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di

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A Lagrangean Heuristic for the Prize Collecting Travelling Salesman Problem

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Abstract

In this paper we consider the Prize Collecting Travelling Salesman Problem (PCTSP), that is a variant of the Travelling Salesman Problem (TSP) where a tour visiting each node at most once in a given graph has to be computed, such that a prize is associated with each node and a penalty has to be paid for every unvisited node; moreover, a knapsack constraint guarantees that a sufficiently large prize is collected. We develop a Lagrangean heuristic and obtain an upper bound in the form of a feasible solution starting from a lower bound to the problem recently proposed in the literature. We evaluate these bounds utilizing both randomly generated instances and real ones with very satisfactory results.

1. Introduction and problem definition

The Prize Collecting Travelling Salesman Problem (PCTSP) can be considered as a variant of the Travelling Salesman Problem (TSP), where the Hamiltonian constraint is not anymore required, but a penalty has to be paid for any node left unvisited in the (simple) tour of the salesman that collects prizes at every visited node.

More formally, the PCTSP can be defined as follows. A weighted digraph $G = (V, A)$ is given, where V is the set of nodes of size n and A is the set of arcs of size m. Let us suppose that $V = \{1,2,\dots,n\}$ and that node 1 is the depot or home city of the salesman. A cost c_{ij} is associated with each arc $(i,j) \in A$, and a prize p_i is associated with each node $i \in V$. Moreover, a penalty γ_i to be paid if node i is not visited is given $\forall\, i \in V$. Node 1 is then such that $p_1 = 0$ and $\gamma_1 = +\infty$.

The PCTSP consists in finding a cycle $\chi = (V_\chi, A_\chi)$, $V_\chi \subseteq V$, $A_\chi \subseteq A$, with $1 \in V_\chi$, visiting each node at most once, such that the total cost and penalties to be paid are minimized and the collected prize is not less than a given amount B.

Introducing the binary variables $x_{ij} = 1$ if arc $(i,j) \in A_\chi$ and 0 otherwise, and $y_i = 1$ if node $i \in V_\chi$ and 0 otherwise, the PCTSP can be formulated as an Integer Programming Problem as follows.

$$\min \sum_{i\in V}\sum_{j\in V\setminus i} c_{ij}x_{ij} + \sum_{i\in V}\gamma_i(1-y_i) \tag{1.1}$$

$$\text{s.t.} \quad \sum_{j\in V\setminus i} x_{ij} = y_i \qquad \forall i\in V \tag{1.2}$$

$$\sum_{i\in V\setminus j} x_{ij} = y_j \qquad \forall j\in V \tag{1.3}$$

$$y_1 = 1 \tag{1.4}$$

$$\sum_{i\in V} p_i y_i \ge B \tag{1.5}$$

$$\sum_{i\in S}\sum_{j\in V\setminus S} x_{ij} \ge y_h \qquad \forall h\in V\setminus 1 \text{ and } \forall S\subset V: \{1\}\in S, h\in V\setminus S \tag{1.6}$$

$$x_{ij} \in \{0,1\} \ \forall\, (i,j) \in A \tag{1.7}$$

$$y_i \in \{0,1\} \ \forall\, i \in V \tag{1.8}$$

(1.1) is our objective function expressing the minimization of the costs of the arcs included in the cycle and of the penalties to be paid for the unvisited nodes. (1.2) and (1.3) are the well known assignment constraints and (1.4) forces the depot to be included in the cycle. Constraint (1.5) is a knapsack-like constraint and ensures that the total collected prize is not less than a given value, that can be defined as our "goal". (1.6) forces every node in V_χ to be connected to the depot, and give the so-called Subtour Elimination Constraint. Finally, (1.7) and (1.8) are the constraints on the variables of the problem.

The PCTSP has been introduced by Balas and Martin (1985) as a model for scheduling the daily operations of a steel rolling mill. The same optimization problem has been successively addressed by the same authors Balas and Martin (1991) and more recently by Baccus et al. (1995). A more detailed model for the general steel hot rolling process is given in Cowling (1995), where Tabu Search heuristic techniques are utilized.

