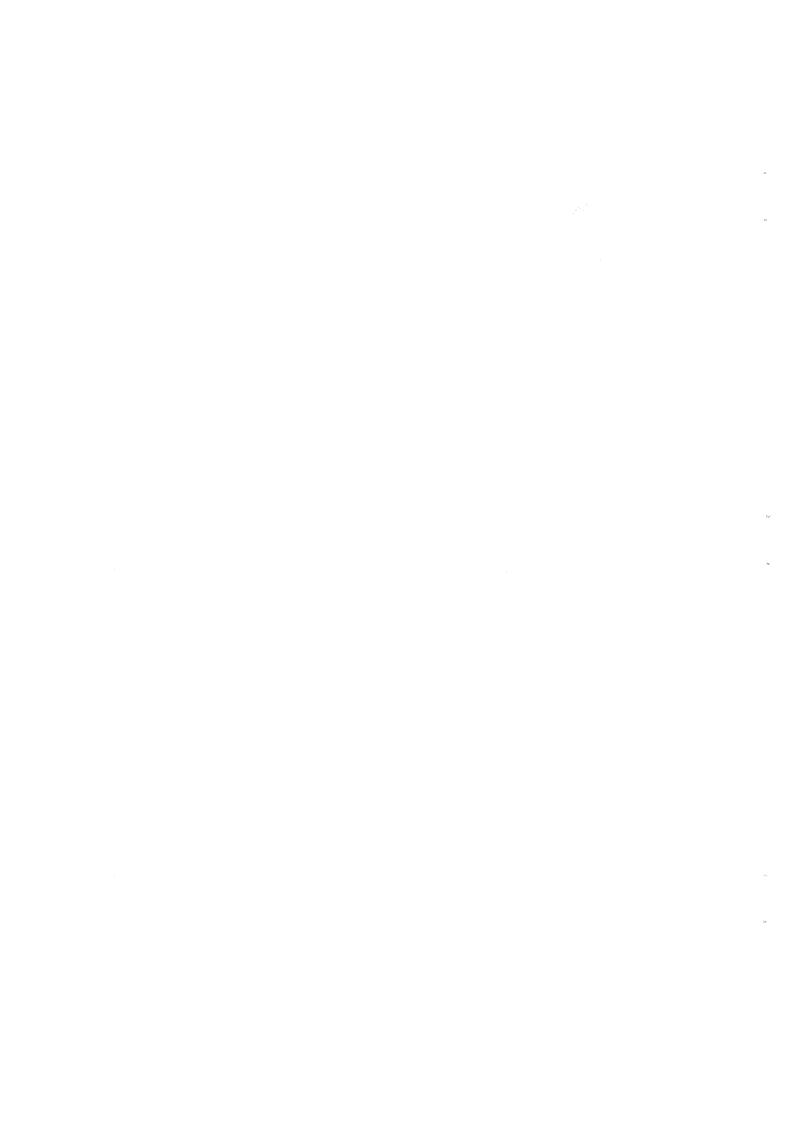\\100\\

Consumption Volatility and Income Presistence
in the Permanent Income Model

di

Mario Forni

Ottobre 1994

Dipartimento di Economia Politica
Viale Berengario, 51
41100 Modena (Italia)

Abstract

Deaton's (1987) "excess smoothness" question can be reformulated by focusing attention on total income rather than labor income: the permanent income theory predicts that the relative volatility of consumption is equal to total income persistence, a fact that is contradicted by empirical evidence. This formulation is more general than the original one in that it is independent of the value of the interest rate, the univariate dynamics of labor income and the information set of the representative consumer. When properly formulated, the excess smoothness problem cannot be solved within Quah's (1990) superior information model; as a consequence, the interest of alternative solutions such as aggregation models is increased.

