567237
8K

\\ 224 \\

On the Microfoundations of
Dynamic Macroeconomics

by

Mario Forni*
Marco Lippi**

Febbraio 1998

* Università degli Studi di Modena
Dipartimento di Economia Politica
Viale Berengario,51
41100 Modena (Italy)
e-mail:forni@unimo.it

** Università degli Studi di Roma
Dipartimento di Scienze Economiche
Via Cesalpino, 14
00161 Roma (Italy)
e-mail:lippi@giannutri.caspur.it

Abstract

We survey a number of important results concerning aggregation of dynamic, stochastic relations. We do not aim at a comprehensive review; instead, we focus heavily on the results collected in Forni and Lippi (1997). We argue that the representative-agent assumption is misleading and the microfoundation of dynamic macroeconomics should be based on explicit modeling of heterogeneity across agents. An unpleasant ascpect of this modeling strategy is that macroeconomic implications of micro theory are difficult to obtain. However, difficulties are reduced by large number results. Moreover, puzzling implications of existing theories could be reconciled with empirical evidence on macro data.

Keywords: Aggregation, heterogeneity, representative agent, linear stochastic process, dynamic factor model.

1. Introduction

This paper reviews some important results on aggregation of linear stochastic dynamic models employed in macroeconomics. A comprehensive discussion is beyond our purposes, so that some interesting contributions will not be mentioned. Rather, we report mainly results by ourselves, illustrated in detail in Forni and Lippi (1997). Rigour is not our primary concern here. Exposition is based on simple examples, with a hint to generalizations, so that a non-specialist reader may get an idea of the problems, the difficulties and extant results. Nonetheless, we assume that the reader is acquainted with elementary theory of ARIMA stochastic processes and the notation based on the lag operator L.

The framework in which our aggregation problem arises is Standard Macroeconomics of aggregate consumption, income, investment, employment. This field has been dominated in the last two decades by two, not necessarily conflicting, approaches. The first is New Classical Macroeconomics, which is characterized by the strong prescription that models linking observable variables must be derived from microeconomic first principles, and in particular that the dynamics of such models must be a consequence of intertemporal optimization, rather than *ad hoc* superimposition to a static maximization scheme. The second is based on VAR and Structural VAR models, in which estimation of a relatively theory-free statistical model comes first, while theory enters at the identification stage.[1]

We shall focus mainly on the former approach, but the latter is also discussed briefly in Section 7. We will claim that irrespective of which approach is taken up, interpretation of dynamic macroequations encounters a very serious aggregation problem: when heterogeneity of agents is allowed, an important change of dynamic shape is likely to occur between micro and macro equations. Basic properties of the micro model do not hold in general for the macromodel: for instance, micro cointegration does not imply macro cointegration, lack of Granger causality in the micro model does not imply the same property at the macro level, static microequations may be transformed by aggregation into dynamic macroequations.

As a consequence, overidentifying restrictions produced by the theory cannot be tested directly using aggregate data. This is unpleasant, since the difficulties involved in formulating a macro model are increased. Aggregate implications of micro theory can only be found by explicit modeling of heterogeneity, which is likely to require a lot of additional information with respect to the traditional strategy. On the other hand, there is also a pleasant implication: existing models which are at odds with aggregate data under the representative-agent assumption could be reconciled with empirical evidence.

Though we will not make specific mention of all of the following authors or works in the sequel, our general ideas are close in spirit to Granger (1980, 1987, 1990), Hildenbrandt (1994), Lewbel (1992, 1994), Lütkepohl (1982, 1984), Pischke (1995), Stoker (1982, 1984, 1986), Trivedi (1985).

[1] For a vast treatment of models based on dynamic objective functions and rational expectations see Hansen and Sargent (1991). A simple presentation of the main topics can be found in Sargent (1987). On Structural VAR models, see Bernanke (1986), Shapiro and Watson (1988), Evans (1989), Giannini (1992).

The paper is organized as follows. In Section 2 we present a most simple example of the micro-macro effect that we want to illustrate in this paper. We have a microequation linking an independent to a dependent variable. The microequation is static, while the independent variable is autocorrelated: if heterogeneity in the microparameters is allowed, the microequation linking the aggregate dependent variable to the aggregate independent variable is dynamic. The example in Section 2 is highly artificial. In Section 3 a more realistic model for the independent variables is discussed. The model is based on the distinction between common and idiosyncratic components. When the number of individuals is huge, common components survive aggregation whereas idiosyncratic components are washed away. Recent empirical work is reported, in which it is shown that major macroeconomic variables are driven by several common components. In Section 4 we give a definition of micro and macromodels that is general enough to accommodate most of the models employed in the literature. We show that the macroparameters are analytic functions of the deep microparameters. As a consequence, restrictions on the macroparameters either hold on the whole microparameter space or hold only on a subset of zero Lebesgue measure. An application of this Alternative Principle is given in Section 5, where we show that cointegration of micromodels does not imply macro cointegration, apart from negligible subsets of the microparameters space. In Section 6 we review an important case in which aggregation has a positive effect. Permanent-income theory of consumption under rational expectations is at odds with aggregate empirical evidence when the representative agent is assumed. Two well-known inconsistencies are "excess sensitivity" and "excess smmothness". However, allowing for some heterogeneity and assuming limited information, aggregate data can be reconciled with the theory. In Section 7 we show some unpleasant consequences of aggregation on Granger non-causality and the interpretation of VAR models. Section 8 concludes.

2. An Example: A Dynamic Macroequation with a Static Microequation

Let us begin with a short review of the theoretical steps involved in the formulation of a typical "microfounded" partial-equilibrium macroeconomic model.
(i) A quadratic intertemporal optimization problem for an economic agent is set up and solved; the result is a linear equation linking the dependent variable y_t to the expected future values of the indepenendent variable x_t and possibly the lagged values of both variables;
(ii) A stochastic linear dynamic equation for x_t is specified. The example

$$x_t = u_t + \alpha u_{t-1}, \tag{1}$$

with u_t white noise, will be useful to fix ideas. Assuming rational expectations, equation (1) is used to replace the expected future values of x_t with present and past values, leading to an equation like

$$y_t = a y_{t-1} + b x_t + c x_{t-1} + \epsilon_t, \tag{2}$$

where only one-period lags have been included for simplicity and ϵ_t is a white-noise residual.

(iii) The agent following the micro model (1)–(2) is assumed to be "representative", meaning that model (1)–(2) can be interpreted as a macro model and therefore can be employed directly for estimation and testing with macro data.

In this paper we depart thoroughly from this microfoundation paradigm. The crucial difference is that we drop the representative agent assumption in step (iii) and assume instead that agents are heterogeneous. We do not place special emphasis on the optimization and the rational expectations steps. Rather, we start by introducing heterogeneity in the micro equations (1) and (2):

$$\begin{aligned} y_{it} &= a_i y_{it-1} + b_i x_{it} + c_i x_{it-1} + \epsilon_{it} \\ x_{it} &= +u_{it} + \alpha_i u_{it-1}. \end{aligned} \tag{3}$$

Then we focus on the following questions: What is the macroequation linking the aggregate variables, i.e. $Y_t = \sum_i y_{it}$ and $X_t = \sum_i x_{it}$? Can we suppose that it is obtained by simply averaging over the coefficients of (3)? Under which conditions the dynamic properties of the micro models hold true in the macro model?

In general the features of the micro models are not preserved at the macro level. The following example should be sufficient to give the reader an idea of the complications arising. Let us assume for simplicity $b_i = c_i = 0$ and $\epsilon_{it} = 0$ for all i, so that we are left with the static, exact micro equations

$$y_{it} = a_i x_{it}. \tag{4}$$

Let us assume also that the independent variables of different agents are orthogonal at any lead and lag, i.e. $\mathrm{cov}(u_{it}, u_{jt-k}) = 0$ for $i \neq j$ and any integer k, and $\mathrm{var}(u_{it}) = 1$ for any i. Finally, let us assume that there are only two agents (or two groups of identical agents), i.e. $Y_t = y_{1t} + y_{2t}$ and $X_t = x_{1t} + x_{2t}$. Now consider the static regression of Y_t on X_t:

$$Y_t = AX_t + \Omega_t. \tag{5}$$

A simple calculation shows that

$$\begin{aligned} A &= \frac{a_1(1+\alpha_1^2) + a_2(1+\alpha_2^2)}{(1+\alpha_1^2) + (1+\alpha_2^2)} \\ \mathrm{var}(\Omega_t) &= (a_1 - A)^2(1+\alpha_1^2) + (a_2 - A)^2(1+\alpha_2^2) \\ \mathrm{cov}(\Omega_t, \Omega_{t-1}) &= (a_1 - A)^2\alpha_1 + (a_2 - A)^2\alpha_2 \end{aligned}$$

Hence $\mathrm{var}(\Omega_t) = 0$ if and only if $a_1 = a_2$, while $\mathrm{cov}(\Omega_t, \Omega_{t-1}) = 0$ if and only if either $a_1 = a_2$, or $\alpha_1 = \alpha_2 = 0$. Thus if the behavioral coefficients are heterogeneous, and there is some autocorrelation in individual incomes, a researcher estimating (5) will find a non-zero residual (whereas no residual is present in the first equation of the micromodel (4)) and detect autocorrelation in Ω_t. If our researcher shares the common representative-agent attitude, he will conclude that the true relationship between the micro counterparts of Y_t and X_t is a dynamic equation like (2) and therefore model (4) must be rejected.

We can stop here the analysis of this very simple example. The message is that aggregation of dynamic multivariate models can produce non-trivial changes in the dynamic shape of each single equation. Starting with a static microequation, like the first of (4), we may end up with a dynamic equation provided that some heterogeneity among the agents is allowed.[2]

3. Independent Variables: Common and Idiosyncratic Components

The examples in Section 2 are partial equilibrium models, in which one or several variables are taken as given both from the agents and the researcher. For general equilibrium models we refer to Forni and Lippi (1997, Chapter 7). Here we shall deal only with partial equilibrium models, which are sufficient for our illustrative purposes.

In the latter models, agents may differ for two reasons: because their independent variables are different, and because their responses to their independent variables are different. Let us begin by modeling the differences in independent variables. In the example of the previous section we have assumed that the variables x_{it}, corresponding to different individuals, are orthogonal at all leads and lags. Although convenient to simplify calculations, this assumption is far from being realistic. Incomes of different agents, wages faced by different firms, although different, are correlated both simultaneously and with lags. A natural way in which both difference and correlation across individual variables can be represented is the dynamic factor model (Sargent and Sims, 1977; Geweke, 1977).

