > y_j'$ $\pi_i(\cdot)$ is decreasing (to the right) at $y_i = y_j$ since $\hat{y}_i(y_j) < y_j$, and the maximum payoff occurs at $y_i = y_j$. Notice that as y_j decreases $\hat{y}_i(y_j) - y_j$ decreases as well, and is equal to zero for $y_j = y_j'$. For $y_j \in [y_j^p, y_j']$ the maximizer $\hat{y}_i(y_j)$ is also the party's payoff maximizer since $\hat{y}_i(y_j) > y_j$, and $\hat{y}_i(y_j)$ is the optimal choice; finally, for $y_j < y_j^p$ the maximizer $\hat{y}_i(y_j)$ violates the constraint of non negative surplus in (5), and the optimal choice is therefore $y_i = |b_i/2|$. The three cases are shown in figure 2.b.

Q.E.D.

[figure 2 about here]

The following Proposition establishes the existence and characterizes the equilibrium in the last stage game.

PROPOSITION 2: For any $S(v) \in \Sigma$ there exists a continuum of symmetric Nash equilibria $y_1 = y_2 = y^*$ with

$$(13) \quad y^* \in [\max \{ \max (|b_1/2|, |b_2/2|), 1 - (b_2 - b_1) / 2S'(1/2) \}, 1]$$

PROOF: First of all it should be noticed that no asymmetric equilibrium can exist since, if $y_1 < y_2$, for any y_2 it is always optimal to set $y_1 \geq y_2$. Consider a candidate equilibrium in which both parties' reaction functions have the form $R_i(y_j) = \hat{y}_i(y_j)$: we are in the region $[|b_1/2|, y_1'] [|b_2/2|, y_2']$, with y_1' as defined in Lemma 1's proof. The equilibrium pair (y_1, y_2) must respect

$$-S(\hat{t}(y_1, y_2)) + (1 - y_1)S' \delta \hat{t} / \delta y_1 = 0$$

$$-(1 - S(\hat{t}(y_1, y_2))) - (1 - y_2)S' \delta \hat{t} / \delta y_2 = 0$$

Solving for a symmetric equilibrium ($S(\hat{t}) = S(1/2) = 1/2$) given (8) we obtain

$$(14) \quad y_1 = y_2 = 1 - (b_2 - b_1) / 2S'(1/2)$$

Since $\hat{y}_i(y_j)$ are contractions, (14) is the unique solution in the region $[|b_1/2|, y_1'] [|b_2/2|, y_2']$. Moreover, global concavity of π_1 ensures that the second order conditions hold. Notice that

(14) corresponds to (y_1', y_2') . Hence, for higher values of y_j , $R_1(y_j) = y_j$. We conclude that any symmetric pair

$$1 \geq y_1 = y_2 = y^* \geq 1 - (b_2 - b_1) / 2S'(1/2)$$

can be an equilibrium, as shown in figure 3.a. Finally, we have to analyze the equilibrium for those pairs (b_1, b_2) at which the lower bound of the equilibrium values, (14), does not respect the constraint of non negative surplus in (5). Suppose (b_1, b_2) are such that (5) is binding for party i given (14), i.e. $1 - (b_2 - b_1) / 2S'(1/2) = |b_i/2|$. It follows from Lemma 1 that if party i is constrained, the equilibrium level of (y_1, y_2) is given by this party's constraint: consequently, $y_2 = y_1 = |b_i/2|$, as can be observed in figured 3.b.

Q.E.D.

[figure 3 about here]

We can notice that the lower bound of the equilibrium values of y_i increases as $S'(1/2)$ moves from its lower bound $S'(1/2)=1$ to higher values. As $S'(1/2) \rightarrow \infty$ the equilibrium interval collapses to a single point $y_1=y_2=1$, and all the surplus is distributed to the voters. For bounded values of $S'(1/2)$ we have a high multiplicity of equilibria. However, they all can be pairwise ranked according to a Pareto criterion: hence, we will consider as the focal result of the last stage game the lowest

equilibrium value of y , i.e. $\max \{ \max (-b_1/2, b_2/2), 1-(b_2-b_1)/2S'(1/2) \}$, which gives the highest payoff to both parties and is unique. Moreover, notice that if $b_1=b_2$, $y^*=1$, so that the equilibrium obtained in Proposition 2 under the assumption of $b_1 < b_2$ converges continuously to the equilibrium established in Proposition 1 when $b_1=b_2$.