CONSUMPTION VOLATILITY AND INCOME PERSISTENCE IN THE PERMANENT INCOME MODEL

Introduction

Deaton (1987) argues that if labor income is I(1) the permanent income theory (PIH) in its modern version (Hall 1978) "... fails to predict the fact that consumption is smooth, the very fact that it was invented to explain in the first place" (p. 22). The argument can be summarized as follows. Assume that the univariate Wold representation of labor income change is $\Delta y_t = a(L)u_t$ and that the representative consumer predicts future income by means of past income alone. Then it is a well known result that under the PIH consumption change is $\Delta c_t = a(\beta)u_t$, where $\beta = 1/(1+r)$, r being a constant interest rate. Now, for reasonable values of the interest rate, say 1% per quarter, $a(\beta)$ is very near to $a(1)$, implying that consumption change is approximately equal to $a(1)u_t$, the revision in the long-run expectation of labor income. If labor income is modeled as a trend-stationary process $a(1) = 0$, so that the change in consumption is predicted to be very small. If on the other hand labor income is a difference-stationary process, consumption volatility may be very large. With U.S. data the estimates of income persistence $a(1)$ are typically greater than unity. If $a(1) > 1$ consumption change is predicted to be greater than income innovation, in sharp contrast with empirical evidence (see also Deaton 1992).

The shortcoming of Deaton's argument is that it relies heavily on the assumption that the representative agent makes univariate predictions. As shown in West (1988), if consumers' information set is larger, the implied variance of consumption change is smaller. Hence with superior information we could have a smooth consumption even if labor income is persistent. This point is investigated in Quah (1990). Quah models labor income as the sum of two components, permanent and transitory, and assumes that the representative consumer can distinguish between them. With these assumptions, both the empirical variance of consumption and the empirical shape of a difference-stationary labor income can be matched. The central message is that excess smoothness is not really a problem for the PIH, but only for a very particular version of the PIH, where the representative consumer makes univariate predictions. In the general case,

the volatility of consumption predicted by the theory "is not determined in any essential way by the magnitude of long-run persistence" (p.460). The latter point is dealt with in detail also in Christiano and Eichenbaum (1989): Quah's conclusion that consumption volatility is not related to labor income persistence strongly corroborates the authors' argument that economists should not care too much about the unit root issue.

In the present paper I argue that Deaton's paradox can be formulated in such a way as to avoid Quah's criticism, simply by focusing the attention on total disposable income rather than labor income. As is well known, two important implications of the PIH are serial independence of consumption changes (Hall 1978) and cointegration of total income and consumption (Campbell 1987). It is easily seen that these properties imply that the ratio of the variance of consumption change to the variance of total income change is equal to Cochrane persistence of total income (Cochrane 1988). Similarly, the ratio of the standard deviation of consumption change to that of the univariate innovation of total income is equal to Beveridge and Nelson persistence of total income (Beveridge and Nelson 1981). Hence consumption volatility is linked to total income persistence by a precise relation. This relation holds true irrespective of which particular value the interest rate takes on, not only in the limit for r approaching zero, as is the case if labor income instead of total income is considered. More important, it holds true irrespective of the information set of the representative consumer, so that Quah's superior information model cannot solve the problem.

Notice that in the formulation proposed here Deaton's paradox takes on a somewhat different meaning since total income, unlike labor income, is endogenous in the PIH model. Hence saying that consumption is predicted to be too volatile *if* total income persistence is *assumed* high would make little sense. We have to say instead that the representative agent PIH model cannot produce both the observed volatility of consumption and the estimated persistence of income. Either consumption is smooth or income is persistent: we cannot have both things together.

Some evidence on consumption volatility and total income persistence is provided in Section 3, since the existing estimates do not concern the relevant variables. It is questionable if the predicted equality of income persistence and relative volatility of consumption can be rejected by data; for instance, Christiano and Eichenbaum (1989) argue that persistence estimates are not reliable. I do not address this question here. Of course, I cannot agree with the other argument by Christiano and Eichenbaum (1989) that persistence is theoretically irrelevant in consumption theory.

The fact that Deaton's paradox, when properly formulated, cannot be solved within Quah's model increases the interest of alternative explanations

2

for excess smoothness, and particularly aggregation models like those of Galí (1990), Clarida (1991), Pisckhe (1991), Forni and Lippi (1994).