To fix ideas suppose that the independent variable is income and that y_{it} is income of agent i. A very simple dynamic factor model for y_{it} is

$$y_{it} = b_i U_t + \xi_{it}, \tag{6}$$

where we assume that:
(a) U_t and ξ_{it} are stationary stochastic variables;
(b) ξ_{it} is orthogonal to U_{t-k} for any integer k;
(c) ξ_{it} is orthogonal to ξ_{jt-k} for any $j \neq i$ and any integer k. Indeed, this model is so simple that it could be confused with a static factor model; notice however that, although the response of the variables to U_t is static, the orthogonality conditions are dynamic. The variable U_t, the *common factor* or *common shock*, is a source of variation affecting all micro incomes, even though with different impacts as measured by the coefficient b_i. The term $b_i U_t$ will be called the *common component* of y_{it}. Changes in U_t can represent for instance fiscal or monetary policy changes, as well as changes in productivity or labor supply that affect the whole economy. By contrast, each of the variables ξ_{it}, the *idiosyncratic components*, represents events affecting only one individual, like health or luck, and are therefore uncorrelated to one another at any lead and lag.

Model (6) can be generalized by introducing more than one common shock and add lags in the response of y_{it} to the common shocks, thus obtaining

$$y_{it} = b_{i1}(L)U_{1t} + b_{i2}(L)U_{2t} + \cdots + b_{ih}(L)U_{ht} + \xi_{it}, \tag{7}$$

[2] The effects of aggregation on the dynamic shape of economic relations have been firstly highlighted by Lippi (1988). The emergence of an "aggregation error" is studied in detail in Lippi and Forni (1990). Similar phenomena can emerge with time aggregation, seasonal adjustment, aggregation of unobserved components, omitted variables, errors in variables; see Sims (1971, 1974), Tiao and Wei (1976), Nerlove *et al.* (1979), Lütkepohl (1982), Forni (1990).

where ξ_{it} is orthogonal to U_{st-k} for any integer k and any $s = 1, h$. Notice that we are not assuming that the variables U_{st}, or the variables ξ_{it}, are white noises. However, if the U_{st}'s are assumed to be costationary, model (7) can be rewritten as

$$y_{it} = a_{i1}(L)u_{1t} + a_{i2}(L)u_{2t} + \cdots + a_{ih}(L)u_{ht} + \xi_{it}, \tag{8}$$

where $(u_{1t} \quad u_{2t} \quad \cdots \quad u_{ht})$ is an othonormal white-noise vector, i.e. $\mathrm{var}(u_{it}) = 1$ for any i, $\mathrm{cov}(u_{it}, u_{jt-k}) = 0$ for $i \neq j$ and any integer k.

Moreover, since empirical incomes are non-stationary while income changes are stationary, then either we interpret the variables y_{it} in (8) as deviations from a deterministic trend, or we modify (8) in such a way that the stationary RHS drives the changes of income:

$$(1 - L)y_{it} = a_{i1}(L)u_{1t} + a_{i2}(L)u_{2t} + \cdots + a_{ih}(L)u_{ht} + \xi_{it}. \tag{9}$$

Factor models like (8) or (9) have been recently employed in macroeconomic literature as parsimunious representations when the number of variables is large with respect to the number of available observations over time.[3] With a small number of factors, models (8) or (9) can provide a considerable reduction of the number of parameters to be estimated as compared to an unrestricted VAR model, in which each of the variables is regressed on itself and lagged values of the others. Despite parsimony, model (9) is flexible enough to allow for a substantial amount of heterogeneity.

Models (8) and (9) have a very important property when the number of individuals is huge. Let us fix ideas on (8). Formally, it is convenient to assume that there exists a countable infinity of agents. We assume also that $\mathrm{var}(\xi_{it})$ is bounded, i.e. there exists a real Λ such that $\mathrm{var}(\xi_{it}) \leq \Lambda$ for any i. Finally, for simplicity, we set $a_{is}(L) = b_s(L) \neq 0$ for any i and $s = 1, h$. It is easily seen that when n tends to infinity the variance of the aggregated common component

$$n\,(b_1(L)u_{1t} + \cdots + b_h(L)u_{ht})$$

tends to infinity as n^2, whereas the variance of the aggregated idiosyncratic component $\sum_{i=1}^{n} \xi_{it}$ cannot tend to infinity faster than n. Thus, if per-capita income $y_t = Y_t/n$ is considered, the idiosyncratic component disappears as n gets larger.[4] For large n we have approximately

$$y_t = b_1(L)u_{1t} + b_2(L)u_{2t} + \cdots + b_h(L)u_{ht},$$

or, more in general,

$$y_t = \bar{a}_1(L)u_{1t} + \bar{a}_2(L)u_{2t} + \cdots + \bar{a}_h(L)u_{ht},$$

[3] See for instance Quah and Sargent (1993) and Forni and Reichlin (1995).

[4] This large-numbers effect is well-known for static models, particularly in the finance literature (see e.g. Chamberlain and Rotshild, 1983). Some important implications for aggregation and macroeconomics are discussed in Granger (1987, 1990). Forni and Lippi (1997, Chapter 1) provide conditions under which the result can be extended to the dynamic case.

where $\bar{a}_s(L)$ is the cross-sectional average of the $a_{is}(L)$.

The elimination of the idiosyncratic components has crucial consequences for both theory and empirical work. From a theoretical point of view, it provides a nice way to reconcile macroeconomics with micro heterogeneity. Individual variables corresponding to different agents may be almost orthogonal to one another owing to big idiosyncratic components as compared to the common components. Therefore, individual variables can be viewed as spanning a vector space with a huge number of dimensions. Nonetheless, this is perfectly consistent with a very basic idea of macroeconomic theory—that aggregate variables, rather than depending on all the corresponding microvariables, can be represented as driven by a relatively small number of macroeconomic sources of variation.

Regarding empirical work, the large-numbers effect can be exploited in order to estimate the model and to study the problem of how many independent common components drive the micro- and the macrovariables. On estimation we refer to Forni and Reichlin (1995) and Forni, Hallin, Lippi and Reichlin (1998).[5] Regarding inference on the number of common shocks, some work can be found in Forni and Lippi (1977) and in both of the papers quoted above.

In Forni and Lippi (1997, Chapter 2) data on incomes and wages relative to US states are employed to show that the number of common components in a representation of the form (9) for individual incomes is definitely bigger or equal to two. This evidence is at odds with with some recent applied macroeconomic work on consumption, in which heterogeneity of incomes is introduced but the micromodel contains only one common component (see Section 6). This outcome, i.e. more than one common component in individual major economic series is interesting both *per se*, and because, as we will see, many of the effects of aggregation on dynamic micromodels depend on (1) heterogeneity of different agents' behaviors, (2) at least two common components driving the independent variables.

Now let us come back to the mutual orthogonality assumption for the idiosyncratic components. The following two examples show that this condition rules out interesting economic models and should be relaxed in some way.

As a first example, consider an n-industry constant-returns economy, where production of industry i at time t is given by the equation

$$y_{it} = d_{i1}y_{1t-1} + d_{i2}y_{2t-1} + \cdots + d_{in}y_{nt-1} + c_i u_t + \chi_{it}, \tag{10}$$

where u_t is a demand common shock, χ_{it} is an idiosyncratic shock fulfilling the orthogonality assumption, i.e. $\mathrm{cov}(\chi_{it}, \chi_{jt-k}) = 0$ for $i \neq j$ and any integer k, d_{ij} is the quantity of the i-th product necessary as a means of production to produce one unit of the j-th product, the one-period lag on the RHS meaning that industry i is partly producing to replace means of productions employed by other industries in the previous period. Starting with (10) and inverting the input-output matrix we obtain an equation like (8):

$$y_{it} = a_i(L)u_t + \xi_{it},$$

[5] Forni and Reichlin (1995) propose to estimate the factors and the common components by using simple averages. Forni and Reichlin (1997) and Forni, Hallin, Lippi and Reichlin (1998) propose procedures based on principal components. An estimator which is a linear combination of the observable variables is developed by Stock and Watson (1997) for a static model with time-varying coefficients.

where

$$\begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \vdots \\ \xi_{nt} \end{pmatrix} = (I + DL + D^2L^2 + \cdots) \begin{pmatrix} \chi_{1t} \\ \chi_{2t} \\ \vdots \\ \chi_{nt}, \end{pmatrix}$$

D being the matrix having d_{ij} in place i, j. However, unlike in (8), unless D is diagonal, the assumption $\xi_{it} \perp \xi_{jt-k}$ for $j \neq i$ and any integer k is no longer valid. The problem stems from the fact that shocks originated in sector i, while affecting direcly only y_{it}, propagate through the system via the autoregressive linkages.

As a second example, consider the model

$$y_{it}^j = a_i^j(L)u_t + b_i^j(L)v_t^j + c_i^j(L)\chi_{it}^j.$$

Here the income of region i in nation j is driven by a world-wide shock u_t, a national shock v_t^j and a local shock χ_{it}^j. A model like this is employed in Forni and Reichlin (1997) in order to study comovements between European regions and to compare them with US counties. The national shocks could be accomodated in model (8) as common shocks; however, if the number of nations in the model is large, parsimony would be lost. By contrast, absorbing the national component into the idiosyncratic term would violate orthogonality, since the variance-covariance matrix would be block-diagonal rather than diagonal.

Now, if the orthogonality assumption on the idiosyncratic terms is dropped in (8) or (9), the theoretical distinction between idiosyncratic and common components is lost and boundedness of $\mathrm{var}(\xi_{it})$ is no longer sufficient to ensure that the per-capita idiosyncratic variance tends to zero. However, mutual orthogonality and bounded variance can be substituted by the following more general condition.

Let Σ_n^ξ be the variance-covariance matrix of the vector $(\xi_{1t} \quad \xi_{2t} \quad \cdots \quad \xi_{nt})$, and let λ_n^ξ be its maximum eigenvalue. If λ_n^ξ is bounded then the variance of

$$\frac{1}{n}\sum_{i=1}^{n} \xi_{it}$$

tends to zero as n tends to infinity. This is very easy to show. Indicating by w the n-dimensional vector with all its components equal to unity, we have

$$\mathrm{var}\left(\frac{1}{n}\sum_{i=1}^{n}\xi_{it}\right) = \frac{1}{n^2}w'\Sigma_n^\xi w \leq \frac{1}{n^2}|w|^2\lambda_n^\xi = \frac{1}{n}\lambda_n^\xi.$$

The bounded eigenvalue condition has been introduced in place of the traditional orthogonality assumption by Chamberlain (1983) and Chamberlain and Rothschild (1983), in a static context. The resulting model is named "approximate factor model". A somewhat different assumption, based on dynamic eigenvalues (Brillinger, 1981), is introduced in Forni and Lippi (1988), where the "approximate dynamic factor model" is proposed and the representation theorems in Camberlain and Rothshild (1983) are generalized to the dynamic framework.

