# Dynamic common factors in large cross-section

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#### Abstract

This paper develops a method to analyse large cross-sections with non trivial time dimension. The method (i) identifies the number of common shocks in a factor analytic model; (ii) estimates the unobserved common dynamic component; (iii) shows how to test for fundamentalness of the common shocks; (iv) quantifies positive and negative comovements at each frequency. We illustrate how the proposed techniques can be used for analysing features of the business cycle and economic growth. KEYWORDS: business cycle, sectoral comovements, factor analysis, principal components.

JEL classification nos.: E32, O30, C51.

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#### 1. Introduction<sup>1</sup>

Many questions about growth and cyclical fluctuations can only be answered by looking at data which contain information on several nations, regions, sectors or firms, i.e. large cross-sections with time dimension large enough so as to be the object of dynamic analysis. Unfortunately, when the number of cross-sectional observations is large, traditional methods such as VAR or VARMA techniques are not appropriate, since the number of parameters to be estimated grows as the square of the cross-sectional dimension. A possible strategy to achieve parameter reduction is to use factor analytic models since, if the number of common factors is fixed, then the number of parameters to estimate grows only linearly with the number of cross-sectional observations. However, a large cross-sectional dimension poses problems for estimation (see, for example, the discussion in Quah and Sargent (1994) and the related comment by Geweke). The aim of this paper is to develop a method for the estimation and identification of a factor analytic model when the number of cross-sectional observations is large.

By exploiting results related to the Laws of Large Numbers, we show how to identify the number of common shocks in the data set and how to estimate them. The basic idea is the following. Since, by assumption, the idiosyncratic components are mutually orthogonal at all leads and lags, whereas the common components are not, if the number of cross-sectional observations is large, the former nearly die out relatively to the latter when averaging across sections. Therefore, the dimension of the common factor space h can be recovered by constructing a vector of aggregates with q > h entries: since the rank of the spectral density matrix of this vector is equal to the number of common factors h, we can determine the latter by computing dynamic principal components as in Brillinger (1981). Moreover, we can estimate the common shocks by constructing h cross-sectional averages, estimating a VAR or a VARMA model and taking the residuals resulting from suitable identifying restrictions. Having recovered the common shocks, the model can be consistently estimated by applying OLS equation by equation. This is a great simplification with respect to what is suggested in the literature.

In order to get the structural parameters, the shocks have to be identified. As in the structural VAR literature, the problem is to find

<sup>&</sup>lt;sup>1</sup> Thanks for helpful comments are due to Renato Flores, Marco Lippi, Marc Watson, and the participants to the ECARE-CEPR conference on empirical macroeconomics.

enough restrictions to select a particular Moving Average respresentation. The task is enormously simplified by assuming that the shocks belong to the space spanned by the present and the past values of the vector of the variables of interest, i.e. the shocks are fundamental. In structural VAR analysis fundamentalness is generally assumed with no discussion. However, as noticed by Lippi and Reichlin (1993), this is arbitrary and can produce misleading results. Here we show that in factor analytic models, unlike in VAR models, fundamentalness can be tested. This provides a further motivation for our framework.

If either fundamentalness is rejected or we are not interested in the structural response functions, we can distinguish the common from the idiosyncratic component in each sector or region by regressing on the present, past and future of the q averages equation by equation. We can then remove the idiosyncratic components and analyse the common ones in order to see whether comovements between regions, nations or sectors are mainly positive, indicating a prevalence of complementarities in economic activity or negative, indicating a prevalence of substitution effects. For this purpose we use an index for the relative weight of negative over positive covariances at all frequencies. This index is constructed from the computation of co-spectra of the common components for all cross-sectional units.

The paper is organized as follows. The next three sections are devoted to identification of the number of common shocks, estimation and test for fundamentalness. In section five we illustrate the proposed methodology on a sample of manufacturing output and hours worked for 450 sectors in the US from 1958 to 1986. The paper ends with a discussion of open questions and possible developments.