The exposition is organized as follows. Section 1 presents the basic model. In Section 2 the main result is derived. Section 3 is devoted to empirical analysis. Section 4 concludes.

1. The model

Let us begin by illustrating the main features of Hall's (1978) model in its standard formulation. An infinitely lived representative consumer maximizes at time $t = 0$ the expected utility of consumption

$$E_0 \sum_{t=0}^{\infty} \theta^t u(c_t), \qquad (1)$$

subject to the sequence of budget constraints

$$A_{t+1} = (1 + r)A_t + y_t - c_t. \qquad (2)$$

In equations (1) and (2) r is the interest rate, which is assumed constant; θ is an individual utility discount factor; c_t is consumption; A_t is assets; y_t is labor income, which is exogenous; E_0 is expectation at time $t = 0$ conditional on the information set I_0. We do not make specific assumptions on I_t other than I_t includes at least y_{t-k}, $k \geq 0$. We assume that Δy_t is stationary and normally distributed.

As is well known, the first order condition for this problem is $\theta E_{t-1} u'(c_t) = \beta u'(c_{t-1})$, where $\beta = (1 + r)^{-1}$. To further simplify matters, let us assume quadratic utility, so that marginal utility is linear; moreover, let us assume that the 'impatience' parameter θ is equal to the market discount factor β. Under these hypotheses, the above equation becomes

$$E_{t-1} c_t = c_{t-1}. \qquad (3)$$

This martingale property is the best known result of Hall's model. Let us rephrase it in the following way.

Property 1 *(excess sensitivity). Consumption change is independent of the variables in the information set I_{t-1}; in particular,*
1a. Δc_t is independent of Δc_{t-k}, $k > 0$;
1b. Δc_t is independent of Δy_{t-k}, $k > 0$.[1]

---

[1] The above properties have been tested by many authors, starting with Hall (1978) and Flavin (1981). As is well known, the overall result of this literature is that both (1a) and (1b) are rejected by empirical data; this finding has become known as 'excess sensitivity' of consumption.

A further important implication of Hall's theory is shown in Campbell (1987): if Δy_t is stationary, as we have assumed, then consumption and total income, defined as the sum of labor and capital income, are cointegrated with the cointegrating vector (1 -1). The cointegration result can be obtained in the following way. Solving forward the budget constraint (2) gives

$$A_t = \frac{\beta}{1 - \beta F} c_t - \frac{\beta}{1 - \beta F} y_t, \tag{4}$$

where F is the forward operator. Taking expectation at time t on both sides and noting that the martingale property of consumption implies $E_t c_{t+k} = c_t$, $k \geq 0$, we get

$$c_t = r \left[A_t + E_t \frac{\beta}{1 - \beta F} y_t \right]. \tag{5}$$

Equation (5) clearly illustrates how modern permanent income theory reinterprets Friedman's original intuition. The expression in square brackets on the right-hand side is total wealth, which includes both assets and human wealth, defined as the present value of the expected labor income stream. Consumption equals permanent income, defined as the flow of rental income from total wealth.

Now let us take the first differences of (4), multiply by F and apply expectations at time t on both sides. By recalling that expectation at time t of future consumption changes is zero, we get

$$s_t = -E_t \frac{\beta F}{1 - \beta F} \Delta y_t, \tag{6}$$

where $s_t = \Delta A_{t+1}$ is saving. Equation (6) is Campbell's 'saving for a rainy day' equation: savings anticipate future income falls. An important implication of (6) is that if the change in labor income is stationary, as we have assumed, then saving is the conditional expectation of a stationary variable and therefore is stationary. Now define total income x_t as the sum of capital income and labor income, i.e.

$$x_t = r A_t + y_t.$$

Hence

$$x_t = c_t + s_t, \tag{7}$$

as it is immediately seen from the budget constraint (2). Since consumption is I(1) total income is also I(1), so that stationarity of saving implies that

Property 2 *(cointegration). x_t and c_t are cointegrated and the cointegrating vector is* (1 -1).