Structural properties of the PCTSP related to the TSP polytope and to the knapsack polytope have been presented by Balas (1989, 1995). Bounding procedures, based on different relaxations, have been developed by Fischetti and Toth (1988) and Dell'Amico, Maffioli and Värbrand (1995).

A variant of the PCTSP is given by the *Profitable Tour Problem* (PTP) which is obtained from PCTSP by removing constraints (1.4) and (1.5) and setting $\gamma_1 = 0$. In the PTP we allow the "empty" solution $x_{ij} = 0 \ \forall \ (i,j) \in A$. In practice, one can choose either the solution which visits zero nodes and pays $\sum_{i=2}^{n} \gamma_i$ or a solution which performs a cycle including node 1 and pays a mixture of penalties γ_i and costs c_{ij}. The most profitable cycle is chosen.

Related problems are the *Selective TSP* (Laporte and Martello (1990)) and the *Orienteering Problem* (Golden et al. (1987, 1988)), which are obtained by PCTSP substituting the objective function (1.1) with the maximization of the prizes collected and substituting the knapsack constraint with a bound on the length of the cycle.

In this paper we present a Lagrangean heuristic for solving problem (1.1)-(1.8). In particular, in Section 2 we briefly describe the lower bound for PCTSP presented in Dell'Amico, Maffioli and Värbrand (1995) that gives our starting solution and we give the basic steps for making this starting solution feasible. Our Lagrangean heuristic based on an *Extension and Collapse* procedure is described in Section 3. Section 4 reports on our experimental results based on both random instances and instances of steel rolling mill derived from real applications.

2. Finding feasible solutions

In this section we first briefly describe the lower bound for PCTSP that is used in the present work for generating starting solutions for the heuristic procedures proposed in the following sections. Then we show how we make these starting solutions feasible.

2.1 Our starting solution: a lower bound for PCTSP

The lower bound LB that is used for generating our starting solutions derives from the work presented in Dell'Amico, Maffioli and Värbrand (1995).

The approach used to obtain a lower bound for PCTSP originates from the following more compact formulation of the problem (see Balas (1989)) obtained by eliminating variables y_i, i = 1,...,n, by substituting them with $x_{ii} = 1 - y_i \ \forall \ i \in V$.

$$\min \quad \sum_{i \in V} \sum_{j \in V \setminus i} b_{ij} x_{ij} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \tag{2.2}$$

$$\sum_{i\in V} x_{ij} = 1 \qquad \forall j \in V \tag{2.3}$$

$$\sum_{i\in V} p_i x_{ii} \le \sum_{i\in V} p_i - B \tag{2.4}$$

$$\sum_{i\in S} \sum_{j\in V\setminus S} x_{ij} \ge 1 - x_{hh} \quad \forall h \in V\setminus 1 \text{ and } \forall S \subset V\text{: } \{1\} \in S, h \in V\setminus S \tag{2.5}$$

$$x_{ij} \in \{0,1\} \ \forall\ (i,j) \in A \tag{2.6}$$

where $b_{ij} = c_{ij}$ if $i \neq j$, otherwise $b_{ij} = +\infty$ if $i = j = 1$ and $b_{ij} = \gamma_i$ if $i = j \neq 1$.

Note that the new variables x_{ii} have now the "reverse" interpretation, i.e. $x_{ii} = 1$ if node i is <u>not</u> included in the cycle and 0 otherwise. In the graph theory therminology, $x_{ii} = 1$ identifies a loop incident to node i.

Model (2.1)-(2.6) is relaxed in a Lagrangean fashion by embedding constraint (2.4) in the objective function (2.1), by means of a parameter $\lambda \ge 0$. The new objective is then

$$L(\lambda,x) = \sum_{i\in V} \sum_{j\in V\setminus i} d_{ij} x_{ij} + \lambda\left(B - \sum_{i\in V} p_i\right) \tag{2.7}$$

with $d_{ij} = c_{ij}$ if $i,j \in V$, $i \neq j$, $d_{ii} = b_{ii} + \lambda p_i$ for $i \in V$.