4. Micro and Macromodels

In the previous section we have established and commented upon a model for the independent variables. Here we give a general representation of micromodels linking dependent to independent microvariables.[6] Let us begin by an example. Assume that agent i determines the variable y_{it} in the following way:

$$y_{it} = d_i \mathrm{E}(x_{it}|I_{t-1}), \tag{11}$$

where x_{it} is an independent variable, d_i is a deep parameter, e.g. a parameter of the utility or of the production function of agent i, $\mathrm{E}(\cdot|I_{t-1})$ being the expectation conditional on the information set available at time $t-1$. Lastly, assume that

$$x_{it} = u_t + \alpha u_{t-1} + \xi_{it}$$

where u_t is a common white noise with unit variance, while ξ_{it} is an order-one idiosyncratic moving average:

$$\xi_{it} = \eta_{it} + \beta_i \eta_{it-1}.$$

For simplicity we assume η_{it} and η_{jt} orthogonal at all leads and lags. The model of agent i contains four parameters: d_i, α, β_i and $\sigma^2_{\eta_i}$, determining both the independent and the dependent variable. Notice that the parameter α is not agent specific, i.e. is equal for all the agents. To solve for the conditional expectation appearing in (11) we have to make some assumption about the set I_{t-1}. Two simple alternatives are:

(A) Agent i observes and employs separately both the common and the idiosyncratic component of x_{it}. If this is the case

$$\begin{aligned} y_{it} &= d_i \alpha u_{t-1} + d_i \beta_i \eta_{it-1} \\ x_{it} &= u_t + \alpha u_{t-1} + \eta_{it} + \beta_i \eta_{it-1}. \end{aligned} \tag{12}$$

(B) Agent i observes and employs only the variable x_{it}, with no distinction between the components of y_{it}. If this is the case, in order to determine $\mathrm{E}(x_{it}|I_{t-1})$ we must resort to the univariate Wold representation of x_{it}

$$x_{it} = u_t + \alpha u_{t-1} + \eta_{it} + \beta_i \eta_{it-1} = \epsilon_{it} + \delta_i \epsilon_{it-1}, \tag{13}$$

where δ_i and $\sigma^2_{\epsilon_i} = \mathrm{var}(\epsilon_{it})$ are determined by the equations

$$\begin{aligned} \mathrm{var}(x_{it}) &= \sigma^2_{\epsilon_i}(1+\delta_i)^2 = 1 + \alpha^2 + \sigma^2_{\eta_i}(1+\beta_i^2) \\ \mathrm{cov}(x_{it}, x_{it-1}) &= \sigma^2_{\epsilon_i}\delta_i = \alpha + \sigma^2_{\eta_i}\beta_i. \end{aligned} \tag{14}$$

This system will provide two reciprocal values for δ_i, the one smaller than unity in modulus being the solution. Thus we end up with the equations

$$\begin{aligned} y_{it} &= d_i \delta_i \epsilon_{it-1} \\ x_{it} &= \epsilon_{it-1} + \delta_i \epsilon_{it-1}, \end{aligned}$$

[6] For more details see Lippi and Forni (1997, Chapters 6 and 7).

i.e., using (13):

$$\begin{aligned} y_{it} &= \frac{d_i \delta_i L}{1+\delta_i L}(u_t + \alpha u_{t-1} + \eta_{it} + \beta_i \eta_{it-1}) \\ x_{it} &= u_t + \alpha u_{t-1} + \eta_{it} + \beta_i \eta_{it-1}. \end{aligned} \tag{15}$$

We can stop here with this example. What we want to retain is that in both cases we end up with a couple of linear dynamic equations linking y_{it} and x_{it} to the individual and common shocks u_t and η_{it}, whose coefficients are functions of the common and individual parameters d_i, α, β_i, $\sigma^2_{\eta_i}$. Under assumption (A) above such functions are elementary manipulations, whereas under assumption (B) they imply taking the roots of the algebraic system of equations (14).

Now, these features of our example are quite general. Even though the micromodels employed as microfoundations of dynamic macromodels can be much more complicated than the example, the general procedure already outlined at the beginning of Section 2 can now be described more precisely:

(a) One starts with intertemporal objective functions and dynamic equations for the independent variables, both depending on deep microparameters.

(b) Then one must solve algebraic equations resulting from the intertemporal optimization and possibly from the procedure necessary to obtain the Wold representation of the independent variables (as in the example, case (B)). The coefficients of such algebraic equations are simple functions of the deep microparameters (as in (14)).

(c) The final result is a system of linar dynamic equations, like (12) or (15), linking the variables of interest to the micro shocks, whose coefficients result from algebraic combinations of the deep parameters and the roots of the algebraic equations just mentioned in (b).

Thus, sticking for simplicity to the case of two variables y_{it} and x_{it}, a *micromodel* can be defined as

(i) A set $\Gamma \subseteq R^c \times R^s$. This is the admissibility region for the microparameters, where c is the number of common microparameters while s is the number of individual microparameters. We can assume that Γ is open and connected.

(ii) A function associating with any element of Γ a system S of linear dynamic equations linking the variables y_{it} and x_{it} to the common and idiosyncratic shocks. The coefficients of system S are real functions defined on Γ, continuous on Γ and analytic on Γ with the exception of a subset of Lebesgue measure zero.

In the example above we have one common parameter, namely α, so that $c = 1$, and three individual parameters, namely d_i, β_i and $\sigma^2_{\eta_i}$, so that $s = 3$. A possible definition for Γ is given by the constraints $1 > \alpha > -1$, $1 > \beta_i > -1$, $\sigma^2_{\eta_i} > 0$.

Moreover, in case (A) the coefficients of (12) are elementary functions of the deep parameters, while in case (B), equations (15) contain both deep parameters and δ_i, which is a function of deep microparameters through system (14). To see why in both cases the coefficients of system S are analytic we must recall that the roots of algebraic equations are analytic as functions of their coefficients, with the exception of those values of the coefficients corresponding to multiple roots. It is reasonable to require that multiple roots may occur only for a negligible subset of Γ. Lastly, notice that, according to the above definition, in our example we have two different micromodels, depending on whether we make assumption (A) or (B) on the information set I_{t-1}.

Once the micromodel has been defined we must define the macromodel. Let us firstly go back to our example. By aggregation over n agents we have

$$Y_t = \alpha u_{t-1} \sum_{i=1}^{n} d_i + \sum_{i=1}^{n} d_i \beta_i \eta_{it-1}$$
$$X_t = n(u_t + \alpha u_{t-1}) + \sum_{i=1}^{n} (\eta_{it} + \beta_i \eta_{it-1})$$

in case (A), while in case (B)

$$Y_t = \sum_{i=1}^{n} \frac{d_i \delta_i L}{1+\delta_i L}(u_t + \alpha u_{t-1}) + \sum_{i=1}^{n} \frac{d_i \delta_i L}{1+\delta_i L}(\eta_{it} + \beta_i \eta_{it-1})$$
$$X_t = n(u_t + \alpha u_{t-1}) + \sum_{i=1}^{n} (\eta_{it} + \beta_i \eta_{it-1})$$

The natural definition of the *macromodel* corresponding to n agents and a given micromodel is:

(I) Assuming for simplicity that $\Gamma = \mathbb{R}^c \times \mathbb{R}^s$, the set

$$\Gamma_n = \overbrace{\mathbb{R}^s \times \mathbb{R}^s \times \cdots \times \mathbb{R}^s}^{n \text{ times}} \times \mathbb{R}^c,$$

is the admissibility set of the macromodel.

(II) The function, obtained by simply summing both sides of the micromodel, associating with any element p of Γ_n a system AS of dynamic equations linking Y_t and X_t to the common and to the idiosyncratic shocks. The coefficients of AS are real functions defined on Γ_n, continuous on Γ_n, analytic on Γ_n with the exception of a subset of Lebesgue measure zero.

In Section 3 we have seen that the idiosyncratic component becomes negligible as compared to the common component when n tends to infinity. Thus, assuming a huge number of agents, we can drop the idiosyncratic component in the macromodel. In our example we find

$$Y_t = \alpha u_{t-1} \sum_{i=1}^{n} d_i$$
$$X_t = n(u_t + \alpha u_{t-1}),$$

and

$$Y_t = \sum_{i=1}^{n} \frac{d_i \delta_i L}{1+\delta_i L}(u_t + \alpha u_{t-1})$$
$$X_t = n(u_t + \alpha u_{t-1})$$

in case (A) and (B) respectively. Notice that dropping the idiosyncratic components in the equations just above does not mean that the idiosyncratic component does

not play any role in the macromodel. This is what happens in case (A), but in case (B) the coefficients of the macromodel depend on the δ_i's and therefore, through the system (14), on $\sigma^2_{\eta_i}$ and β_i.

More in general, when there are h common components we can write the macromodel as

$$\begin{aligned} Y_t &= a_1(p,L)u_{1t} + a_2(p,L)u_{2t} + \cdots + a_h(p,L)u_{ht} \\ X_t &= b_1(p,L)u_{1t} + b_2(p,L)u_{2t} + \cdots + b_h(p,L)u_{ht} \end{aligned} \tag{16}$$

where $a_s(p,L)$ and $b_s(p,L)$, $s = 1, h$, are power expansions in L, whose coefficients are functions defined on Γ_n, continuous on Γ_n, analytic on Γ_n with the exception of a subset of Lagrange measure zero.

The property that the coefficients of the power expansions in (16) are analytic as functions defined on Γ_n is crucial. For, as we are going to show by some examples in the following sections, basic properties like cointegration, Granger non-causality, etc., result in simple algebraic relationships between the coefficients of equations like (16). And since such coefficients are analytic on Γ_n, we can resort to an important proposition, the *Alternative Principle*, stating that such relationships hold either everywhere on Γ_n or only for a zero Lagrange-measure subset. Equivalently, if we may find a point $\tilde{p}$ in Γ_n such that cointegration, Granger non-causality, etc., does not hold for $\tilde{p}$, then cointegration, Granger non-causality, etc., holds only for a subset of Γ_n of zero Lebesgue measure.

5. Cointegration

The results presented in this section are drawn from Lippi (1988), Gonzalo (1993) and Forni and Lippi (1977, Chapter 9). Cointegration has been shown to hold for important macrovariables; the consumption-income example, which is discussed in the following section, is perhaps the most celebrated. Let us recall the definition. Let $(\, y_t \quad x_t \,)$ be a I(1) vector, i.e. $(\, y_t \quad x_t \,)$ is non-stationary, but $(1-L)(\, y_t \quad x_t \,)$ is stationary. The variables y_t and x_t are cointegrated if there exists a real c such that $y_t - cx_t$ is stationary.