4

2. Excess smoothness

Now let us come to the main point of the present paper, that is the relation linking consumption volatility and income persistence. Equation (7) decomposes x_t into a random walk and a stationary component. In such a case the variance of the random walk component is equal to the spectral density of Δx_t at zero frequency; in symbols

$$\text{var}(\Delta c_t) = S_{\Delta x}(0), \tag{8}$$

where $S_{\Delta x}(\lambda)$, $-\pi \le \lambda < \pi$ is the spectral density of Δx_t. To obtain the above equation simply take the first differences of (7), equate the spectra and notice that the spectrum of Δs_t and the cross spectra of Δc_t and Δs_t vanish at zero frequency because of stationarity of savings.

Notice that Properties 1a and 2 are sufficient for equation (8). By Property 1a consumption change is a white-noise process, so that its variance is equal to its spectral density at zero frequency; Property 2 implies that consumption and total income changes have the same spectral density at zero frequency; hence the variance of consumption change is equal to the spectral density of total income change evaluated at zero frequency.

Dividing both sides of (8) by the variance of Δx_t gives the following result.

Property 3 *(smoothness-persistence relation). The ratio of the variance of consumption change to the variance of total income change is equal to the standardized spectral density of total income change evaluated at zero frequency, that is Cochrane persistence of total income (Cochrane 1988). In symbols,*

$$\frac{\text{var}(\Delta c_t)}{\text{var}(\Delta x_t)} = \frac{S_{\Delta x}(0)}{\text{var}(\Delta x_t)} = 1 - 2\sum_{1}^{\infty} \rho_k,$$

where $S_{\Delta x}(\lambda)$, $-\pi \le \lambda < \pi$, is the spectral density of Δx_t and ρ_k is the k-th order autocorrelation of Δx_t.

An obvious alternative formulation of Property 3 is as follows.

Property 3'. *Assume that $\Delta x_t = b(L)e_t$ is the (univariate) Wold representation of Δx_t. Then*

$$\sqrt{\frac{\text{var}(\Delta c_t)}{\text{var}(e_t)}} = b(1);$$

i.e. the ratio of the standard deviation of consumption change to the fundamental univariate innovation of total income is equal to Beveridge and Nelson persistence of total income (Beveridge and Nelson 1981).

5

Although they are a simple consequence of the PIH, Properties 3 and 3' have been completely ignored in the excess smoothness literature.[2] They say that under the PIH either consumption is smooth or total income is persistent: we cannot have both things together.

The following Section provides empirical evidence on consumption volatility and total income persistence; before moving to empirical analysis, however, let us make two observations. Firstly, Properties 3 and 3' hold true whatever the value of the interest rate, not only in the limit for the interest rate approaching zero, as would be the case if labor income rather than total income were considered. Secondly, the smoothness-persistence relation is completely independent of the information set of the representative agent, so that it holds true even in Quah's superior information model, as should be clear by observing that Properties 1a and 2 do not depend on information and are retained in Quah's model.

3. Empirical estimates

Available empirical results concerning U.S. income persistence are not very appropriate for the aim of this paper, primarily because they concern variables such as labor income or GNP, or because data are expressed in logs. Hence it is useful to perform new persistence estimates concerning disposable income. In this Section I essentially reiterate the exercise of Campbell and Mankiw (1987) by using the same data set employed in Forni and Lippi (1994). The source is NIPA, the frequency is quarterly (at quarterly rates), the period is 1947:1 1991:4. Data are seasonally adjusted, taken in per-capita terms and expressed in thousands of 1987 U.S. dollars. The variables considered are total disposable income (x_t) and total consumption expenditures (c_t).[3] It must be recognized that total consumption expenditures is not the most appropriate series to use in consumption theory since for durable goods it is the service flows rather than purchases that should be included in the aggregate. Hence as in Forni and

---

[2] Properties 3 and 3' are not unknown: they are clearly stated in Cochrane and Sbordone (1988) and Cochrane (1994). However their application to the excess smoothness question has not been noticed. In the papers quoted above Property 3 is assumed and used to obtain an estimate of GNP persistence. Not surprisingly, very low persistence estimates are obtained. In Cochrane (1990) a Section is devoted to excess smoothness, but the original labor income formulation is maintained.