The resulting problem is an Asymmetric PTP (APTP). The APTP can be polynomially reduced to the Asymmetric TSP on a larger digraph G' (see Dell'Amico, Maffioli and Värbrand (1995)), such that there is an one-to-one correspondence between the subtours in G which include node 1 and the Hamiltonian tours in G', and their costs are identical.

In many cases the NP-hard problem ATSP can be solved within small computational times, as shown in Carpaneto, Dell'Amico and Toth (1995) where the FORTRAN listing of a branch and bound algorithm called CDT is given. Since our enlarged graph G' has $2n$+1 nodes, but only n^2+3n-1 arcs, we modified code CDT so as to take advantage from sparsity. Algorithm CDT, at the root node, solves an assignment problem on a full density matrix, then defines a sparse matrix to be used during the exploration of the branch decision tree. Thus we substituted the original procedure used to solve the assignment problem with a FORTRAN version of the algorithm SPJV (Jonker and Volgenant (1987)).

Let Ω be the set of all feasible solutions to APTP: a subgradient technique can be used to solve the dual problem $max\{L(\lambda,x)\colon \lambda \ge 0, x \in \Omega\}$. We started using as initial value for λ the dual value associated with the optimal solution to the continuous knapsack problem, in minimization form: *min* $\left\{\sum_{i=1}^{n} w_i y_i : \sum_{i=1}^{n} p_i y_i \ge B, 0 \le y_i \le 1, i = 1,\ldots,n\right\}$, where w_i = *min* $\{c_{hi} : h = 1,\ldots,n, h \neq i\}$- γ_i is the minimum cost that one has to pay to include node i in the cycle instead of leaving it unrouted.

2.2 Obtaining feasibility

In this section we describe the basic steps for obtaining feasible solutions for PCTSP starting from the cycle χ obtained in Section 2.1 when it is not optimal. Note that, as it will be explained in more details while showing our computational results, when $\gamma_i = 0 \ \ \forall\ i \in V$ almost all the starting cycles are not feasible, while they are frequently feasible when the value of γ becomes larger.

Our first proposed step for obtaining feasible solutions is procedure ***Adding-Nodes*** which works as follows: add to χ one node $j \in V \setminus V_\chi$ at a time until the sum of the prizes of the nodes belonging to V_χ is at least B.

In the following we will use the ordered sequence of nodes $C = (v_1, v_2, \ldots, v_k, v_{k+1})$ with $v_{k+1} = v_1$ and $(v_i, v_{i+1}) \in A_\chi$ for i = 1, ..., k, to identify cycle χ.

In order to choose node $j \in V \setminus V_\chi$ to be added to C the following two inserting rules have been evaluated.

Rule R1. Choose node $j \in V \setminus V_\chi$ to add to C, in such a way that the resulting cycle cost is minimized, i.e. add j $\notin C$ between v_i and $v_{i+1} \in C$ with

$$j, v_i : c_{v_i j} + c_{j v_{i+1}} - c_{v_i v_{i+1}} - \gamma_j = min\{c_{v_h k} + c_{k v_{h+1}} - c_{v_h v_{h+1}} - \gamma_k : k \notin C, v_h \in C\} \quad (2.8)$$

This rule is a straightforward adaptation of the classical "cheapest insertion" rule used in greedy algorithms for TSP.

A rule which takes into account also the prizes of the nodes (and implicitly the knapsack constraint) is

Rule R2. Choose node $j \in V \setminus V_\chi$ to add to C, in such a way that the ratio "prize / cost" is mazimized, i.e. add j $\notin C$ between v_i and $v_{i+1} \in C$ with

$$j, v_i : \frac{p_j}{c_{v_i j} + c_{j v_{i+1}} - c_{v_i v_{i+1}} - \gamma_j} = max\{p_k / (c_{v_h k} + c_{k v_{h+1}} - c_{v_h v_{h+1}} - \gamma_k) : k \notin C, v_h \in C\} \quad (2.9)$$

As a complexity analysis, note that each iteration of ***Adding-Nodes Procedure*** requires computational time **O**(n^2).