It is natural to ask whether micro cointegration implies macro cointegration. If this is the case, when micro cointegration is a property of the micromodel, testing for cointegration on aggregate data would make sense as a test of micro theory. Let us consider again a very simple example. Suppose that the micromodel is:

$$(1-L)y_{it} = \delta_i \alpha_i u_{1t} + \delta_i \beta_i u_{2t} + (1-L)a_i u_{3t}$$
$$(1-L)x_{it} = \alpha_i u_{1t} + \beta_i u_{2t} + (1-L)b_i u_{3t}.$$

For simplicity here we have no idiosyncratic component. Two of the common shocks, namely u_{1t} and u_{2t}, are permanent, whereas the third is transitory. We assume that the coefficients α_i, β_i, δ_i, a_i and b_i are the deep microparameters (there are no common microparameters). The microvariables y_{it} and x_{it} are cointegrated for any i, since $y_{it} - \delta_i x_{it}$ is obtained by integrating $(1-L)(a_i - \delta_i b_i)u_{3t}$ and is therefore stationary. Summing over individuals both sides of the above equation it is seen that cointegration of the macrovariables requires the existence of a real c such that

$$c\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \delta_i \alpha_i, \qquad c\sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \delta_i \beta_i. \tag{17}$$

There are two important cases in which a c fulfilling (17) exists:
(a) If $\delta_i = \delta_j$ for any i and j. In this case $c = \delta_i$.
(b) If there exists a τ such that $\beta_i = \tau\alpha_i$ for any i. In this case $c = \sum_i \delta_i\alpha_i / \sum_i \alpha_i$.

The economic meaning of the above conditions is simple. If x_t is consumption and y_{it} is income, condition (a) means that all agents share the same long-run propensity to consume. On the other hand, when condition (b) holds, the shocks u_{1t} and u_{2t} are 'redundant', this meaning that all incomes are driven by the shock $u_{1t} + \tau u_{2t}$. This in turn implies that all individual incomes are paiwise cointegrated.

More generally, if a c fulfilling (17) exists then

$$\sum_{i=1}^{n} \delta_i\beta_i \sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \delta_i\alpha_i \sum_{i=1}^{n} \beta_i, \tag{18}$$

which has conditions (a) and (b) as particular cases. Since we have assumed five individual microparameters and no common microparameters, the set Γ_n is an open connected subset of $\mathbb{R}^{5n}$. Since (18) describes an algebraic hypersurface in $\mathbb{R}^{5n}$, the subset of Γ_n fulfilling (18) is negligible.

This result is fairly obvious. We have three parameters (a_i and b_i play no role) that are locally free to vary with respect to one another (Γ is an open set). Then we take all possible combinations of points of Γ, and therefore a thick subset of $\mathbb{R}^{5n}$. As an easy consequence the points of Γ_n that fulfill (18) form a negligible subset. However, a less obvious result is the following theorem which is based on the Alternative Principle mentioned in Section 4. Let us modify the example by assuming that α_i, β_i, δ_i, a_i and b_i are not deep parameters themselves but functions of the deep parameters, i.e. functions defined on Γ, analytic on Γ with the exception of a negligible subset. Then the following results hold:

(i) Each of the conditions (a) and (b) either holds for the whole Γ or for a negligible subset of Γ.
(ii) If there exists a point of Γ such that neither (a) nor (b) holds, then Y_t and X_t are cointegrated only for a negligible subset of Γ_n.
(iii) If (a) or (b) holds for the whole Γ then Y_t and X_t are cointegrated for any point of Γ_n.

Some remarks are in order. First, the aggregation effect summarized in the theorem above, statement (ii), requires that there are at least two non-redundant common permanent shocks driving the microvariables (condition (b) can hold only for a negligible subset of Γ). On the other hand, as we have reported in Section 3, we have strong evidence that several non-redundant permanent common shocks drive the microvariables corresponding to major macrovariables. Second, we have already observed that the construction of Γ_n consists in taking all possible combinations of agents picked up from Γ. We impose no restriction whatsoever on the agents of a population. Moreover, many of our results refer to subsets of zero Lebesgue measure of Γ_n. This means that we assume a state of profound ignorance about the distribution of the microparameters in empirical populations. If informations were available, these might lead to restrictions on the set Γ_n. However, such restrictions would not necessarily imply fulfillment of (18).

To conclude this section, let us go back to the example and introduce the following modification:

$$y_{it} = \tilde{\alpha}_i u_{1t} + \tilde{\beta}_i u_{2t} + (1-L)a_i u_{3t}$$
$$x_{it} = \alpha_i u_{1t} + \beta_i u_{2t} + (1-L)b_i u_{3t}.$$

Here $\tilde{\alpha}_i$, $\tilde{\beta}_i$, α_i, β_i, a_i and b_i are the microparameters. Now the microvariables y_{it} and x_{it} are no longer cointegrated, apart from a negligible subset of Γ. However, this does not imply the impossibility of macro cointegration. The condition is

$$\sum_{i=1}^{n} \tilde{\alpha}_i \sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \tilde{\beta}_i \sum_{i=1}^{n} \alpha_i,$$

which describes a $(6n-1)$-dimensional subset of Γ_n, which is $6n$-dimensional. Thus, unless there is a good reason to assume that condition (a) or (b) holds, macro cointegration is not more likely when microvariables are cointegrated than when micro cointegration does not occur.

6. Aggregation is not Necessarily Bad: The Case of Consumption

There is a very important case in recent literature in which aggregation effects contribute to reconciling theory and empirical evidence. Assume that labour income obeys the equation

$$\Delta x_t = a(L)\epsilon_t,$$

where $a(L) = 1 + a_1L + a_2L^2 + \cdots$, and ϵ_t is a white noise. According to the Life-Cycle Permanent-Income theory in its simplest version consumption should obey

$$\Delta c_t = a(\beta)\epsilon_t,$$

where $\beta = 1/(1+r)$, r being a risk-free interest rate. This is a famous result by Hall (1978). There are three remarkable features in Hall's result:
(1) Consumption changes follow a white noise process ucorrelated with labour income changes at time $t-k$, for any $k > 0$.
(2) As noticed by Deaton (1987), if labour income is persistent according to the measure proposed by Cochrane (1988), i.e. $a(1)^2/\sum a_h^2$, and β is near to unity, then consumption is more volatile than income.
(3) Consumption and income changes are driven by the same white noise, so that the vector $(\Delta x_t \quad \Delta c_t)$ has rank one as a stochastic vector.
(4) As shown by Campbell (1987), total consumption and income, defined as the sum of labour income and asset returns, are cointegrated with cointegrating vector $(1 \quad -1)$.

The first three features of Hall's model are at odds with aggregate empirical evidence. Consumption changes are positively autocorrelated and correlated with past values of income: this fact is known as "excess sensitivity" (Flavin 1981). The variance of consumption changes is much smaller than the variance of income changes; indeed, this is what Friedman's (1956) original permanent-income theory was designed to explain. At the same time, there is evidence indicating that income is persistent. This problem is known as "excess smoothness" of consumption or "Deaton's paradox". Lastly, consumption changes form a rank two stochastic vector; some evidence on this is presented in Lippi and Forni (1977, Chapter 13).

Regarding (4), evidence is not clear-cut. However, cointegration beetween aggregate income and consumption is accepted by several authors.[7]

Let us show how a very simple heterogeneity assumption can solve the excess smoothness and sensitivity problems. Suppose that income of agent i evolves according to

$$\Delta x_{it} = (1 + aL)u_t + (1 + bL)\chi_{it}, \tag{19}$$

which is a simplification of the example used in Section 4. We suppose that the idiosyncratic shocks χ_{it} are orthogonal to one another and that they share the same variance: $\sigma^2_{\chi_i} = \sigma^2_{\chi}$. Notice that there are two common parameters, a and b, no individual parameters and that different incomes differ only for the idiosyncratic term χ_{it}. Then assume as in Section 4, Assumption (B), that agent i observes only Δx_{it}, rather than its components. There exist a real d, $|d| \leq 1$, and a white noise η_{it} such that

$$\Delta x_{it} = (1 + dL)\eta_{it} = (1 + aL)u_t + (1 + bL)\chi_{it}$$

(notice that since a, b and σ^2_{χ} are common, the coefficient d does not depend on i). Lastly, apply Hall's theory to agent i to obtain

$$\begin{aligned} \Delta x_{it} &= (1 + dL)\eta_{it} = (1 + aL)u_t + (1 + bL)\chi_{it} \\ \Delta c_{it} &= (1 + d\beta)\eta_{it} = \frac{1 + d\beta}{1 + dL}(1 + aL)u_t + (1 + bL)\chi_{it}. \end{aligned}$$

Indicating by x_t and c_t per-capita magnitudes we get

$$\begin{aligned} \Delta x_t &= (1 + aL)u_t \\ \Delta c_t &= \frac{1 + d\beta}{1 + dL}(1 + aL)u_t. \end{aligned} \tag{20}$$

In general $d \neq a$, so that Δc_t is not a white noise and is correlated with the past of Δx_t. Moreover, it is not difficult to see that, if d is negative and a is positive, then we have both a persistent income and a smooth consumption.

What about cointegration? For brevity, we do not introduce explicitly assets and total income in the model. However, the discussion in the previous Section will be sufficient to convince the reader that, despite heterogeneity, cointegration is retained in the macro model, consistently with empirical evidence. This because in this model, by property (4), all individual long-run propensities to consume are equal to 1.

The possibility that heterogeneity and incomplete information might explain the excess sensitivity and/or the excess smoothness puzzles, has been noted in Lippi (1990), Goodfriend (1992) and Pischke (1995).[8] In Pischke (1995) an important step forward has been obtained by using information coming from micro data. Pischke found that model (19), estimated for a USA panel of household data, provided a positive a, a negative b, and a σ^2_{χ} much bigger than σ^2_u, so that negative autocorrelation prevails in individual incomes. As a consequence d is negative, so that per-capita consumption changes, according to model (20), should exhibit positive autocorrelation, posive correlation with past income change and low variance, consistently with empirical evidence.

[7] See for instance Davidson *et al.* (1978), Campbell and Deaton (1989), Engle and Granger (1987).