#### 2. Specification

Let us begin by assuming a countable infinity of sectors or regions  $i = 1, ..., \infty$ . We specify a dynamic factor analytic model, as for instance in Sargent and Sims (1977), Geweke and Singleton (1981) and, more recently, Quah and Sargent (1994), Forni and Reichlin (1995). Precisely, we assume that the vector of cross-sectional variables  $y_t^i = (y_{1t}^i, y_{2t}^i, \dots, y_{mt}^i)'$  can be written as

$$y_t^i = A^i(L)u_t + \epsilon_t^i, \tag{1}$$

where

$$\epsilon_t^i = (\epsilon_{1t}^i, \epsilon_{2t}^i, \cdots, \epsilon_{mt}^i)'$$

is a vector of sector-specific factors - the idiosyncratic components - possibly autocorrelated but mutually orthogonal at all leads and lags, with variances bounded above by the reals  $\sigma_h$ ;

$$u_t^i = (u_{1t}, u_{2t}, \cdots, u_{qt})'$$

is a vector of unit variance white noises, the common shocks, identical for all sectors and variables, mutually orthogonal and orthogonal to  $\epsilon_t^i$  for all i;  $A^i(L)$  is a  $m \times q$  matrix of rational functions in the lag operator L. All the variables are zero-mean, wide-sense stationary and linearly regular, with rational spectral density matrix.

The methodology proposed here exploit an important property of factor models: due to orthogonality, when aggregating across a large number of sectors the idiosyncratic component vanishes relatively to the common component  $A(L)u_t$  (Granger (1987), Forni and Lippi (1995)).

To better clarify what we mean, let us introduce for each variable h a sequence of real numbers  $\omega_h^i$ ,  $i=1,\ldots\infty$ , such that we can find positive reals  $L_h$  and  $U_h$  fulfilling

$$L_h \leq \omega_h^i \leq U_h$$
.

Now consider a strictly increasing sequence of positive integers  $i_k$ ,  $k = 1, ..., \infty$  and let  $D_n = \{i_1, ..., i_n\}$ . The variance of the aggregate idiosyncratic component

$$\bar{\epsilon}_{ht}^n = \frac{\sum_{i \in D_n} \omega_h^i \epsilon_{ht}^i}{\sum_{i \in D_n} \omega_h^i}$$

is bounded above by  $n^{-1}(U_h^2\sigma_h/L_h^2)$ . Hence  $\lim_{n\to\infty} \bar{\epsilon}_{ht}^n=0$ .

On the other hand, the common components  $y_{ht}^i - \epsilon_{ht}^i$  are not mutually orthogonal, so that in general they will not vanish asymptotically. A positive lower bound for all but a finite number of cross-covariances between the common components is a sufficient (but not necessary) condition for this to be true.<sup>2</sup> It follows that for n large the weighted average

$$\bar{y}_{ht}^n = \frac{\sum_{i \in D_n} \omega_h^i y_{ht}^i}{\sum_{i \in D_n} \omega_h^i}$$

is approximately equal to  $B_h^n(L)u_t$ , where

$$B_h^n(L) = \sum_{i \in D_n} \omega_h^i A_h^i(L) / \sum_{i \in D_n} \omega_h^i$$

Necessary conditions are given in Forni and Lippi (1995).

and  $A_h^i(L)$  is the h-th raw of the matrix  $A^i(L)$ . More precisely, as stated in the following Proposition, the percentage of the total variance explained by the common component is close to unity.

**Proposition 1.** As 
$$n \to \infty$$
,  $\operatorname{var}(B_h^n(L)u_t)/\operatorname{var}(\bar{y}_{ht}^n) \to 1$ .

The above result provide the justification for a method of identification of the number of common shocks in a factor model having large cross-sectional dimension.