Both rules R1 and R2 have been evaluated on the same random instances with n ranging from 20 to 500, $c_{ij} \in [0,1000]$ $\forall$ $(i,j) \in A$, $p_i \in [0,100]$, γ_i $\forall$ $i \in V$. The results related to feasible solutions obtained by applying rules R1 and R2 are reported in Table 1, where **R1** and **R2** give the average ratio (20 entries each) between the objective function value of the solution obtained after applying ***Adding-Nodes Procedure*** and the value LB of the lower bound obtained as described in Section 2.1.

	α = 0.2		α = 0.5		α = 0.8	
n	**R1**	**R2**	**R1**	**R2**	**R1**	**R2**
20	1.7296	1.7984	1.3368	1.2454	1.0803	1.0958
40	1.5253	1.4775	1.4200	1.2601	1.1813	1.1369
60	1.9103	1.8283	1.1744	1.1416	1.0815	1.0807
80	1.5114	1.5410	1.1505	1.1358	1.0904	1.1146
100	1.4650	1.4206	1.3331	1.1962	1.0924	1.0615
200	1.5508	1.5421	1.1196	1.0899	1.0445	1.0358
300	1.5936	1.4644	1.1579	1.1197	1.0875	1.0639
400	2.3807	2.0305	1.0445	1.0553	1.0548	1.0332
500	1.9085	1.7227	1.1258	1.0602	1.0306	1.0261

Table 1. *Average ratio between our first upper bound (rules R1 and R2) and the lower bound.*

In Table 1 and in all the following ones, we denote by α the percentage of the sum of the prizes used for determining the knapsack constraint, i.e. the value for constraint (1.5) is computed according to relation

$$B = \alpha \sum_{i \in V} p_i \quad (2.10)$$

In all our computational results we fix α = 0.2, 0.5 and 0.8.

Looking at Table 1, we can see that rule R1, that is based on the cost minimization, gives generally worse results than R2 for all values of n and α except for few cases. Note also how the solutions provided by ***Adding-Nodes Procedure*** are closer to LB when α increases.

As a further comparison between the two proposed inserting rules, it is worth mentioning that the number of iterations, i.e. the number of nodes added to C, performed by our procedure in the case of rule R1 and R2 is definitely in favour of R2; in fact, R1 requires on the average from 24% to 88% more iterations than rule R2, except in a single case (n = 100 and α = 0.2). Moreover, note that at most $\frac{19}{100}n$ and $\frac{16}{100}n$ iterations are performed by ***Adding-Node Procedure*** when using R1 and R2, respectively.

On the basis of this last evaluation and on the results given in Table 1, rule R2 has been selected and used in all our next computations. In the following, we will give all the results obtained starting from feasible solutions obtained by ***Adding-Node Procedure*** using rule R2; however, in the experiments with the Lagrangean heuristic presented in the next section rule R1 has also been evaluated but with very unsatisfactory results.

3. Improving feasible solutions

Once a feasible cycle is obtained, the next step is to try to improve its objective function value.

Our first idea has been to apply a local search procedure to look for the less convenient nodes in V_χ and substituting them for some nodes in $V\backslash V_\chi$ having the most favorable ratio according to rule R2, thus performing the so-called X-changes on the current feasible cycle. However, experimenting with this approach we got only at most a 3.45% average improvement with respect to the solutions reported in Table 1 and that improvement was related only to about 6% of the instances.

Therefore, a different approach aimed at enlarging the solution space has been developed. Such an approach, described in the next section, has been subsequently embedded in a Lagrangean heuristic.

3.1 Extension and collapse

Our idea for further improving feasible solutions for PCTSP is to explore a much wider neighbourhood of the current solution than that considered in an X-change. Then the next step is to first enlarge C and successively reduce it according to some criterion in such a way that our goal is still preserved.