[3] Following Blinder and Deaton (1985), we made some adjustments to NIPA definitions. Firstly, the 1975:2 tax rebate is removed from the income data; the numbers for this correction are taken from Blinder (1981), table 2. Secondly, personal 'nontax' payments to state and local governments are added to both consumption and income. Lastly, interest paid by consumers to business is subtracted from disposable income.

Lippi (1994) I also consider an alternative measure of total consumption (c'_t) that includes an estimate of the imputed rent on the stock of consumer durables, clothing and shoes.[4] In relation with c'_t I use an estimate of disposable income, x'_t, which is the sum of x_t and net interests on durables, clothing and shoes. For the sake of comparison I also consider an estimate of labor income (y_t).[5]

Tab. 1: Beveridge and Nelson persistence of total income

$ARMA(p,q)$	$b(1)$	$s(\Delta c_t)/s(e_t)$	$b'(1)$	$s(\Delta c'_t)/s(e'_t)$	$a(1)$
(0,1)	1.11 (.08)	.73	1.12 (.08)	.44	1.12(.08)
(0,2)	1.04 (.11)	.73	1.05 (.11)	.44	1.11(.10)
(0,3)	1.21 (.17)	.73	1.22 (.17)	.44	1.24(.16)
(1,0)	1.10 (.09)	.73	1.11 (.09)	.44	1.13(.10)
(1,1)	1.10 (.04)	.74	1.10 (.04)	.45	1.09(.05)
(1,2)	1.07 (.09)	.74	1.07 (.09)	.45	1.12(.09)
(1,3)	1.12 (.13)	.74	1.13 (.13)	.45	1.16(.12)
(2,0)	1.05 (.11)	.73	1.06 (.11)	.44	1.14(.13)
(2,1)	1.08 (.09)	.74	1.09 (.09)	.45	1.13(.10)
(2,2)	1.09 (.15)	.74	1.10 (.15)	.45	1.19(.17)
(2,3)	1.14 (.18)	.74	1.15 (.17)	.44	1.21(.17)
(3,0)	1.16 (.17)	.74	1.17 (.17)	.44	1.23(.18)
(3,1)	1.14 (.15)	.74	1.15 (.15)	.45	1.22(.17)
(3,2)	1.18 (.13)	.74	1.19 (.13)	.45	1.22(.14)
(3,3)	1.17 (.07)	.75	1.17 (.07)	.45	1.14(.07)
(4,4)	1.13 (.14)	.75	1.14 (.14)	.45	1.17(.15)

$\Delta x_t = b(L)e_t; \ \Delta x'_t = b'(L)e'_t; \ \Delta y_t = a(L)u_t.$

---

[4] The stock of durables is calculated by accumulating the spending flow, starting with the NIPA net stock of consumer durables for the end of 1946, and assuming a depreciation rate of 5 per cent per quarter. In calculating the imputed rent, a user cost is assumed of 6 per cent per quarter. Clothing and shoes are treated in a similar way, but assuming a depreciation rate of 20 per cent, a zero initial value and a user cost of 21 per cent per quarter. Consumption c'_t is the imputed rent on durables, clothing and shoes plus expenditures on services and nondurables other than clothing and shoes.

[5] Labor income y_t is constructed following Pisckhe (1991). Define W = wage and salary disbursements; O = other labor income; I = personal interest income; D = personal dividend income; R = rental income of persons; P = proprietors' income; T = transfer payments to persons; C = personal contributions for social insurance; τ = personal tax and nontax payments; N = personal nontax payments to state and local governments; $q = (W+O)/(W+O+I+D+R)$. Then $y_t = W + O + T - C + N + q(P - \tau)$. The tax rebate of 1975:2 is removed.