Although extremely interesting, model (20) is a rank-one vector. For that matter, only one common shock is present in the micromodel, so that both the macrovariables are driven by the same shock. On the other hand, we know that only one common shock in individual incomes is unrealistic. Thus let us consider a micromodel for incomes with two common shocks:

$$\Delta x_{it} = (1 + a_{1i}L)u_{1t} + (1 + a_{2i}L)u_{2t} + (1 + b_iL)\chi_{it}. \tag{21}$$

Again, we employ the univariate representation

$$\Delta x_{it} = (1 + d_iL)\eta_{it} = (1 + a_{1i}L)u_{1t} + (1 + a_{2i}L)u_{2t} + (1 + b_iL)\chi_{it}$$

to obtain individual consumption

$$\Delta c_{it} = (1 + d_i\beta)\eta_{it} = \frac{1 + d_i\beta}{1 + d_iL}(1 + a_{1i}L)u_{1t} + (1 + a_{2i}L)u_{2t} + (1 + b_iL)\chi_{it}.$$

Taking per-capita magnitudes

$$\begin{aligned} \Delta x_t &= (1 + \bar{a}_1L)u_{1t} + (1 + \bar{a}_2L)u_{2t} \\ \Delta c_t &= \frac{1}{n}\sum_{i=1}^{n} \frac{1 + d_i\beta}{1 + d_iL}\left((1 + \bar{a}_1L)u_{1t} + (1 + \bar{a}_2L)u_{2t}\right), \end{aligned} \tag{22}$$

where $\bar{a}_k = \sum_i a_{ki}/n$. The rank of $(\,\Delta x_t \quad \Delta c_t\,)$ is less than two if and only if

$$\det \begin{pmatrix} 1 + \bar{a}_1L & 1 + \bar{a}_2L \\ \sum_{i-1}^{n} \frac{1 + d_i\beta}{1 + d_iL}(1 + a_{1i}L) & \sum_{i-1}^{n} \frac{1 + d_i\beta}{1 + d_iL}(1 + a_{2i}L) \end{pmatrix} = 0. \tag{23}$$

It is not difficult to get convinced that if the coefficients a_{1i}, a_{2i}, b_i and σ^2_k, σ^2_{χ} have sufficient freedom of variation with respect to one another, then (23) will hold only for a negligible subset of Γ_n. Formally, this is an application of the Alternative Principle: since (23) is an algebraic equations between the coefficients of (22), and since such coefficients are analytic functions defined on Γ_n, apart from a negligible subset, then if (23) does not hold for a point $\tilde{p}$ of Γ_n it holds only on a negligible

[8] Similar results in an overlapping generation framework are found by Clarida (1991) and Galí (1990).

subset of Γ_n. On the other hand, under mild heterogeneity conditions, a point $\tilde{p} \in \Gamma_n$ such that (23) does not hold is very easy to find.[9]

In conclusion, allowing for heterogeneity of agents in a common-idiosyncratic micromodel, more than one common shock, and limited information of individual agents, only cointegration is implied at the macro level by Hall's model, whereas the puzzling implications are avoided. Even though several alternative solutions for these puzzles have been proposed,[10] the one outlined above has the advantage of an explicit consideration of heterogeneity and aggregation.

7. Wold and autoregressive representations of the aggregate vector

Dealing with cointegration and with the consumption model we had only to consider representation (16) of the aggregate vector, in which on the LHS we have the aggregate variables and on the RHS the common microshocks. Now, in order to outline some further results we need the Wold representation of the aggregate vector. Unlike (16), which is structural or semi-structural and has on the RHS a number of shocks usually bigger than the number of variables on the LHS, the Wold representation has the form

$$\begin{aligned} Y_t &= A_{11}(p,L)v_{1t} + A_{12}(p,L)v_{2t} \\ X_t &= A_{21}(p,L)v_{1t} + A_{22}(p,L)v_{2t}, \end{aligned} \tag{24}$$

where:
(1) $A_{ij}(p,L)$ is a power series in L whose coefficients are functions of $p \in \Gamma_n$. Moreover, $A_{11}(p,0) = A_{22}(p,0) = 1$, $A_{12}(p,0) = A_{21}(p,0) = 0$.
(2) $(v_{1t} \quad v_{2t})$ is a vector white noise. Comparison of (24) to (16) shows that v_{1t} and v_{2t} depend on p; however we do not need to further complicate notation.
(3) $A_{11}(p,L)A_{22}(p,L) - A_{12}(p,L)A_{21}(p,L)$ does not vanish in the open circle $|z| < 1$.

Forni and Lippi (1997, Chapter 10) show that the coefficients of $A_{ij}(p,L)$ are continuous on Γ_n and analytic on Γ_n with the exception of a negligible subset. Therefore the Alternative Principle, stated in Section 3, can be applied. Furthermore, it is easily seen that the same Principle can also be applied to the coefficients of the power series $B_{ij}(p,L)$ in the autoregressive representation obtained by inverting (24), i.e.

$$\begin{aligned} B_{11}(p,L)Y_t + B_{12}(p,L)X_t &= v_{1t} \\ B_{21}(p,L)Y_t + B_{22}(p,L)X_t &= v_{2t}. \end{aligned} \tag{25}$$

This result has important consequences on several aggregation problems. Here we shall discuss two of them, namely Granger-causality and structural VAR models. The fact that causality relations are destroyed by aggregation has firstly been pointed out in Lippi (1988). Results on VAR models are presented in Blanchard and Quah (1989). Here we follow Lippi and Forni (1997, Chapters 11 and 12), in which further references can be found.

Since representation (25) has been obtained by inverting (24), we have that $B_{11}(p,0) = B_{22}(p,0) = 1$, $B_{12}(p,0) = B_{21}(p,0) = 0$, i.e. the equations are the prjections of Y_t and X_t respectively on past values of Y_t and X_t. By definition, Y_t

[9] See Lippi and Forni, 1997, Section 13.7.

[10] For a comprehensive review see Deaton (1992).

does not Granger-cause X_t if $B_{21}(p,L) = 0$, this meaning that past values of Y_t do not help in predicting X_t once the information contained in the past of X_t has been fully exploited. The Alternative Principle entails that if there exists a $\tilde{p} \in \Gamma_n$ such that Y_t Granger-causes X_t, then the subset of Γ_n where Y_t does not Granger-cause X_t is negligible.

Now the question is: assuming that y_{it} does not Granger-cause x_{it}, can we conclude that Y_t does not Granger-cause X_t ? The answer is negative and an illustration of the result can be given by using the example of Section 2:

$$\begin{aligned} y_{it} &= a_i x_{it} \\ x_{it} &= (1+\alpha_i L)u_{it}. \end{aligned}$$

Like in Section 2 we assume that the u_{it} are mutually orthogonal at any lead and lag and that there are two agents. The aggregate equations are

$$\begin{aligned} Y_t &= a_1(1+\alpha_1 L)u_{1t} + a_2(1+\alpha_2 L)u_{2t} \\ X_t &= (1+\alpha_1 L)u_{1t} + (1+\alpha_2 L)u_{2t}. \end{aligned}$$

The corresponding Wold representation is obtained by normalizing:

$$\begin{aligned} \begin{pmatrix} Y_t \\ X_t \end{pmatrix} &= Q \begin{pmatrix} a_1(1+\alpha_1 L) & a_2(1+\alpha_2 L) \\ 1+\alpha_1 L & 1+\alpha_2 L \end{pmatrix} \begin{pmatrix} 1 & -a_2 \\ -1 & a_1 \end{pmatrix} \begin{pmatrix} a_1 & a_2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} \\ &= Q \begin{pmatrix} a_1(1+\alpha_1 L) - a_2(1+\alpha_2 L) & a_1 a_2(\alpha_1 - \alpha_2)L \\ (\alpha_1 - \alpha_2)L & a_1(1+\alpha_2 L) - a_2(1+\alpha_1 L) \end{pmatrix} \begin{pmatrix} \tilde{v}_{1t} \\ \tilde{v}_{2t} \end{pmatrix}, \end{aligned}$$

where $Q = (a_1 - a_2)^{-1}$. The (2,1) entry of the corresponding autoregressive representation is

$$\frac{(\alpha_2 - \alpha_1)L}{(a_1 - a_2)(1+\alpha_1 L)(1+\alpha_2 L)}.$$

Thus if $a_1 \neq a_2$ and $\alpha_1 \neq \alpha_2$ the macrovariable Y_t Granger-causes the macrovariable X_t, even though no such Granger-causation occurs at the micro level. Here again we see how aggregation effects occur when both the independent variables and the responses of the agents are heterogeneous. On the other hand, given a micromodel, if there exists a point in Γ_n for which such aggregation effect occurs, that same effect occurs almost everywhere in Γ_n.

Let us now turn to VAR models. Suppose that a VAR is estimated for the macrovariables Y_t and X_t:

$$B(L)\begin{pmatrix} Y_t \\ X_t \end{pmatrix} = \begin{pmatrix} V_{1t} \\ V_{2t} \end{pmatrix}.$$

Let $A(L) = B(L)^{-1}$ and write

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} = A(L)\begin{pmatrix} V_{1t} \\ V_{2t} \end{pmatrix}.$$

Suppose also that according to our theory the variables Y_t and X_t are driven by a supply shock W_{1t} and a demand shock W_{2t}, and that (i) W_{1t} and W_{2t} have unit variance and are orthogonal at any lead and lag, (ii) the shock W_{1t} has no contemporaneous effect on X_t. This is sufficient to identify W_{1t} and W_{2t}, and the matrix $C(L)$ such that

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} = C(L) \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix}, \tag{26}$$

where $C_{21}(0) = 0$. Now assume that the microvariables are driven by h common supply shocks and k common demand shocks

$$\begin{pmatrix} y_{it} \\ x_{it} \end{pmatrix} = \begin{pmatrix} c_{11i}(L) & c_{12i}(L) \\ c_{21i}(L) & c_{22i}(L) \end{pmatrix} \begin{pmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{ht} \\ \eta_{1t} \\ \vdots \\ \eta_{kt} \end{pmatrix}, \tag{27}$$

where $c_{11i}(L)$ and $c_{21i}(L)$ are $1 \times h$ while $c_{21i}(L)$ and $c_{22i}(L)$ are $1 \times k$. Moreover, assume that $c_{21i}(0) = 0$, so that our identification criterion is "correct", i.e. not inconsistent with the underlying micromodel. Aggregating (27) and using (26) we obtain

$$\begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} = C(L)^{-1} \begin{pmatrix} \sum c_{11i}(L) & \sum c_{12i}(L) \\ \sum c_{21i}(L) & \sum c_{22i}(L) \end{pmatrix} \begin{pmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{ht} \\ \eta_{1t} \\ \vdots \\ \eta_{kt} \end{pmatrix},$$

where W_{1t} and W_{2t} appear as linear combination of the ϵ_{st}'s and the η_{st}'s. The question is: can we state that W_{1t} is a combination of the micro supply shocks ϵ_{st} only, so that it is reasonable to call W_{1t} an aggregate supply shock, or a mixing occurs? Similarly, can we state that W_{2t} is a demand shock? The answer is that, if the mixing occurs for one point of Γ_n, then it occurs almost everywhere. Moreover, a mild heterogeneity is sufficient to generate the mixing. Thus, under heterogeneity, the aggregate supply shock is a combination of both the supply and demand microshocks.