In particular, for each node $j \in V\backslash V_\chi$ the ratio between its prize and its insertion cost is evaluated according to an inserting rule similar to (2.9). This ratio is defined as the *gain* of node j and it is given by:

$$R_j = min\left\{p_j / (c_{v_h j} + c_{j v_{h+1}} - c_{v_h v_{h+1}} - \gamma_j) : v_h \in C\right\} \qquad (3.1)$$

Let $\overline{\mathrm{R}}$ be the average gain, that is $\overline{\mathrm{R}} = \sum_{i \in V\backslash V_\chi} \frac{R_i}{(n-k)}$. Then, all the nodes $j \in V\backslash V_\chi$ such that $R_j > \overline{\mathrm{R}}$ are possibly added to C, whereas we disregard nodes with gain less than $\overline{\mathrm{R}}$. In particular, any time a node j with $R_j > \overline{\mathrm{R}}$ is added to C the value R_k for all $k \in V\backslash V_\chi$ is updated taking into account the new cycle so that new nodes previously excluded from consideration can be now added to C. Value $\overline{\mathrm{R}}$, instead, is not changed from one iteration to the other.

We denote this phase, in which we try to improve the overall profit of the current feasible cycle, the "***Extension phase***".

Figure 1 gives an example of how the original cycle is enlarged in the ***Extension phase***. The average gain in the starting cycle of the digraph depicted in *Figure 1* is $\overline{\mathrm{R}} \approx 1.24$, and the gain associated with the nodes belonging to $V \backslash V_{\chi}$ is, respectively, $R_8 = 0.5$, $R_9 = 2.\bar{7}$ and $R_{10} = 0.\overline{45}$. Therefore, in the first expansion step node 9 is added to C. After the insertion of node 9 we have a new value for the gain of node 10: $R_{10} = 1.\bar{6} > \bar{R}$ so that also node 10 is added to C. Note that the new enlarged cycle still satisfies the knapsack constraint (1.5) since the collected prize has been increased.

In the next step of our approach we focus on the cost reduction of the cycle as it has been augmented in the previous phase, trying to remove from C the most expensive nodes. The possible cost reduction is performed as follows. We select a path $P = (v_i, \ldots, v_j)$ of χ, with $1 \in P$, such that $\sum_{h \in P} p_h < B$ and $\sum_{h \in P} p_h + p_{v_{j+1}} \geq B$. Then, in order to close the cycle we consider the set $F \subseteq V \backslash P$ of nodes such that $\sum_{h \in P} p_h + p_r \geq B \; \forall \; r \in F$. We select node $k \in F$ which minimizes the cost $c_{v_j k} + c_{k v_j}$. Let us denote node k the "*collapse point*".

The search for the collapse point is computed by the following ***Collapse Procedure***:

Procedure Collapse(V,A, χ)

begin

  Set initial node $i = 1$;
  Set Best $\chi = \chi$;
  do
    begin
      Follow the cycle from i until the feasibility is obtained (i.e. (1.5) is satisfied);
      Let f be the node where the feasibility is obtained;
      Go back to the predecessor $j = b(f)$ of node f;
      Set $P = (i, \ldots, 1, \ldots, j)$;
      if (node 1 does not belong to P) **then** Return(Best χ);
      Find a node $k \in V \backslash P$ for collapsing at minimum cost while satisfying (1.5);
      Let be the new collapsed cycle;
      if cost() < cost(Best χ) **then** set Best $\chi = \bar{\chi}$;
      Set $i = b(i)$
    end
  while ($i \neq 1$);
  Return(Best χ)

end.

The second phase of our procedure is consequently denoted "***Collapse phase***". We can see that the above ***Collapse Procedure*** moves backward in the cycle until node 1, representing the depot, is included. The main steps performed in the ***Collapse phase*** are depicted in *Figure 2*.

We execute iteratively "***Extension and Collapse Procedure***" (obtained by applying in sequence the above ***Extension phase*** and ***Collapse Procedure***) until no further improvement is obtained. Each execution of this procedure has computational cost $\mathbf{O}(n^2)$.

Note that other possibilities for enlarging the cycles have been evaluated, but the consequently much higher computational time has suggested to consider only the above search step. Moreover, in order to overcome the locality of ***Extension and Collapse Procedure***, different starting solutions have been generated; however, while we were able to find significant improvements for very isolated instances, the average behaviour was worst than that obtained with the lower bound procedure described in Section 2.1.