Table 1 shows parametric estimates of Beveridge and Nelson persistence for x_t, x'_t and y_t when first differences are modeled as various low-order ARMA processes. Persistence estimates of x_t are compared with the estimates of the ratio of the standard deviation of consumption change Δc_t to that of the innovation of Δx_t; the same is done with $\Delta c'_t$ and $\Delta x'_t$. Persistence estimates of x_t range from 1.04 to 1.21, while the estimates of the consumption-income variance ratio range from .73 to .75. The difference is even larger with x'_t and c'_t: persistence estimates of x'_t range from 1.05 to 1.22 while estimates of $\sqrt{\text{var}(\Delta c'_t)/\text{var}(\Delta x'_t)}$ range from .44 to .45. For all models the estimate of consumption volatility is largely outside the standard confidence interval, particularly when $\Delta c'_t$ is concerned.

Tab. 2: Cochrane persistence of total income

	10	20	window 30	size 40	60	ψ	ψ'
Δx_t	.98 (.27)	.93 (.36)	.96 (.46)	.97 (.53)	.93 (.79)	.52	–
$\Delta x'_t$	1.00 (.27)	.95 (.36)	.97 (.46)	.98 (.53)	.93 (.62)	–	.19
Δy_t	1.22 (.33)	1.24 (.48)	1.26 (.60)	1.26 (.69)	1.13 (.75)	–	–

Table 2 shows Bartlett nonparametric estimates of Cochrane persistence for x, x' and y. These are compared with the ratio of the variance of consumption change to that of income change, reported in the last column; ψ is the ratio $\text{var}(\Delta c_t)/\text{var}(\Delta x_t)$, while $\psi' = \text{var}(\Delta c'_t)/\text{var}(\Delta x'_t)$. Persistence estimates of x range from .93 to .98 while that of x' range from .93 to 1.00. As usual, standard errors are quite large; notice, however, that for x', c' and lag window size 20 the consumption-income variance ratio is still outside the 95% confidence interval.

4. Conclusion

Within the permanent income model of consumption we cannot establish a relation between labor income persistence and consumption volatility unless we make particular assumptions about the form of the univariate dynamics of labor income, the level of the interest rate, and the information set of the representative agent. By contrast, none of these assumptions is required if total income rather than labor income is considered. Stationarity of savings along with the random walk characterization of consumption implies that the relative

volatility of consumption is equal to total income persistence, irrespective of whether labor income is difference-stationary or trend-stationary, whether the interest rate is high or low or whether consumers have superior information.

This result provides a very convenient reformulation of Deaton's (1987) 'excess smoothness' question: in empirical data either consumption is too smooth or total income is too persistent with respect to theoretical predictions. Quah's (1990) superior information model cannot explain Deaton's paradox in this more general version. This failure increses the interest of alternative explanations such as aggregation models like Galí (1990), Clarida (1991), Pisckhe (1991), Forni and Lippi (1994). The fact that consumption volatility is linked to total income persistence partially render void the arguments by Quah (1990) and Christiano and Eichenbaum (1989) about the theoretical significance of persistence measures.

References

Beveridge, S. and Nelson, C.R., 1981, "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics* 7, 151-174.

Blinder, A.S. and Deaton, A., 1985, "The Time Series Consumption Function Revisited," *Brooking Papers on Economic Activity* 2,465-521.

Blinder, A.S., 1981, "Temporary Income Taxes and Consumer Spending," *Journal of Political Economy* 89, 26-53.

Campbell, J., 1987, "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica* 55, 1249-73.

Campbell, J.Y. and Mankiw, N.G., 1987, "Are Output Fluctuations Transitory?" *Quarterly Journal of Economics* 102, 857-80.

Christiano, L.J. and Eichenbaum, M., 1989, "Unit Roots in Real GNP — Do We Know and Do We Care?" Paper presented at the Carnegie Rochester Public Policy Conference.

Clarida, R.H., 1991, "Aggregate Stochastic Implications of the Life-Cycle Hypothesis," *Quarterly Journal of Economics* 106, 851-67.

Cochrane, J.H., 1988, "How Big is the Random Walk in the GNP?," *Journal of Political Economy* 96, 893-920.