Notice that for any pair b_1, b_2 , $b_1 < b_2$, only symmetric equilibria exist in the last stage game, such that $t=1/2$. Then, it is immediate to establish the following proposition.

PROPOSITION 3: There is a countinuum of subgame perfect equilibria in the electoral game, with (b_1, b_2) defined according to the following expressions:

$$\{(b_1, b_2) \mid b_1 \in [-2, 2S'/(2+S')], b_2 = 2S' + b_1(1+S')\}$$

(15)

$$\{(b_1, b_2) \mid b_1 \in [2S'/(2+S'), 2], b_2 = 2S'/(1+S') + b_1/(1+S')\}$$

where $S' = S'(1/2)$.

PROOF: In general we obtain corner solutions and (b_1, b_2) are determined by the constraints of non negative surplus $P_1(\cdot; t) \geq 0$ $t \in [0, 1]$ and the condition $b_1 \leq b_2$. Notice that the condition $1-(b_2-b_1)/2S'(1/2) \geq -b_1/2$, ($P_1(t=1) \geq 0$), can be rewritten as $b_2 \leq 2S' + b_1(1+S')$, while $1-(b_2-b_1)/2S'(1/2) \geq b_2/2$, ($P_2(t=0) \geq 0$) corresponds to $b_2 \leq 2S'/(1+S') + b_1/(1+S')$. Hence the two constraints and the condition $b_1 \leq b_2$ identify two regions in

(b_1, b_2) space, B_1 and B_2 , respectively: in the interior of B_1 party i is not constrained at the equilibrium defined by (14), while (5) strictly binds at the boundaries of B_1 . In $B_1 \cap B_2$ $\delta\pi_1/\delta b_1 < 0$ and $\delta\pi_2/\delta b_2 > 0$, so that party 1 will set the lowest b_1 consistent with its constraint and party 2 the greatest. B_1 and B_2 do not completely overlap; hence there are two regions in which one party is constrained while the other is not. However, when only party i is constrained, the equilibrium level of (y_1, y_2) is determined uniquely by b_1 and not influenced by b_j : we conclude that in these regions $\delta\pi_j/\delta b_j = 0$. This implies that if only party i is at the boundary of its unconstrained region, it has no other feasible electoral platform with a further variation in b_1 , while party j is indifferent among a large set of platforms with different b_j given b_1 , which ensure the same payoff. Hence, we can restrict our attention to the intersection $B_1 \cap B_2$. Finally, the condition $b_1 \leq b_2$ sets the extremes the regions, $b_1 \geq -2$ and $b_2 \leq 2$, as is shown in figure 4.

Q.E.D.

[figure 4 about here]

REMARK: All the subgame perfect equilibria defined in (15) can be ranked according to the level of payoffs to the two parties: since the iso-payoffs curves have slopes equal to -1 , the highest level of payoffs is obtained at the symmetric equilibrium

$$(16) \quad b_2 = -b_1 = 2S'/(2+S')$$

Evaluating the surplus distributed to the voters at the equilibrium (16) gives

$$(17) \quad y_1 = y_2 = S'/2+S'$$

Now we compare the equilibrium in the electoral game obtained under the assumption of SMP with a different set of institutional rules, that we can define as Lump Sum Majority Premium (LSMP): the distinctive element of this class of rules is the existence of a discrete premium for the party who wins the election, independent of the amount of votes collected. Let α be the premium for the party who has the majority of votes. The parties' payoffs are modified according to the following expressions:

$$\pi_1 = \min[(1-y_1)(S+\alpha); 1-y_1] ; \quad \pi_2 = \max[(1-y_1)(S-\alpha); 0] \quad t > 1/2$$

$$\pi_1 = \pi_2 = (1-y_1)/2 \quad t = 1/2$$

$$\pi_1 = \max[(1-y_2)(S-\alpha); 0] ; \quad \pi_2 = \min[(1-y_2)(S+\alpha); 1-y_2] \quad t < 1/2$$

where $S=S(v_i) \in \Sigma$ $i=1,2$ and $\alpha \in (0,1/2)$. Then, it is easy to establish the following result:

PROPOSITION 4: For any $S \in \Sigma$ and $\alpha \in (0,1/2)$ there exists a unique equilibrium in the last stage of the electoral game, with

$$(18) \quad y_1 = y_2 = 1$$

invariant in (b_1, b_2) .