3.2 A Lagrangean heuristic.

Motivated by the results of the previous section, we decided to develope a Lagrangean heuristic by applying ***Extension and Collapse Procedure*** during the computation of the lower bound of Section 2. In this way we look for a better solution throughout ***Extension and Collapse Procedure*** at each step of the computation of the Lagrangean multiplier. In particular, we determine an interval $[\lambda_1,\lambda_2]$ such that the value λ^*, representing the optimal value of the multiplier, belongs to this interval. Successively, we reduce it iteratively by computing the gradient in the two extremes λ_1 and λ_2, then use the intersection point as the new value for one of the two extremes of the interval, determined in such a way that λ^* belongs to the new interval. Note that the average number of iterations required for finding λ^* are 4, 5 and 7 respectively for $\alpha = 0.2$, 0.5 and 0.8.

The computational results obtained when applying the Lagrangean heuristic while executing ***Extension and Collapse Procedure*** at each iteration of the computation of the multiplier are reported in Table 2. The computational experiments are related to the same instances specification used for Table 1 on the basis of 20 entries / value. Column headings in Table 2 are as follows. **UB/LB** gives for each value of α the ratio between the present solution value, i.e our upper bound, and the lower bound; **I-Val** is the average percentage improvement of the solution values with respect to the solutions of Table 1 (in the columns related to R2), and **CPU** is the CPU time (in seconds) required for obtaining our solutions on a PC 486/66 Mhz.

	$\alpha = 0.2$			$\alpha = 0.5$			$\alpha = 0.8$		
n	**UB/LB**	**I-Val**	**CPU**	**UB/LB**	**I-Val**	**CPU**	**UB/LB**	**I-Val**	**CPU**
20	1.3597	24.40	0.12	1.1344	8.91	0.14	1.0617	3.11	0.17
40	1.3722	7.13	0.36	1.1317	10.19	0.51	1.1059	2.73	0.85
60	1.3241	27.58	0.41	1.1036	3.33	0.94	1.0765	0.39	1.30
80	1.2930	16.09	0.87	1.1161	1.74	1.39	1.0564	5.22	3.18
100	1.2719	10.47	3.30	1.1351	5.11	6.31	1.0519	0.90	4.80
200	1.1812	23.40	13.26	1.0535	3.34	13.00	1.0280	0.75	21.88
300	1.1051	24.54	37.73	1.0667	4.73	44.24	1.0293	3.25	70.65
400	1.2754	37.19	82.18	1.0335	1.89	79.72	1.0223	1.05	159.76
500	1.1872	31.08	131.50	1.0473	1.22	111.49	1.0210	0.50	265.83

Table 2. *Extension and CollapseProcedure in the Lagrangean heuristic: average values.*

Looking at Table 2 it can be easily noted that larger improvements of the solutions, up to 37%, are related to instances with $\alpha = 0.2$, while no more than a 5% improvement in the values of the solutions with respect to the feasibility phase is obtained after ***Extension and Collapse Procedure*** in the Lagrangean heuristic when $\alpha = 0.8$; however, note that our upper bounds for $\alpha = 0.5$ and 0.8 are very close to the lower bound, expecially for large instances ($n \geq 200$).

It is worth mentioning that the CPU time includes the running time required for computing the lower bound and the time for executing both ***Adding-Nodes Procedure*** and ***Extension and Collapse Procedure*** at each step of the computation of the Lagrangean multiplier. Note that up to $n = 300$ all the computations are performed within one minute of CPU time, and only in one case ($n = 500$ and $\alpha = 0.8$) four minutes are required.

4. Further computational experiments

As a further evaluation of the goodness of our solutions, a comparison between our upper bounds and the optimal solutions of the problem under consideration has been performed. In particular, we were able to solve up to the optimality the same set of random instances of the previous sections with n ranging in [20, 80] by using the IBM Optimization Subroutine Library

(OSL) on an IBM RISC 6000 model 550. The corresponding results are reported in Table 3, where **UB/z*** gives for each value of α the average ratio between our upper bounds and the optimal solutions, and **Opt** is the percentage of the number of optimal solutions found executing the proposed procedures among all the instances (20 entries / value).