8. Conclusions

In this paper we have shown a number of unpleasant aggregation effects. If agents are heterogeneous, a macroequation can be dynamic even though the corresponding microequations are static. Cointegration is destroyed by aggregation, unless either the micro cointegration coefficients are equal, or there is only one permanent common shock driving the microvariables. When consumers have limited information, the cross-correlation and volatility properties of consumption and income changes implied by Hall's model are lost at the macro level. In general, unidirectional Granger-causality is not robust with respect to aggregation. Lastly, the

shocks appearing in a structural VAR representation result from a mixing of both corresponding and non-corresponding micro shocks.

We can summarize all of these results by saying that when the representative-agent assumption is dropped and heterogeneity among individuals is introduced, we cannot expect that a macro model shares the same dynamic properties as the underlying micro model. As a consequence, given an estimated macroequation, we cannot interpret its dynamic shape or other features as revealing something about the behaviour of individual agents. In the same way, if the estimated macro parameters fail to fulfill the restrictions implied by the micro theory, this is not a good reason to reject the micro theory. Thus, on one hand, aggregation is not so bad after all, since theory could be reconciled with evidence, as we have seen for the permanent income model. But, on the other hand, it should be recognized that additional difficulties arise in macroeconomic modeling. In order to obtain testable implications at the macro level, information on the joint behaviour of individual independent variables is needed.

Here we have proposed a model for the independent variables: the dynamic factor analytic model, generalized to allow for cross-correlated idiosyncratic components. This model is flexible enough to accomodate a large amount of heterogeneity, while retaining a reasonably parsimonious parameterization. A remarkable feature of the model is that, when the number of individuals is large, the idiosyncratic components die out with aggregation. This enormously simplifies things, since microvariables moving in a huge-dimensional space are made consistent with macrovariables driven by a small-dimensional vector of shocks.

Unfortunately, long series of individual data are seldom available, so that information on the number of common shocks and the cross-sectional distribution of individual response functions may be very difficult to collect. Nevertheless, we do not think that such difficulties should convince us to stick to the representative-agent practice, which amounts to transforming a complete lack of information about a distribution in the assumption that such distribution is concentrated on a single point of the microparameter space.

References

Bernanke, B. (1986), "Alternative Explanations of the Money-Income Correlation," *Carnegie–Rochester Conference Series on Public Policy,* 25, 49–99.

Blanchard, O. J. and Quah, D. (1989), "The Dynamic Effects of Aggregate Supply and Demand Disturbances", *American Economic Review 79*, 655–73.

Brillinger, D. R. (1981), *Time Series. Data Analysis and Theory* (Expanded Edition). San Francisco: Holden-Day.

Campbell, J. Y. (1987), "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica 55*, 1249–73.

Campbell, J. Y. and Deaton, A. S. (1989), "Why Is Consumption so Smooth?" *Review of Economic Studies 56*, 357–74.

Chamberlain, G. (1983), "Funds, Factors, and Diversification in Arbitrage Pricing Models", *Econometrica,* 51, 1305–1323.

Chamberlain, G. and Rothschild, M. (1983), "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets", *Econometrica,* 51, 1281–1304.

Clarida, R. H. (1991), "Aggregate Stochastic Implications of the Life-Cycle Hypothesis," *Quarterly Journal of Economics 106,* 851–67.

Cochrane, J. H. (1988), "How Big is the Random Walk in the GNP?," *Journal of Political Economy 96,* 893–920.

Davidson, J. E. H., Hendry, D. F., Srba, F. and Yeo, S. (1978), "Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom," *Economic Journal 88,* 661–92.

Deaton, A. S. (1987), "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" in Bewley, T.F., (ed.), *Advances in Econometrics, Fifth World Congress,* Volume II. Cambridge: Cambridge University Press, 121–46.

—— (1992), *Understanding Consumption.* Oxford: Oxford University Press.

Engle, R. F. and Granger, C. W. J. (1987), "Cointegration and Error Correction: Representation, Estimation and Testing," *Econometrica 55,* 251–76.

Evans, G. W. (1989), "Output and Unemployment Dynamics in the United States: 1950-1985," *Journal of Applied Econometrics,* 4, 213–37.

Flavin, M. (1981), "The Adjustment of Consumption to Changing Expectations about Future Income," *Journal of Political Economy 89,* 974–1009.

Forni, M. (1990), "Misspecification in Dynamic Models," Materiali di Discussione n. 65, Dipartimento di Economia Politica, Modena, forthcoming on *Statistica.*

Forni, M., Hallin, M., Lippi, M. and Reichlin, L. (1988) "Principal Components Estimation for Dynamic Factor Models", Paper presented to the CEPR Conference "New Approaches to the Study of the Business Cycle", Madrid, 30/31 January 1988.

Forni, M. and Lippi, M. (1998) "Notes on a Dynamic Generalization of Factor Representation Theory", mimeo.

Forni, M. and Reichlin, L. (1995) "Let's get Real: A Dynamic Factor Analytical Approach to Disaggregated Business Cycle Dynamics", CEPR working papers series n. 1244, forthcoming on *Review of Economic Studies.*

—— (1997), "National Policies and Local Economies: Europe and the United States," CEPR working papers series n. 1632.

Friedman, M. (1957), *A Theory of the Consumption Function.* Princeton: Princeton University Press.

Galí J., 1990, "Finite Horizons, Life Cycle Savings and Time Series Evidence on Consumption," *Journal of Monetary Economics 26,* 433-52.

Geweke, J. B. (1977), "The Dynamic Factor Analysis of Economic Time Series," in Aigner, D. J. and Goldberger, A. S. (eds.) *Latent Variables in Socioeconomic Models.* Amsterdam: North-Holland, 365–83.

Giannini, C. (1992), *Topics in Structural VAR Econometrics.* Berlin: Springer Verlag.

Goodfriend, M. (1992), "Information-Aggregation Bias," American Economic Review 82, 508–19.

Gonzalo, J. (1993), "Cointegration and Aggregation," *Ricerche Economiche 47,* 281–91.

Granger, C. W. J. (1980), "Long Memory Relationships and the Aggregation of Dynamic Models," *Journal of Econometrics,* 14, 227–38.

—— (1987), "Implication of Aggregation with Common Factors," *Econometric Theory 3,* 208–22.

—— (1990), "Aggregation of Time-Series Variables — a Survey," in Barker, T. and Pesaran, M. H. (eds.), *Disaggregation in Econometric Modelling.* London: Routledge, 17–34.

Hall, R. E. (1978), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy 96,* 971–87.

Hansen, L. P. and Sargent, T. J., eds., (1991), *Rational Expectations Econometrics.* London: Westview.

Hildenbrand, W. (1994), *Market Demand.* Princeton, NJ: Princeton University Press.

Lewbel, A., 1992, "Aggregation with Log-Linear Models," *Review of Economic Studies 59,* 635-42.

——, 1994, "Aggregation and Simple Dynamics," *American Economic Review 84,* 905-18.

Lippi, M. (1988), "On the Dynamic Shape of Error Correction Models," *Journal of Economic Dynamics and Control 12,* 561–85.

—— (1990), "Issues on Aggregation and Microfundations of Macroeconomics," Paper presented to the Workshop "Rational Behavior and Aggregation: Theory and Tests", EUI, Florence.

Lippi, M. and Forni, M. (1990), "On the Dynamic Specification of Aggregated Models," in Barker, T. and Pesaran, M. H. (eds.), *Disaggregation in Economic Modelling.* London: Routledge, 35–72.

Lütkepohl, H., 1982, "Non-Causality due to Omitted Variables," *Journal of Econometrics 19,* 367-78.

——, 1984, "Linear Transformations of Vector ARMA Processes," *Journal of Econometrics 26,* 283-93.

Nerlove, M., Grether, D. M., and Carvalho, J. L. (1979), *Analysis of Economic Time Series.* New York: Academic Press.

Pischke, J.-S. (1995), "Individual Income, Incomplete Information, and Aggregate Consumption," *Econometrica,* 63, 805–40.

Quah, D. and Sargent, T. J. (1994), "A dynamic index model for large cross sections" in Stock, J. H. and Watson, M. W. (eds.), *Business Cycles, Indicators and Forecasting.* Chicago: The University of Chicago Press, 285–309.

Sargent, T. J. (1987), *Macroeconomic Theory* (2nd Edition). New York: Academic Press.

Shapiro, M. and Watson, M. W. (1988), "Sources of Business Cycle Fluctuations," in Fisher, S. (ed.), *NBER Macroeconomics Annual*, 1988. Cambridge, MA: MIT Press, 111–48.

Sims, C. A. (1971), "Discrete Approximations to Continuous Distributed Lags in Econometrics," *Econometrica,* 39, 545–64.

—— (1974), "Seasonality in Regression," *Journal of the American Statistical Association,* 69, 618–26.

Stock, J. H., and Watson, M. W. (1997) "Diffusion Indexes", mimeo.

Stoker, T. M., 1982, "The Use of Cross-Section Data to Characterize Macro Functions", *Journal of the American Statistical Association 77*, 369-80.

——, 1984, "Completeness, Distribution Restrictions, and the Form of Aggregate Functions", *Econometrica 52*, 887-907.

——, 1986, "Simple tests of Distributional Effects on Macroeconomic Equations", *Journal of Political Economy 94*, 763-95.

Tiao, G. and Wei, W. (1976), "Effects of Temporal Aggregation on the Dynamic Relationships of Two Time-Series Variables," *Biometrika,* 63, 513–23.

Trivedi, P. K. (1985), "Distributed Lags, Aggregation and Compounding: Some Econometric Implications," *Review of Economic Studies,* 52, 19–35.