PROOF: From the definition of the payoff functions in the LSMP case it is immediate that the best reply function is

$$R_1(y_2) = y_2 + \epsilon \quad \forall y_2 \in [0, 1)$$

Hence, the last stage game is a usual Bertrand game whose solution is given by (18), independently of (b_1, b_2) .

Q.E.D.

The choice of the parameter b_1 does not influence the party's payoff. Hence, any pair (b_1, b_2) consistent with the constraints $P_1(\cdot) \geq 0$ is an equilibrium point.

6. Concluding remarks

In the context that we have analyzed the outcome of the electoral competition comes out to be strongly influenced by the institutional rules that determine how the surplus appropriated is shared among the parties as a function of the votes gathered. Two classes of such rules are considered, according to whether a discrete or a smooth majority premium is assigned to the winner. In the former case equilibrium entails distribution of the whole surplus to the voters, while parties are indifferent among all platforms; in the latter case parties offer different electoral platforms and are able to partially appropriate the surplus⁷.

When a smooth majority premium is assigned to the winner the model generates a multiplicity of equilibria. However, they can

be ranked according to a Pareto-dominance criterion in such a way that a unique subgame perfect Pareto superior Nash equilibrium can be singled out. This equilibrium prescribes that the two parties choose opposite platforms with the steepest schedule: hence they are "ideologically" identified and distinct.

The intuition is that, by choosing the steepest platform, each party reduces the number of votes that the rival can obtain by increasing slightly the share of surplus distributed, y_1 . In other words, the maximum payoff is obtained by selecting a distributive parameter (b_1) in the platform in such a way as to reduce the incentive to compete in total surplus (y_1).

We notice that a more pronounced smooth majority premium, as represented by a higher value of $S'(1/2)$, increases the degree of electoral competition and implies a greater distribution of surplus, i.e. a lower payoff to the parties in equilibrium. This remark suggests that the institutional rules and habits expressed by the function $S(\cdot)$ could emerge from the common advantage of both parties in an ex-ante agreement that softens political competition by sufficiently rewarding the loser.

Hence we suggest that the existence of ex-post sharing rules can be theoretically explained not only by the aim of softening the behaviour of minority after the election, when the political platforms are implemented, as suggested in Stigler (1972) and discussed in section 3, but also by the aim of softening the degree of political competition before the election, when political platforms are designed.

NOTES

¹ "Besides, when I go forth naked as I do, there is no other proof needed to show that I have governed like an angel". (Translation: J.Ormsby, Encyclopaedia Britannica, 1952).

² Within the limits of our model there is no reason to distinguish between a "representative" and a "party". A recent economic theory of political parties (see A.Breton and G.Galeotti, 1985, and G.Galeotti and A.Breton, 1986) describes a party as a reputation mechanism intended to solve the incentive problem which arises from the temporal lag between the quid and the quo of the political exchange (a vote today in exchange for a promise to be kept tomorrow).

³ We suppose, for the sake of simplicity, that there is a unique optimal way of producing the public good, common to all parties platforms.

⁴ A.Hirschman, 1977, gives a splendid account of this historical perspective.

⁵ See A.Breton and G.Galeotti , 1985, and G.Galeotti and A.Breton, 1986.

⁶ A general strike, and the toughness of it, is a possible instance.

⁷ Notice that this result does not imply that parties are colluding, and is obtained in a non cooperative setting.