Let us consider a data set concerning n sectors. Now take a partition of s subsets  $G_1, G_2, \dots, G_s$ , call  $n_1, n_2, \dots, n_s$  the number of elements in these sets and define the ms vector of aggregates:

$$Z_{t} = \begin{pmatrix} \sum_{i \in G_{1}} y_{t}^{i}/n_{1} \\ \sum_{i \in G_{2}} y_{t}^{i}/n_{2} \\ \vdots \\ \sum_{i \in G_{s}} y_{t}^{i}/n_{s} \end{pmatrix}$$
(2).

If the corresponding vector of idiosyncratic components is zero,  $Z_t$  has a (possibly infinite) moving average representation driven by  $u_t$ , say  $C(L)u_t$ . Hence it will be singular whenever the dimension of  $u_t$ , q, is less than ms; i.e. the spectral density of  $Z_t$ ,  $f_Z(\lambda) = C(e^{-i\lambda})C(e^{i\lambda})'$ , will have reduced rank, equal to q.

But then, to identify the number of common shocks, we can just compute the ms dynamic principal components of  $Z_t$  and check how many we need to capture most of its variance at each frequency.

The proposed procedure consists of the following steps.

**STEP 1** Select randomly l different partitions of the sectors in the data set and compute the corresponding vectors  $Z_t^j$ ,  $j = 1, \dots, l$ .

**STEP 2** For each j, compute the spectral density of  $Z_t^j$ , and decompose it in the following way:

$$f_Z(\lambda) = P(\lambda)D(\lambda)\overline{P(\lambda)}'$$

where  $D(\lambda)$  is a diagonal matrix with eigenvalues

$$[\mu_1(\lambda), \cdots, \mu_{ms}(\lambda)]$$

on the principal diagonal and

$$\operatorname{rank} D(\lambda) = \operatorname{rank} f_Z(\lambda)$$

(see Brillinger 1981).

**STEP 3** Order the  $\mu_k(\lambda)$ 's from the largest to the smallest and compute the ratio between the variance of the sum of the first r principal components and the variance of the sum of all principal components:

$$\frac{\int_0^\pi \sum_{k=1}^r \mu_k(\lambda) d\lambda}{\int_0^\pi \sum_{k=1}^q \mu_k(\lambda) d\lambda}$$
 (3)

for  $r = 1, \dots, ms$ . This gives us the percentage of the trace of the covariance matrix of  $Z_t^j$  accounted for by the first r principal components.

**STEP 4** Set q = r when r is such that the explained variance captures at least 95 % of the total. If r components are sufficient and if this result is robust across the l experiments, conclude that there are q = r common shocks in the data set.

## 3. Estimation and diagnostic checking

Another implication of Proposition 1 is that the common dynamic component of the model can be estimated by simply using sectoral averages. In particular, if the common shock  $u_t$  has dimension q, the common component can be estimated by using q aggregates. This is an estimation procedure much simpler than the one based on the EM algorithm, which has been suggested in this context (e.g. Quah and Sargent 1994).

Notice that in model (1)  $A^i(L)u_t$  is the projection of  $y_t^i$  on the present, past and future of  $u_t$  since, by assumption,  $u_t$  is orthogonal to  $\epsilon_t^i$ . Now take any vector of q averages  $Y_t$  such that the idiosyncratic component has died out. From Section 2, we know that the components of  $Y_t$  span the same linear space than the components of  $u_t$ . Hence, we can recover the common and the idiosyncratic components simply by regressing equation by equation the  $y_{ht}^i$  on the present, past and future of  $Y_t$ .

Let us assume for simplicity that m = q (this assumption will be relaxed below). Our proposed procedure is the following.

#### STEP 1 Set

$$Y_t = [y_{1t}, y_{2t}, \cdots, y_{mt}]'$$

where

$$y_{ht} = \frac{\sum_{i=1}^{n} \omega_h^i y_{ht}^i}{\sum_{i=1}^{n} \omega_h^i}.$$

Then choose the weights  $\omega_h^i$  so as to maximize the chance for the sector-specific component to go to zero. Under the orthogonality assumption, the weights minimizing the variance of the aggregate local component are:

$$\omega_h^i = 1/\sigma_h^i$$

where  $\sigma_h^i$  is the variance of the unobserved idiosyncratic component of sector i for variable h. To compute the  $\sigma_h^i$ 's proceed iteratively.