1. Maria Cristina Marcuzzo [1985] "Yoan Violet Robinson (1903-1983)", pp. 134
2. Sergio Lugaresi [1986] "Le imposte nelle teorie del sovrappiù", pp. 26
3. Massimo D'Angelillo e Leonardo Paggi [1986] "PCI e socialdemocrazie europee. Quale riformismo?", pp. 158
4. Gian Paolo Caselli e Gabriele Pastrello [1986] "Un suggerimento hobsoniano su terziario ed occupazione: il caso degli Stati Uniti 1960/1983", pp. 52
5. Paolo Bosi e Paolo Silvestri [1986] "La distribuzione per aree disciplinari dei fondi destinati ai Dipartimenti, Istituti e Centri dell'Università di Modena: una proposta di riforma", pp. 25
6. Marco Lippi [1986] "Aggregations and Dynamic in One-Equation Econometric Models", pp. 64
7. Paolo Silvestri [1986] "Le tasse scolastiche e universitarie nella Legge Finanziaria 1986", pp. 41
8. Mario Forni [1986] "Storie familiari e storie di proprietà. Itinerari sociali nell'agricoltura italiana del dopoguerra", pp. 165
9. Sergio Paba [1986] "Gruppi strategici e concentrazione nell'industria europea degli elettrodomestici bianchi", pp. 56
10. Nerio Naldi [1986] "L'efficienza marginale del capitale nel breve periodo", pp. 54
11. Fernando Vianello [1986] "Labour Theory of Value", pp. 31
12. Piero Ganugi [1986] "Risparmio forzato e politica monetaria negli economisti italiani tra le due guerre", pp. 40
13. Maria Cristina Marcuzzo e Annalisa Rosselli [1986] "The Theory of the Gold Standard and Ricardo's Standard Comodity", pp. 30
14. Giovanni Solinas [1986] "Mercati del lavoro locali e carriere di lavoro giovanili", pp. 66
15. Giovanni Bonifati [1986] "Saggio dell'interesse e domanda effettiva. Osservazioni sul cap. 17 della General Theory", pp. 42
16. Marina Murat [1986] "Betwin old and new classical macroeconomics: notes on Lejonhufvud's notion of full information equilibrium", pp. 20
17. Sebastiano Brusco e Giovanni Solinas [1986] "Mobilità occupazionale e disoccupazione in Emilia Romagna", pp. 48
18. Mario Forni [1986] "Aggregazione ed esogeneità", pp. 13
19. Sergio Lugaresi [1987] "Redistribuzione del reddito, consumi e occupazione", pp. 17
20. Fiorenzo Sperotto [1987] "L'immagine neopopulista di mercato debole nel primo dibattito sovietico sulla pianificazione", pp. 34
21. M. Cecilia Guerra [1987] "Benefici tributari nel regime misto per i dividendi proposto dalla commissione Sarcinelli: una nota critica", pp. 9
22. Leonardo Paggi [1987] "Contemporary Europe and Modern America: Theories of Modernity in Comparative Perspective", pp. 38
23. Fernando Vianello [1987] "A Critique of Professor Goodwin's 'Critique of Sraffa'", pp. 12
24. Fernando Vianello [1987] "Effective Demand and the Rate of Profits. Some Thoughts on Marx, Kalecki and Sraffa", pp. 41
25. Anna Maria Sala [1987] "Banche e territorio. Approccio ad un tema geografico-economico", pp. 40
26. Enzo Mingione e Giovanni Mottura [1987] "Fattori di trasformazione e nuovi profili sociali nell'agricoltura italiana: qualche elemento di discussione", pp. 36
27. Giovanna Procacci [1988] "The State and Social Control in Italy During the First World War", pp. 18
28. Massimo Matteuzzi e Annamaria Simonazzi [1988] "Il debito pubblico", pp. 62
29. Maria Cristina Marcuzzo (a cura di) [1988] "Richard F. Kahn. A discipline of Keynes", pp. 118
30. Paolo Bosi [1988] "MICROMOD. Un modello dell'economia italiana per la didattica della politica fiscale", pp. 34
31. Paolo Bosi [1988] "Indicatori della politica fiscale. Una rassegna e un confronto con l'aiuto di MICROMOD", pp. 25
32. Giovanna Procacci [1988] "Protesta popolare e agitazioni operaie in Italia 1915-1918", pp. 45
33. Margherita Russo [1988] "Distretto Industriale e servizi. Uno studio dei trasporti nella produzione e nella vendita delle piastrelle", pp. 157
34. Margherita Russo [1988] "The effect of technical change on skill requirements: an empirical analysis", pp. 28
35. Carlo Grillenzoni [1988] "Identification, estimations of multivariate transfer functions", pp. 33
36. Nerio Naldi [1988] "'Keynes' concept of capital", pp. 40
37. Andrea Ginzburg [1988] "locomotiva Italia?", pp. 30
38. Giovanni Mottura [1988] "La 'persistenza' secolare. Appunti su agricoltura contadina ed agricoltura familiare nelle società industriali", pp. 40
39. Giovanni Mottura [1988] "L'anticamera dell'esodo. I contadini italiani della 'restaurazione contrattuale' fascista alla riforma fondiaria", pp. 40
40. Leonardo Paggi [1988] "Americanismo e riformismo. La socialdemocrazia europea nell'economia mondiale aperta", pp. 120
41. Annamaria Simonazzi [1988] "Fenomeni di isteresi nella spiegazione degli alti tassi di interesse reale", pp. 44
42. Antonietta Bassetti [1989] "Analisi dell'andamento e della casualità della borsa valori", pp. 12
43. Giovanna Procacci [1989] "State coercion and worker solidarity in Italy (1915-1918): the moral and political content of social unrest", pp. 41
44. Carlo Alberto Magni [1989] "Reputazione e credibilità di una minaccia in un gioco bargaining", pp. 56
45. Giovanni Mottura [1989] "Agricoltura familiare e sistema agroalimentare in Italia", pp. 84
46. Mario Forni [1989] "Trend, Cycle and 'Fortuitous cancellation': a Note on a Paper by Nelson and Plosser", pp. 4
47. Paolo Bosi , Roberto Golinelli , Anna Stagni [1989] "Le origini del debito pubblico e il costo della stabilizzazione", pp. 26
48. Roberto Golinelli [1989] "Note sulla struttura e sull'impiego dei modelli macroeconometrici", pp. 21
49. Marco Lippi [1989] "A Shorte Note on Cointegration and Aggregation", pp. 11
50. Gian Paolo Caselli e Gabriele Pastrello [1989] "The Linkage between Tertiary and Industrial Sector in the Italian Economy: 1951-1988. From an External Dependence to an International One", pp. 40
51. Gabriele Pastrello [1989] "Francois quesnay: dal Tableau Zig-zag al Tableau Formule: una ricostruzione", pp. 48
52. Paolo Silvestri [1989] "Il bilancio dello stato", pp. 34
53. Tim Mason [1990] "Tre seminari di storia sociale contemporanea", pp. 26
54. Michele Lalla [1990] "The Aggregate Escape Rate Analysed throught the Queueing Model", pp. 23
55. Paolo Silvestri [1990] "Sull'autonomia finanziaria dell'università", pp. 11

56. Paola Bertolini, Enrico Giovannetti [1990] "Uno studio di 'filiera' nell'agroindustria. Il caso del Parmigiano Reggiano", pp. 164

57. Paolo Bosi, Roberto Golinelli, Anna Stagni [1990] "Effetti macroeconomici, settoriali e distributivi dell'armonizzazione dell'IVA", pp. 24

58. Michele Lalla [1990] "Modelling Employment Spells from Emilia Labour Force Data", pp. 18

59. Andrea Ginzburg [1990] "Politica Nazionale e commercio internazionale", pp. 22

60. Andrea Giommi [1990] "La probabilità individuale di risposta nel trattamento dei dati mancanti", pp. 13

61. Gian Paolo Caselli e Gabriele Pastrello [1990] "The service sector in planned economies. Past experiences and future prospectives", pp. 32

62. Giovanni Solinas [1990] "Competenze, grandi industrie e distretti industriali,. Il caso Magneti Marelli", pp. 23

63. Andrea Ginzburg [1990] "Debito pubblico, teorie monetarie e tradizione civica nell'Inghilterra del Settecento", pp. 30

64. Mario Forni [1990] "Incertezza, informazione e mercati assicurativi: una rassegna", pp. 37

65. Mario Forni [1990] "Misspecification in Dynamic Models", pp. 19

66. Gian Paolo Caselli e Gabriele Pastrello [1990] "Service Sector Growth in CPE's: An Unsolved Dilemma", pp. 28

67. Paola Bertolini [1990] "La situazione agro-alimentare nei paesi ad economia avanzata", pp. 20

68. Paola Bertolini [1990] "Sistema agro-alimentare in Emilia Romagna ed occupazione", pp. 65

69. Enrico Giovannetti [1990] "Efficienza ed innovazione: il modello "fondi e flussi" applicato ad una filiera agro-industriale", pp. 38

70. Margherita Russo [1990] "Cambiamento tecnico e distretto industriale: una verifica empirica", pp. 115

71. Margherita Russo [1990] "Distretti industriali in teoria e in pratica: una raccolta di saggi", pp. 119

72. Paolo Silvestri [1990] " La Legge Finanziaria. Voce dell'enciclopedia Europea Garzanti", pp. 8

73. Rita Paltrinieri [1990] "La popolazione italiana: problemi di oggi e di domani", pp. 57

74. Enrico Giovannetti [1990] "Illusioni ottiche negli andamenti delle Grandezze distributive: la scala mobile e l''appiattimento' delle retribuzioni in una ricerca", pp. 120

75. Enrico Giovannetti [1990] "Crisi e mercato del lavoro in un distretto industriale: il bacino delle ceramiche. Sez I", pp. 150

76. Enrico Giovannetti [1990] " Crisi e mercato del lavoro in un distretto industriale: il bacino delle ceramiche. Sez. II", pp. 145

78. Antonietta Bassetti e Costanza Torricelli [1990] "Una riqualificazione dell'approccio bargaining alla selezioni di portafoglio", pp. 4

77. Antonietta Bassetti e Costanza Torricelli [1990] "Il portafoglio ottimo come soluzione di un gioco bargaining", pp. 15

79. Mario Forni [1990] "Una nota sull'errore di aggregazione", pp. 6

80. Francesca Bergamini [1991] "Alcune considerazioni sulle soluzioni di un gioco bargaining", pp. 21

81. Michele Grillo e Michele Polo [1991] "Political Exchange and the allocation of surplus: a Model of Two-party competition", pp. 34

82. Gian Paolo Caselli e Gabriele Pastrello [1991] "The 1990 Polish Recession: a Case of Truncated Multiplier Process", pp. 26

83. Gian Paolo Caselli e Gabriele Pastrello [1991] "Polish firms: Pricate Vices Pubblis Virtues", pp. 20

84. Sebastiano Brusco e Sergio Paba [1991] "Connessioni, competenze e capacità concorrenziale nell'industria della Sardegna", pp. 25

85. Claudio Grimaldi, Rony Hamaui, Nicola Rossi [1991] "Non Marketable assets and households' Portfolio Choice: a Case of Study of Italy", pp. 38

86. Giulio Righi, Massimo Baldini, Alessandra Brambilla [1991] "Le misure degli effetti redistributivi delle imposte indirette: confronto tra modelli alternativi", pp. 47

87. Roberto Fanfani, Luca Lanini [1991] "Innovazione e servizi nello sviluppo della meccanizzazione agricola in Italia", pp. 35

88. Antonella Caiumi e Roberto Golinelli [1992] "Stima e applicazioni di un sistema di domanda Almost Ideal per l'economia italiana", pp. 34

89. Maria Cristina Marcuzzo [1992] "La relazione salari-occupazione tra rigidità reali e rigidità nominali", pp. 30

90. Mario Biagioli [1992] "Employee financial participation in enterprise results in Italy", pp. 50

91. Mario Biagioli [1992] "Wage structure, relative prices and international competitiveness", pp. 50

92. Paolo Silvestri e Giovanni Solinas [1993] "Abbandoni, esiti e carriera scolastica. Uno studio sugli studenti iscritti alla Facoltà di Economia e Commercio dell'Università di Modena nell'anno accademico 1990/1991", pp. 30