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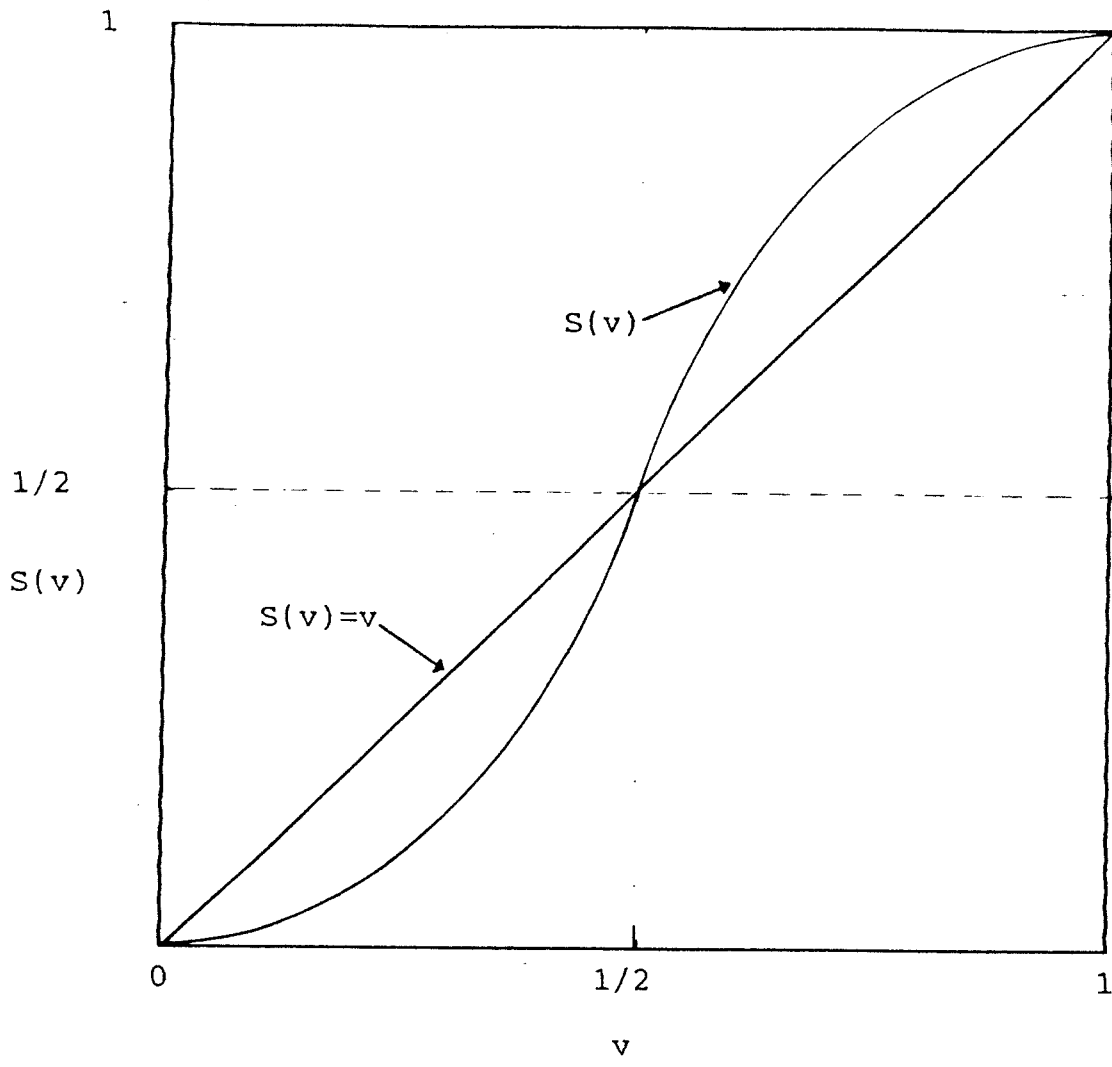