	α = **0.2**		α = **0.5**		α = **0.8**	
n	**UB/z***	**Opt**	**UB/z***	**Opt**	**UB/z***	**Opt**
20	1.168	60	1.095	35	1.009	40
40	1.096	45	1.061	25	1.037	15
60	1.098	35	1.060	25	1.039	10
80	1.041	45	1.058	15	1.015	40

Table 3. *Comparison between our upper bounds and optimal solutions*

It can be easily noted by observing Table 3 that for $\alpha = 0.8$ the upper bounds are very close to the optimal solutions, even if the larger number of optimal solutions found by our procedures (60%) is related to instances with $\alpha = 0.2$

To get a more precise idea of the values of our solutions with respect to the optimal ones, *Figures 3, 4* and *5* report the average distribution of the UB/z* values for all n and for $\alpha = 0.2$, 0.5 and 0.8, respectively. Note how the number of solutions having the UB/z* value less than 1.1 moves from about 70% to 90% when the value of α, related to the knapsack constraint, is increased.

As a further general comment we can see that generally our upper bounds are closer to the optimal solutions than to the lower bounds; in particular, in only 13% of the instances under consideration with n ranging in [20,80], mainly for $\alpha = 0.5$, the lower bounds were closer to the optimal solutions than the UB values. Moreover, only once, among the 35% of the instances having this ratio greater than 1.25 in Figure 3, we got a value UB/z* = 2.1459, that is our worst result.

The computational experiments performed until now have been made with random instances generated similarly to those already introduced and tested in the recent literature (see Fischetti and Toth (1988) and Dell'Amico, Maffioli and Värbrand (1995)). The main characteristic of such instances is to have penalities $\gamma_i = 0 \; \forall \; i \in V$, so we decided to investigate the influence of the penalty on the difficulty of the problem. We have hence reconsidered the same set of instances used for the computational experiments reported in Sections 2 and 3 (with n ranging from 20 up to 500, costs randomly generated from an uniform distribution in [0,1000], prizes also randomly derived from an uniform distribution in [0,100] and $\alpha = 0.2$, 0.5 and 0.8), and with seven different classes of values for the penalties: the first six classes have $\gamma_i \in [1,\beta]$, $\beta = (20, 50, 100, 200, 500, 1000)$ and the last class has $\gamma_i = p_i, \forall \; i \in V$.

	α = **0.2**		α = **0.5**		α = **0.8**	
n	**UB/LB**	**Sol**	**UB/LB**	**Sol**	**UB/LB**	**Sol**
20	1.1226	0	1.1693	0	1.1201	0
40	1.0532	0	1.1075	0	1.0525	0
60	1.0143	2	1.0511	0	1.0473	0
80	1.0008	9	1.0155	1	1.0367	0
100	1	20	1.0173	1	1.0213	0
200	1	20	1	20	1.0050	0
300	1	20	1	20	1.0002	1
400	1	20	1	20	1	20
500	1	20	1	20	1	20

Table 4. *Average solutions for procedure Extension and Collapse for* $\gamma_i \in [1,20]$

In Table 4 we report the results for values of the penalties generated in [1,20]. In columns **UB/LB** we give the average ratios between the upper bounds and the lower bounds, while in columns **Sol** is reported the number of instances, over 20, in which the cycle produced by the lower bound was feasible (and hence optimal).

In this last case the problems result much easier than in the previous ones. In particular, it is possible to note that the number of instances in which the cycle determined by the lower bound is optimal increases with n and after a threashold value of n (increasing with α) all the instances are optimally solved by the lower bound procedure.

From Tables 5 and 6 we can see that when the value of β increases to 50 and 100, respectively, the solutions have similar behaviour than that in the case of $\beta = 20$, but the instances become easier and easier. In particular, all instances with $n > 100$, $\gamma_i \in [1,50]$, and $n > 60$, $\gamma_i \in [1,100]$ (not all reported in the tables) have been solved by the lower bound procedure.