- (i) As starting value for the estimate of  $\sigma_h^i$  take the sample variance of  $y_{ht}^i$ . Then compute the associated weights and  $Y_t$ .
- (ii) Fix a K and, for each h, perfor the n regressions of  $y_{ht}^i$  on the variables in  $Y_{t-k}$ ,  $k = -K, \ldots, K$ , by OLS so as to obtain a first estimate of  $\sigma_h^i$ . Use the associated weights to get a new regressor  $Y_t$ .
- (iii) Repeat step (ii) until all estimates of  $\sigma_h^i$  converge.

STEP 2 After estimation verify (i) pairwise orthogonality between the local components by means of a Q test; (ii) whether the idiosyncratic component has died out in the sample. This can be done by computing, for each of the aggregates used in the estimation of the common shocks, the ratio of the variance of the local component to that of the aggregate. Under the orthogonality assumption this ratio can be estimated by

$$\sum_{i=1}^{n} \frac{1}{\hat{\sigma}_h^i} / s_h, \tag{4}$$

where  $\hat{\sigma}_h^i$  is the final estimate of  $\sigma_h^i$  and  $s_h$  is the sample variance of  $\sum_{i=1}^n y_{ht}^i/\hat{\sigma}_h^i$ .

Let us now come to the case  $q \neq m$ . In this case, we can still apply the above procedure, but STEP 1 must be modified in such a way to ensure that vector  $Y_t$  has q entries. If q < m, we can simply drop m - q components from  $Y_t$ . If q > m, we must partition the sectors as in Section 2 for some or all of the variables h. The indeterminacy problems arising in both cases will be briefly discussed in the last Section.

The sectoral response functions  $A^i(L)$  cannot be estimated without recovering  $u_t$ . Identification of the common shocks will be discussed in the next Section. However, independently of identification, we can analyse the common component by spectral methods. Notice that the common components reflects both positive and negative comovements; a prevalence of the former over the latter would indicate that the propagation of shocks is mainly through complementarities while the opposite

would indicate a larger importance of substitution effects. In particular, we can estimate which effect prevails at all frequencies and obtain information on short and long-run fluctuations. In a separate paper (Forni and Reichlin 1995) we analyse this issue in detail and propose a measure of substitution defined as the ratio between the sum of the negative values of the co-spectra and the sum of the positive values for different frequencies.

To be more precise, let  $s_{ij}(\lambda)$  be the co-spectrum of sectors i and j for variable h. We can decompose  $s_{ij}(\lambda)$  as:

$$s_{ij}(\lambda) = s_{ij}(\lambda)_{-} + s_{ij}(\lambda)_{+}$$

where

$$s_{ij}(\lambda)_{-} = [s_{ij}(\lambda) - |s_{ij}(\lambda)|]/2$$

and

$$s_{ij}(\lambda)_{+} = [\mid s_{ij}(\lambda) \mid +s_{ij}(\lambda)]/2.$$

A measure of the substitution effect of the common shocks can be defined as the ratio:

$$SUBST(\lambda) = -\frac{\sum s_{ij}(\lambda)_{-}}{\sum s_{ij}(\lambda)_{+}}$$
 (5)

#### 4. Identification

As mentioned, estimation of the sectoral response functions requires identification of the vector of the common shocks  $u_t$ . We propose the following two-steps procedure. The first step is to check whether the common shocks are fundamental, i.e. whether  $u_t$  belongs to the space spanned by the present and the past of  $Y_t$ . In traditional structural VAR analysis this cannot be tested and fundamentalness is just assumed. However, it has been pointed out by Lippi and Reichlin (1993) that this is an arbitrary assumption because, in general, we cannot ensure that  $u_t$  belongs to the larger set spanned by the future as well as the present and the past of  $Y_t$ . We will show here that, unlike in VAR models, in factor models fundamentalness can be tested.