Figure 1 - The sharing rule $S(v)$

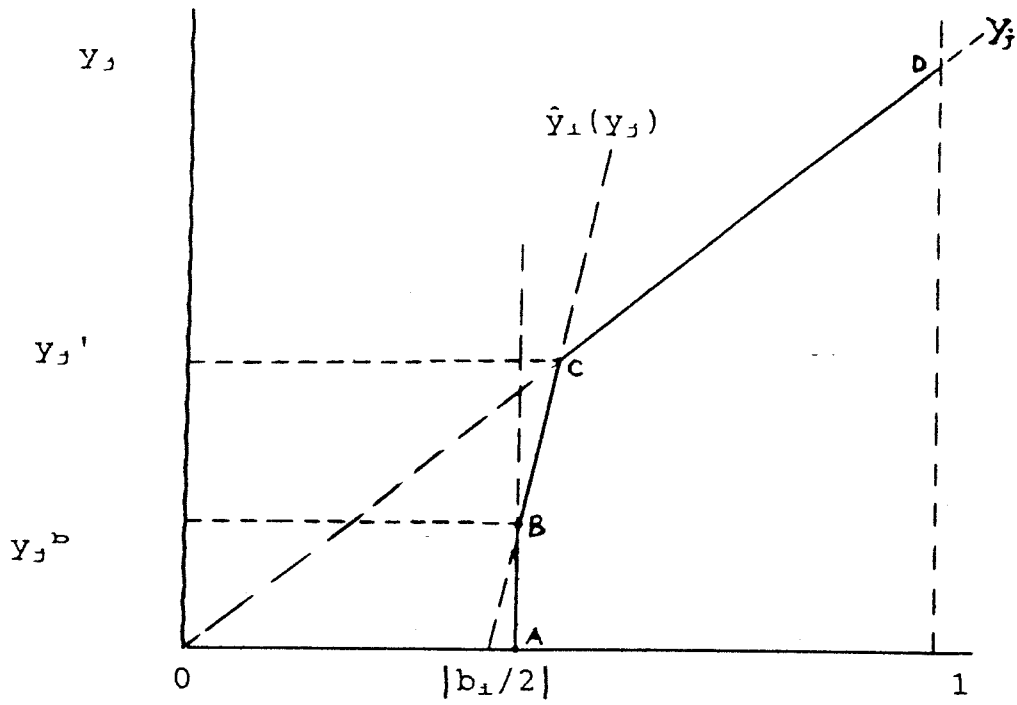


Figure 2.a - Party i's best reply function

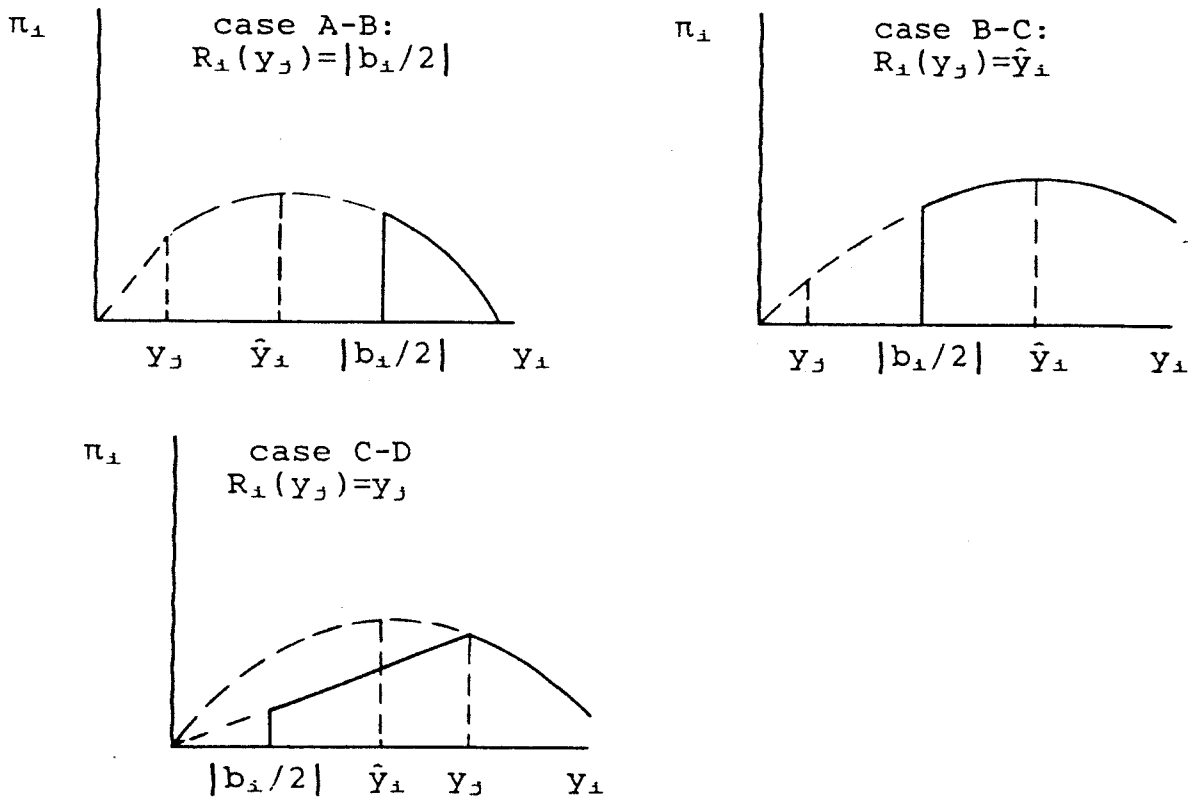


Figure 2.b - Party i's payoff function