Cochrane, J.H., 1990, "Univariate vs. Multivariate Forecasts of GNP Growth Stock Returns," NBER Working Paper 3427.

mobile e l'"appiattimento" delle retribuzioni in una ricerca" pp. 120

75. Enrico Giovannetti [1990] "Crisi e mercato del lavoro in un distretto industriale: il bacino delle ceramiche. Sez. I" pp. 150

76. Enrico Giovannetti [1990] "Crisi e mercato del lavoro in un distretto industriale: il bacino delle ceramiche. Sez. II" pp. 145

77. Antonietta Bassetti e Costanza Torricelli [1990] "Il portafoglio ottimo come soluzione di un gioco bargaining" pp. 15

78. Antonietta Bassetti e Costanza Torricelli [1990] "Una riqualificazione dell'approccio bargaining alla selezioni di portafoglio" pp. 4

79. Mario Forni [1990] "Una nota sull'errore di aggregazione" pp. 6

80. Francesca Bergamini [1991] "Alcune considerazioni sulle soluzioni di un gioco bargaining" pp. 21

81. Michele Grillo e Michele Polo [1991] "Political exchange and the allocation of surplus: a model of two-party competition" pp. 34

82. Gian Paolo Caselli e Gabriele Pastrello [1991] "The 1990 Polish Recession: a Case of Truncated Multiplier Process" pp. 26

83. Gian Paolo Caselli e Gabriele Pastrello [1991] "Polish firms: Pricate Vices Public Virtues" pp. 20

84. Sebastiano Brusco e Sergio Paba [1991] "Connessioni, competenze e capacità concorrenziale nell'industria della Sardegna" pp. 25

85. Claudio Girmaldi, Rony Hamaui, Nicola Rossi [1991] "Non marketable assets and households' portfolio choices: a case study of Italy" pp. 38

86. Giulio Righi, Massimo Baldini, Alessandra Brambilla [1991] "Le misure degli effetti redistributivi delle imposte indirette: confronto tra modelli alternativi" pp. 47

87. Roberto Fanfani, Luca Lanini [1991] "Innovazione e servizi nello sviluppo della meccanizzazione agricola in Italia" pp. 35

88. Antonella Caiumi e Roberto Golinelli [1992] "Stima e applicazioni di un sistema di domanda Almost Ideal per l'economia italiana" pp. 34

89. Maria Cristina Marcuzzo [1992] "La relazione salari-occupazione tra rigiditá reali e rigiditá nominali" pp. 30

90. Mario Biagioli [1992] "Employee financial participation in enterprise results in Italy" pp. 50

91. Mario Biagioli [1992] "Wage structure, relative prices and international competitiveness" pp. 50

92. Paolo Silvestri e Giovanni Solinas [1993] "Abbandoni, esiti e carriera scolastica. Uno studio sugli studenti iscritti alla Facoltá di Economia e Commercio dell'Universitá di Modena nell'anno accademico 1990/91" pp. 30

93. Gian Paolo Caselli e Luca Marinelli [1993] "Italian GNP growth 1890-1992: a unit root or segmented trend representation?" pp. 25

94. Angela Politi [1993] "La rivoluzione fraintesa. I partigiani emiliani tra liberazione e guerra fredda, 1945-1955" pp. 55

95. Alberto Rinaldi [1993] "Lo sviluppo dell'industria metalmeccanica in provincia di Modena: 1945-1990" pp. 70

96. Paolo Emilio Mistrulli [1993] "Debito pubblico, intermediari finanziari e tassi d'interesse: il caso italiano" pp. 30

97. Barbara Pistoresi [1993] "Modelling disaggregate and aggregate labour demand equations. Coin-

tegration analysis of a labour demand function for the Main Sectors of the Italian Economy: 1950-1990" pp. 45

98. Giovanni Bonifati [1993] "Progresso tecnico e accumulazione di conoscenza nella teoria neoclassica della crescita endogena. Una analisi critica del modello di Romer" pp. 50

99. Marcello D'Amato e Barbara Pistoresi [1994]"The Relationship(s) among Wages, Prices, Unemployment and Productivivy in Italy" pp. 30