If fundamentalness is not rejected, the second step then consists in estimating a VAR or a VARMA model for  $Y_t$ , obtaining any vector of orthonormal residuals  $v_t$  and identifying  $u_t$  within the set of orthonormal

transformation of  $v_t$  by imposing economically meaningful restrictions. Since this can be done by standard procedures we do not discuss this point in detail here (for a detailed discussion of this issue, see Forni and Reichlin (1995)).

Let us now show how to develop the first step.

From (2), we know that the vector of the q aggregate variables  $Y_t$  can be written to a good approximation as

$$Y_t = D(L)u_t$$

where the h-th raw of D(L) is

$$D_h(L) = \frac{\sum_{i=1}^{n} 1/\sigma_h^i A_h^i(L)}{\sum_{i=1}^{n} 1/\sigma_h^i}$$

The vector white noise  $u_t$  is fundamental if, and only if,  $\det(D(L))$  does not vanish within the unit circle in the complex plane. An equivalent condition is that the past of  $u_t$  spans the same linear space as the past of  $Y_t$  (see Rozanov 1967). Assuming that the equality  $Y_t = D(L)u_t$  holds exactly, we can state the following result.

**Proposition 2.** If  $u_t$  is fundamental for  $Y_t$ , then none of the sectoral variables  $y_{ht}^i$  Granger-causes  $Y_t$ .

Proof. Let us call  $\mathbf{H}_t$  the Hilbert space spanned by all variables in  $Y_{t-k}$ ,  $k \geq 0$ . Fundamentalness implies that  $u_{h,t-k} \in \mathbf{H_{t-1}} \ \forall \ h, k > 0$ . Now, consider the orthogonal projection of  $Y_t$  on  $\mathbf{H}_{t-1}$  and call  $\eta_t$  the vector of residuals. Clearly,  $\eta_t$  is orthogonal to  $u_{h,t-k} \ \forall \ h, k > 0$ . Moreover,  $\eta_t$  is orthogonal to  $\epsilon_{h,t-k}^i \ \forall \ i,h,k$  since it lies in  $\mathbf{H}_t$ . Hence  $\eta_t$  is orthogonal to  $y_{h,t-k}^i \ \forall \ i,h,k > 0$ . Q.E.D.

Therefore, while in VAR or vector ARMA framework fundamentalness of the structural shocks cannot be tested, in the factor model fundamentalness implies the testable implication that the macrovariables used for estimation cannot be Granger caused by any of the sectoral variables.

#### 5. Empirical Application

We now apply the ideas developed in Sections 1 to 4 to a data set containing output and hours worked of 450 manufacturing sectors in the US from 1958 to 1986.<sup>3</sup> As it will be illustrated, the factor analytic model

<sup>&</sup>lt;sup>3</sup> Further information on the data set is given in the Appendix.

can be used to capture essential features of short and long run fluctuations in manufacturing by reducing the parameter space in a data set of 26200 data points.

#### 5.1 Number of common shocks

We reordered sectors by extracting randomly without replacement natural numbers from 1 to 450 to form the sequence  $i_k$ , k = 1, ..., 450. Then we partitioned the sectors in six groups of 75 sectors each by taking  $G_1 = \{i_1, ..., i_{75}\}, ..., G_6 = \{i_{376}, ..., i_{450}\}$ . We repeated the experiment 50 times to get the vectors  $Z_t^j$ , j = 1, ..., 50. Since we have two variables we have twelve aggregates forming the vector  $Z_t^j$ .

Figure 1 reports the estimated ratio (3) for  $r = 1, \dots, 12$  and for all experiments. The spectra were estimated using a Bartlett window with lag window size equal to five. For all experiments, the result is that 2 principal components are sufficient to capture more than .95 % of the total variance. From this we conclude that there are two common shocks to our 450 sectors.