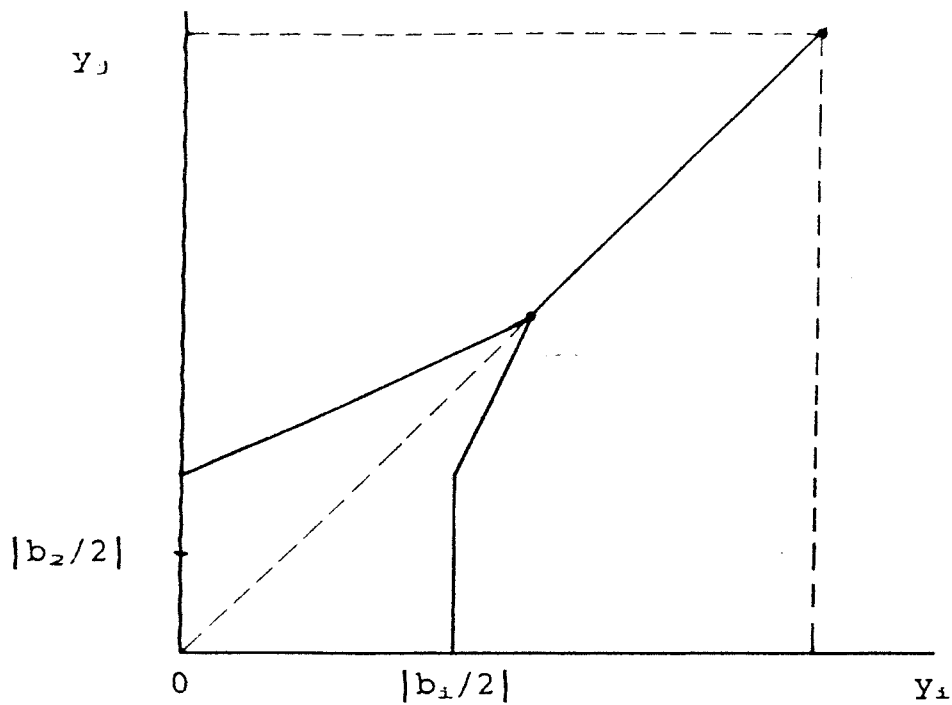


Figure 3.a - Nash equilibria: $y_1=y_2=y^*e[1-(b_2-b_1)/2S'(1/2), 1]$

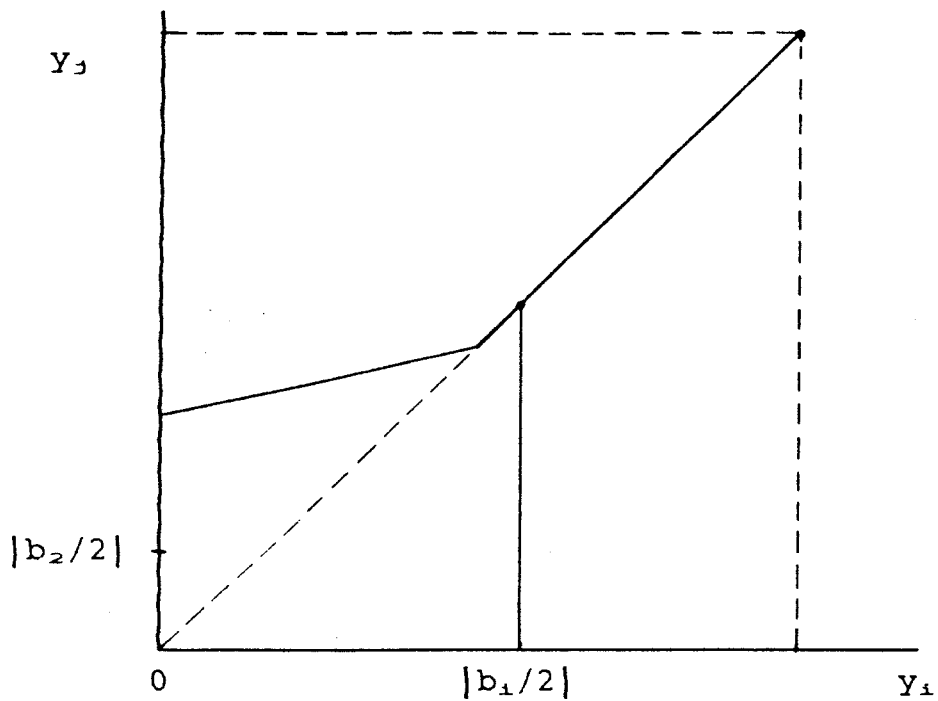