For larger values of β and also in the seven-th case, that is $\gamma_i = p_i$, $\forall\ i \in V$, all the instances were solved by the lower bound.

	$\alpha = 0.2$		$\alpha = 0.5$		$\alpha = 0.8$	
n	**UB/LB**	**Sol**	**UB/LB**	**Sol**	**UB/LB**	**Sol**
20	1.0629	8	1.0579	0	1.0753	0
40	1	20	1.0182	0	1.0341	0
60	1	20	1.0029	15	1.0199	0
80	1	20	1	20	1.0117	0
100	1	20	1	20	1.0106	0
200	1	20	1	20	1	20

Table 5*. Average solutions for procedure Extension and Collapse for* $\gamma_i \in [1,50]$

	$\alpha = 0.2$		$\alpha = 0.5$		$\alpha = 0.8$	
n	**UB/LB**	**Sol**	**UB/LB**	**Sol**	**UB/LB**	**Sol**
20	1.0035	11	1.0184	4	1.0435	0
40	1	20	1.0182	20	1.0166	6
60	1	20	1.0029	20	1.0051	8
80	1	20	1	20	1	20

Table 6*. Average solutions for procedure Extension and Collapse for* $\gamma_i \in [1,100]$

In order to complete our investigation on the performance of ***Exstension and Collapse Procedure*** in the Lagrangean heuristic we considered three sets of data derived from real applications in a steel-mill rolling problem. In particular, we considered problems with 186, 415 and 447 nodes, where the nodes belonging to V represent the jobs to be performed in a given time period. The values c_{ij} between two nodes i,j, $\forall\ (i,j) \in A$, represent the request of performing job j immediately after job i, so that the objective function is in maximization form (we transformed the problems in minimization form in order to use our procedures). The constants γ_i correspond to the "importance" of job i, while the constants p_i are the length of the slab to be cut by job i, $\forall\ i \in V$. A detailed description of these instances can be found in Cowling (1995).

Since the PCTSP is a semplification of a much more complex real problem, many PCTSP instances are solved with different values of β, trying to satisfy most of the real constraints. In our computational experiments we have solved the above three problems with nine values of β, determined by setting $\alpha = (0.1, 0.2, ..., 0.9)$. From an analysis of the cost matrix it results that these instances are quasi-symmetric, that is most of the values $|c_{ij} - c_{ji}|$ are close to zero. As pointed out from the authors, code CDT cannot solve this kind of instances, so we used a weaker lower bound (see Fischetti and Toth (1988)) obtained from model (2.1)-(2.6) by removing constraints (2.5) and embedding in a Lagrangean fashion constraints (2.4). The subproblem to be solved is thus a linear assignment. In *Figures 6* and *7* we give, respectively, the CPU time (**CPU**) in seconds and the ratios (**UB/lb**) of the solutions obtained by ***Extension and Collapse Procedure*** over the values **lb** of the lower bound described above. The exact numerical values are reported in Table 7.

	n = 186		n = 415		n = 447	
α	**UB/lb**	**CPU**	**UB/lb**	**CPU**	**UB/lb**	**CPU**
0.1	1.015	0.49	1.018	7.52	1.042	13.51
0.2	1.034	0.38	1.020	14.39	1.039	22.25
0.3	1.036	0.49	1.018	27.96	1.043	31.75
0.4	1.036	0.93	1.016	34.12	1.039	44.23
0.5	1.038	2.03	1.020	51.64	1.032	62.25
0.6	1.037	2.91	1.017	66.32	1.040	81.04
0.7	1.033	3.73	1.014	71.09	1.034	88.57
0.8	1.026	5.82	1.010	77.25	1.026	93.40
0.9	1.021	6.43	1.007	78.02	1.011	96.96

***Table** 7. Average solutions for procedure Extension and Collapse for real applications*

We observe that the computational time required to solve the three problems greatly increases when we require to collect larger prizes (i.e. when α increases), but the quality of the solutions tends to decrease. The absolute error of the heuristic solutions is always within a 4.5% from a lower bound, which confirms that our approach can be effectively applied to solve subproblems encountered during the solution of real life applications.

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