Figure 2 reports the ratios

$$\frac{\sum_{j=1}^{2} \mu_{j}(\lambda)}{\sum_{j=1}^{q} \mu_{j}(\lambda)} \quad \text{and} \quad \frac{\mu_{1}(\lambda)}{\sum_{j=1}^{q} \mu_{j}(\lambda)}$$

at each  $\lambda$  for the 50 experiments.

Observe that the variance explained by the first two principal components is similar across frequencies and that results are robust across experiments. Observe also that, had we modelled our factor model with one common shock only, we would have missed information about the long-run.

#### 5.2 Estimation results

The OLS estimates were performed on the following specification:

$$y_{ht}^{i} = \beta_{h}^{i} Y_{t+1} + \gamma_{h}^{i} Y_{t} + \delta_{h}^{i} Y_{t-1} + \nu_{ht}^{i}$$
 (6)

On this specification we have followed the procedure described in STEP 1 of Section 3.

We computed the substitution index for the panel of sectoral output growth by estimating the co-spectra of the common components of sectoral outputs with Bartlett window size equal to 5.

Figure 3 reports the values of  $SUBST(\lambda)$  for the common component.

**Figure 1:** Variance of  $Z_t$  expalined by the first 12 principal components (l=50 experiments)

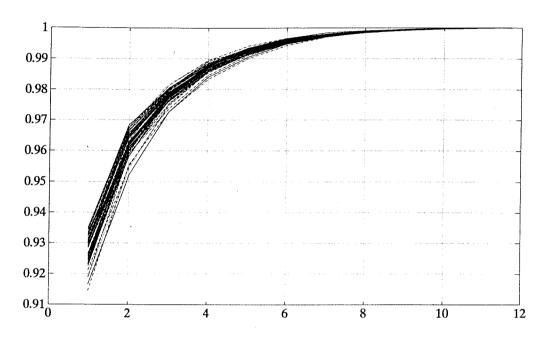


Figure 2: Variance of  $Z_t$  explained by the first two principal components at different frequencies

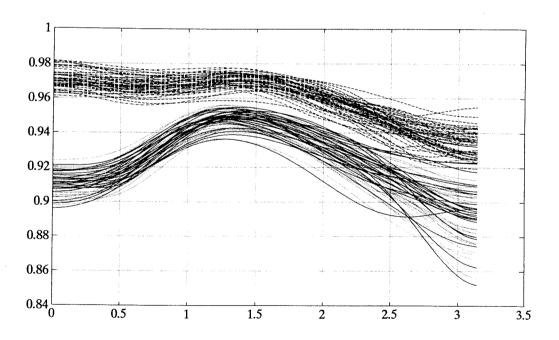


Figure 3 Absolute sum of positive (dashed line) and negative (solid line) co-spectra

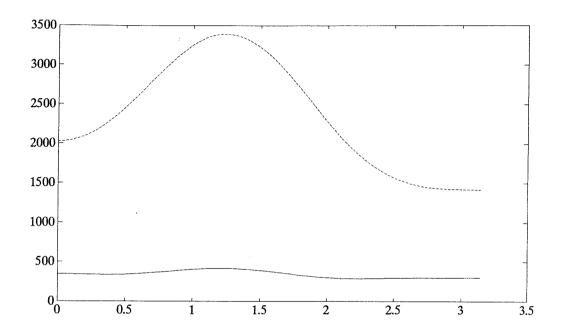


Figure 4 reports positive and negative co-spectra separately.

The Figures illustrate nicely the business cycle features of our data: all the series of the sums of the positive cospectra have peaks at business cycle frequencies, while the series of the negative cospectra are rather flat.