Figure 3.b - Nash equilibria: $y_1=y_2=y^*e[\max(|b_1/2|, |b_2/2|), 1]$

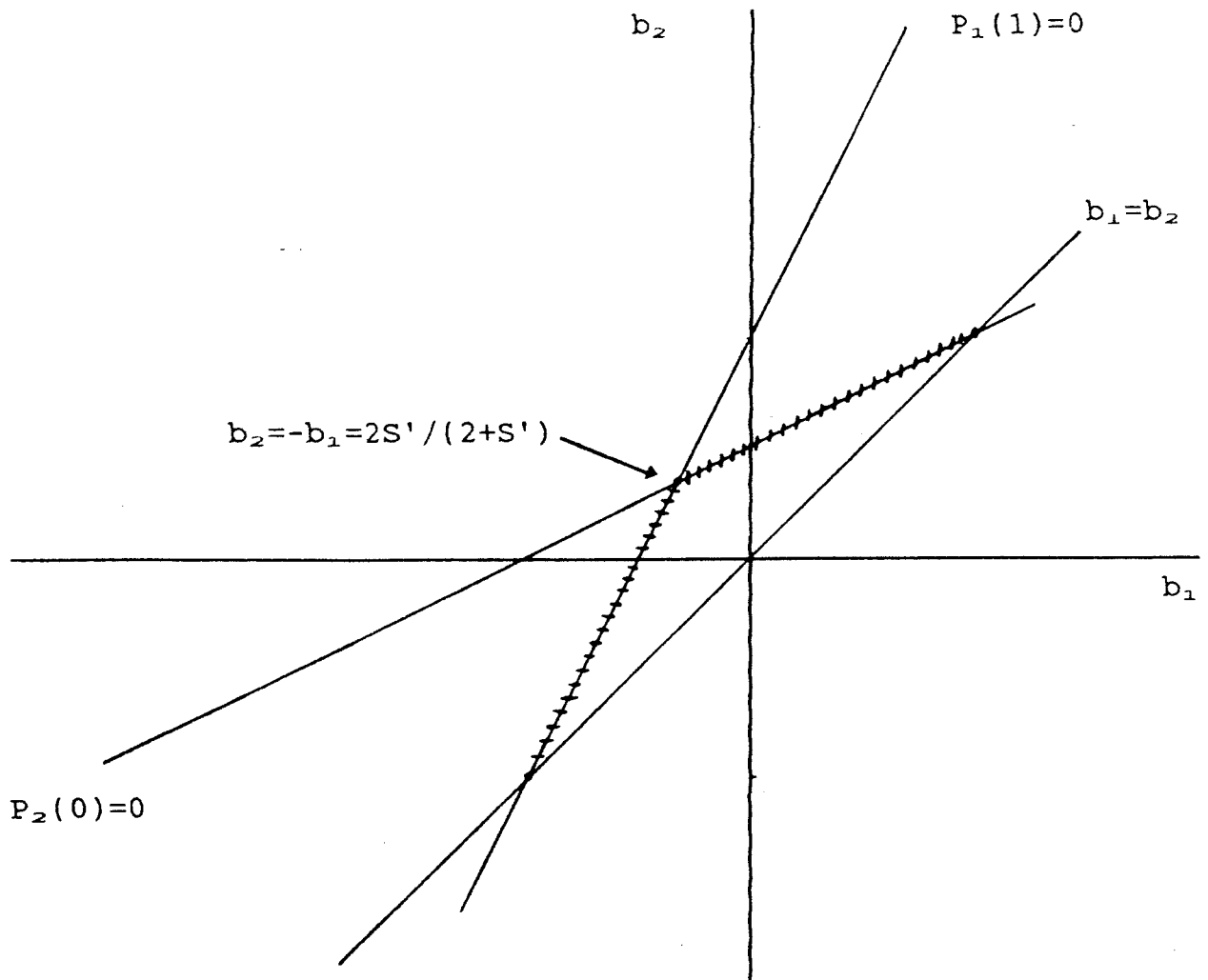


Figure 4: Subgame perfect Nash equilibria

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