### 5.3 Diagnostic

To verify the orthogonality between the sector-specific components we performed a Q test on pre-whitened residuals from the sectoral regressions (6). For each pair of sectors we computed  $T \sum_{k=-3}^{3} r_k^2$ , where T is the time dimension of the residuals and  $r_k^2$  denotes the sample cross-correlation of  $\epsilon_{ht}^i$  and  $\epsilon_{h,t-k}^i$ . Under the null of pairwise orthogonality, the distribution of the test statistic is  $\chi^2(7)$ . Figure 5 reports both the theoretical and the empirical distributions for output residuals.

From the comparison we conclude that there is no evidence of large cross-correlations between the etimated idiosyncratic components.

To verify how rapidly the variance of the idiosyncratic component goes to zero for increasingly larger aggregates, we performed the following exercise. First, we reordered sectors by extracting randomly without replacement natural numbers from 1 to 450 to form the se-

Figure 4: Substitution index for the common component

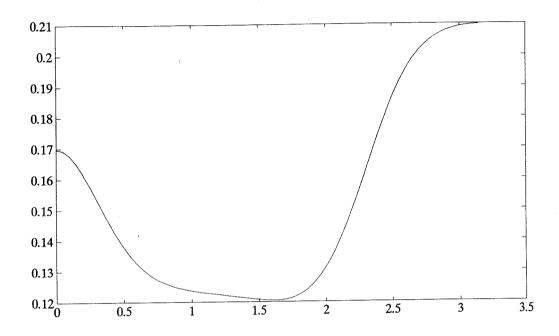
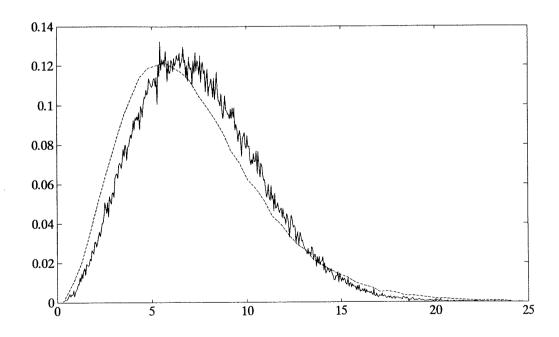
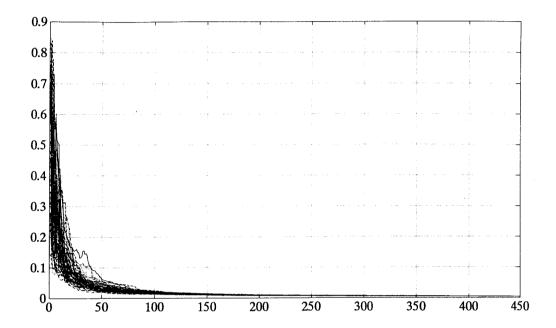


Figure 5: Distribution of the Q-test statistic for residuals of ouput regressions



quence  $i_k$ , k = 1, ..., 450. Second, we computed the ratio (4) for the sets  $\{i_1, ..., i_n\}$ , n = 1, ..., 450. Lastly, we repeated the experiment for 50 different reorderings. Figure 6 illustrates the results for the sample of sectoral output.

**Figure 6:** Ratios of the variance of the idiosyncratic component to the variance of the sub-aggregates - 50 experiments - output data

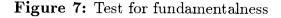


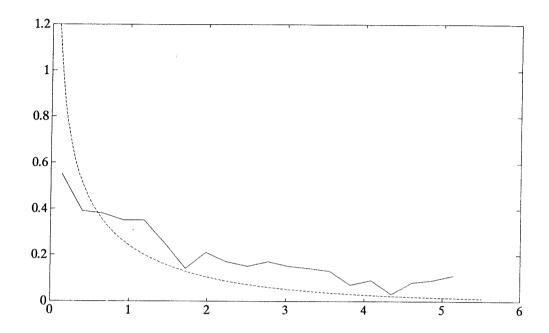
The ratio reached is less than .01 for n=450, which is the cross-sectional dimension of our data set. Notice that n=50 is sufficiently large to get ratios of about .05. By imposing equal weights the same figure is reached with 60-70 sectors; this provides a confirmation for the dimension of  $Z_t$  selected in Section 5.1. Similar results are obtained for hours worked.

## 5.4 Test for fundamentalness

We test for fundamentaness by computing Geweke's measure of causation with one lag (Geweke 1982). Under the null of no causation this statistics multiplied by T-1, where T is the time dimension of  $y_{ht}^i$ , is distributed as  $\chi_{(1)}^2$ . Figure 7 compares the distribution of Geweke's test (solid line) with the theoretical  $\chi_{(1)}^2$  distribution. Fundamentalness is rejected for 95 sectors out of 450 (21 %).

We conclude that fundamentalness is rejected.





### 6. Summary and open questions

The paper has developed a simple method to analyse dynamic factor analytic models in data sets which contain a large number of cross-sectional units. We have shown how to identify the number of common shocks and how to estimate the model by using nothing more than OLS. Moreover, we have developed an index for measuring the importance of negative comovements over positive comovements at all frequencies. A final contribution of the paper is to show formally than in factor analytic models, unlike in VAR models, we can test whether the common shocks are fundamental, i.e. whether they belong to the space spanned by the present and the past of the variables of interest. This is a crucial step for the full identification of the model.

While our method provides a simple strategy for identification of the dimension and estimation of the common dynamic component in factor analytic models, there are at least two problems of indeterminacy in our strategy that should be further analysed.

The first concerns the strategy for the determination of the number of common shocks. Notice that the vector  $Z_t$  can be constructed in a number of different ways and this may lead to different results. In order to differentiate the entries in  $Z_t$ , we have suggested to consider

non-overlapping averages, that is, a partition of the data set. This leads to the question of which partition should be chosen. In particular, we have to decide the number of aggregates s: s should be set as large as possible, since we need sm > q, and q is unknown. However, the larger is s, the smaller are the groups in the partition, and we need large groups in order to wash out the idiosyncratic components. A method for solving this trade-off is needed. The sample considered in our empirical application is very large and the diagnostic of Section 5.3 indicates that for our choice of s = 12, the idiosyncratic component is safely dead (see Figure 6). However, for smaller data sets the trade-off may be more troublesome.

The second problem concerns the estimation of the common component through sectoral averages. Here there are two issues: the choice of the weights in the computation of these averages and the choice of the partition.

As for the former, recall that any weighted average for which the idiosyncratic component goes to zero might be used. In Section 4, we have suggested to chose weights following the criterion of maximizing the chance of the idiosynctaric component going to zero. This is needed when we do not have many cross-sectional observations. However, when the number of sections is huge, other criteria, based on economic considerations, for example, might be chosen.

As for the latter, notice that, if m = q, as in our empirical application, it is natural to do as we have done and consider partitions over all the m variables; in this way we use all the information in the sample. However, if  $m \neq q$ , there is no obvious choice.

In general, in the choice of the aggregates we have two problems: (i) the idiosyncratic component must to go to zero; (ii) we want to chose aggregates for which the vector of shocks pass the test for fundamentalness suggested in Section 4. If the data set is large, problem (i) is easily overcome. Problem (ii), on the other hand, is likely to be more serious and indeed the empirical application shows this to be the case. One possible development of our analysis, which will be the scope of further research, is to device a strategy for searching for those aggregates for which we can accept the fundamentalness hypothesis.

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#### **APPENDIX**

## Data sources and data treatment

The data set used is the Annual Survey of Manufacturers (ASM) which is a survey of manufacturing establishments sampled from those responding to the comprehensive Census of Manufacturers. This database contains information for 4-digit manufacturing industries from 1958 through 1986.

We have used value added data for output and deflated them by the value of shipments.

Logs of sectoral data on output and hours worked were subject to unit root tests. For all data we were not able to reject the null of a unit root (results available on request) at the 5~% level. We then toke the differences and removed the mean.

The electronic computer sector (SIC 357) was found to have a unit root after being detrended by a segmented trend with change in drift in 1972.



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