93. Gian Paolo Caselli e Luca Martinelli [1993] "Italian GPN growth 1890-1992: a unit root or segmented trend representatin?", pp. 30

94. Angela Politi [1993] "La rivoluzione fraintesa. I partigiani emiliani tra liberazione e guerra fredda, 1945-1955", pp. 55

95. Alberto Rinaldi [1993] "Lo sviluppo dell'industria metalmeccanica in provincia di Modena: 1945-1990", pp. 70

96. Paolo Emilio Mistrulli [1993] "Debito pubblico, intermediari finanziari e tassi d'interesse: il caso italiano", pp. 30

97. Barbara Pistoresi [1993] "Modelling disaggregate and aggregate labour demand equations. Cointegration analysis of a labour demand function for the Main Sectors of the Italian Economy: 1950-1990", pp. 45

98. Giovanni Bonifati [1993] "Progresso tecnico e accumulazione di conoscenza nella teoria neoclassica della crescita endogena. Una analisi critica del modello di Romer", pp. 50

99. Marcello D'Amato e Barbara Pistoresi [1994] "The relationship(s) among Wages, Prices, Unemployment and Productivity in Italy", pp. 30

100. Mario Forni [1994] "Consumption Volatility and Income Persistence in the Permanent Income Model", pp. 30

101. Barbara Pistoresi [1994] "Using a VECM to characterise the relative importance of permanent and transitority components", pp. 28

102. Gian Paolo Caselli and Gabriele Pastrello [1994] "Polish recovery form the slump to an old dilemma", pp. 20

103. Sergio Paba [1994] "Imprese visibili, accesso al mercato e organizzazione della produzione", pp. 20

104. Giovanni Bonifati [1994] "Progresso tecnico, investimenti e capacità produttiva", pp. 30

105. Giuseppe Marotta [1994] "Credit view and trade credit: evidence from Italy", pp. 20

106. Margherita Russo [1994] "Unit of investigation for local economic development policies", pp. 25

107. Luigi Brighi [1995] "Monotonicity and the demand theory of the weak axioms", pp. 20

108. Mario Forni e Lucrezia Reichlin [1995] "Modelling the impact of technological change across sectors and over time in manufactoring", pp. 25

109. Marcello D'Amato and Barbara Pistoresi [1995] "Modelling wage growth dynamics in Italy: 1960-1990", pp. 38

110. Massimo Baldini [1995] "INDIMOD. Un modello di microsimulazione per lo studio delle imposte indirette", pp. 37

111. Paolo Bosi [1995] "Regionalismo fiscale e autonomia tributaria: l'emersione di un modello di consenso", pp. 38

112. Massimo Baldini [1995] "Aggregation Factors and Aggregation Bias in Consumer Demand", pp. 33

113. Costanza Torricelli [1995] "The information in the term structure of interest rates. Can stocastic models help in resolving the puzzle?" pp. 25

114. Margherita Russo [1995] "Industrial complex, pôle de développement, distretto industriale. Alcune questioni sulle unità di indagine nell'analisi dello sviluppo." pp. 45

115. Angelika Moryson [1995] "50 Jahre Deutschland. 1945 - 1995" pp. 21

116. Paolo Bosi [1995] "Un punto di vista macroeconomico sulle caratteristiche di lungo periodo del nuovo sistema pensionistico italiano." pp. 32

117. Gian Paolo Caselli e Salvatore Curatolo [1995] "Esistono relazioni stimabili fra dimensione ed efficienza delle istituzioni e crescita produttiva? Un esercizio nello spirito di D.C. North." pp. 11

118. Mario Forni e Marco Lippi [1995] "Permanent income, heterogeneity and the error correction mechanism." pp. 21

119. Barbara Pistoresi [1995] "Co-movements and convergence in international output. A Dynamic Principal Components Analysis" pp. 14

120. Mario Forni e Lucrezia Reichlin [1995] "Dynamic common factors in large cross-section" pp. 17

121. Giuseppe Marotta [1995] "Il credito commerciale in Italia: una nota su alcuni aspetti strutturali e sulle implicazioni di politica monetaria" pp. 20

122. Giovanni Bonifati [1995] "Progresso tecnico, concorrenza e decisioni di investimento: una analisi delle determinanti di lungo periodo degli investimenti" pp. 25

123. Giovanni Bonifati [1995] "Cambiamento tecnico e crescita endogena: una valutazione critica delle ipotesi del modello di Romer" pp. 21

124. Barbara Pistoresi e Marcello D'Amato [1995] "La riservatezza del banchiere centrale è un bene o un male? ,Effetti dell'informazione incompleta sul benessere in un modello di politica monetaria." pp. 32

125. Barbara Pistoresi [1995] "Radici unitarie e persistènza: l'analisi univariata delle fluttuazioni economiche." pp. 33

126. Barbara Pistoresi e Marcello D'Amato [1995] "Co-movements in European real outputs" pp. 20

127. Antonio Ribba [1996] "Ciclo economico, modello lineare-stocastico, forma dello spettro delle variabili macroeconomiche" pp. 31

128. Carlo Alberto Magni [1996] "Repeatable and una tantum real options a dynamic programming approach" pp. 23

129. Carlo Alberto Magni [1996] "Opzioni reali d'investimento e interazione competitiva: programmazione dinamica stocastica in optimal stopping" pp. 26

130. Carlo Alberto Magni [1996] "Vaghezza e logica fuzzy nella valutazione di un'opzione reale" pp. 20

131. Giuseppe Marotta [1996] "Does trade credit redistribution thwart monetary policy? Evidence from Italy" pp. 20

132. Mauro Dell'Amico e Marco Trubian [1996] "Almost-optimal solution of large weighted equicut problems" pp. 30

133. Carlo Alberto Magni [1996] "Un esempio di investimento industriale con interazione competitiva e avversione al rischio" pp. 20

134. Margherita Russo, Peter Börkey, Emilio Cubel, François Lévêque, Francisco Mas [1996] "Local sustainability and competitiveness: the case of the ceramic tile industry" pp. 66

135. Margherita Russo [1996] "Camionetto tecnico e relazioni tra imprese" pp. 190

136. David Avra Lane, Irene Poli, Michele Lalla, Alberto Roverato [1996] "Lezioni di probabilità e inferenza statistica" pp. 288

137. David Avra Lane, Irene Poli, Michele Lalla, Alberto Roverato [1996] "Lezioni di probabilità e inferenza statistica - Esercizi svolti - " pp. 302

138. Barbara Pistoresi [1996] "Is an Aggregate Error Correction Model Representative of Disaggregate Behaviours? An example" pp. 24

139. Luisa Malaguti e Costanza Torricelli [1996] "Monetary policy and the term structure of interest rates" , pp. 30

140. Mauro Dell'Amico, Martine Labbé, Francesco Maffioli [1996] "Exact solution of the SONET Ring Loading Problem", pp. 20

141. Mauro Dell'Amico, R.J.M. Vaessens [1996] "Flow and open shop scheduling on two machines with transportation times and machine-independent processing times in NP-hard, pp. 10

142. M. Dell'Amico, F. Maffioli, A. Sciomechen [1996] "A Lagrangean Heuristic for the Pirze Collecting Travelling Salesman Problem", pp. 14

143. Massimo Baldini [1996] "Inequality Decomposition by Income Source in Italy - 1987 - 1993", pp. 20

144. Graziella Bertocchi [1996] "Trade, Wages, and the Persistence of Underdevelopment" pp. 20

145. Graziella Bertocchi and Fabio Canova [1996] "Did Colonization matter for Growth? An Empirical Exploration into the Historical Causes of Africa's Underdevelopment" pp. 32

146. Paola Bertolini [1996] "La modernization de l'agricolture italienne et le cas de l'Emilie Romagne" pp. 20

147. Enrico Giovannetti [1996] "Organisation industrielle et développement local: le cas de l'agroindutrie in Emilie Romagne" pp. 18

148. Maria Elena Bontempi e Roberto Golinelli [1996] "Le determinanti del leverage delle imprese: una applicazione empirica ai settori industriali dell'economia italiana" pp. 31

149. Paola Bertolini [1996] "L'agriculture et la politique agricole italienne face aux recents scenarios", pp. 20

150. Enrico Giovannetti [1996] "Il grado di utilizzo della capacità produttiva come misura dei costi di transazione: una rilettura di 'Nature of the Firm' di R. Coase", pp. 75

151. Enrico Giovannetti [1996] "Il I° ciclo del Diploma Universitario Economia e Amministrazione delle Imprese", pp. 25

152. Paola Bertolini, Enrico Giovannetti, Giulia Santacaterina [1996] "Il Settore del Verde Pubblico. Analisi della domanda e valutazione economica dei benefici", pp. 35

153. Giovanni Solinas [1996] "Sistemi produttivi del Centro-Nord e del Mezzogiorno. L'industria delle calzature", pp. 55

154. Tindara Addabbo [1996] "Married Women's Labour Supply in Italy in a Regional Perspective", pp. 85

155. Paolo Silvestri, Giuseppe Catalano, Cristina Bevilacqua [1996] "Le tasse universitarie e gli interventi per il diritto allo studio: la prima fase di applicazione di una nuova normativa" pp. 159

156. Sebastiano Brusco, Paolo Bertossi, Margherita Russo [1996] "L'industria dei rifiuti urbani in Italia", pp. 25

157. Paolo Silvestri, Giuseppe Catalano [1996] "Le risorse del sistema universitario italiano: finanziamento e governo" pp. 400

158. Carlo Alberto Magni [1996] "Un semplice modello di opzione di differimento e di vendita in ambito discreto", pp. 10

159. Tito Pietra, Paolo Siconolfi [1996] "Fully Revealing Equilibria in Sequential Economies with Asset Markets" pp. 17

160. Tito Pietra, Paolo Siconolfi [1996] "Extrinsic Uncertainty and the Informational Role of Prices" pp. 42

161. Paolo Bertella Farnetti [1996] "Il negro e il rosso. Un precedente non esplorato dell'integrazione afroamericana negli Stati Uniti" pp. 26

162. David Lane [1996] "Is what is good for each best for all? Learning from others in the information contagion model" pp. 18

163. Antonio Ribba [1996] "A note on the equivalence of long-run and short-run identifying restrictions in cointegrated systems" pp. 10

164. Antonio Ribba [1996] "Scomposizioni permanenti-transitorie in sistemi cointegrati con una applicazione a dati italiani" pp. 23

165. Mario Forni, Sergio Paba [1996] "Economic Growth, Social Cohesion and Crime" pp. 20

166. Mario Forni, Lucrezia Reichlin [1996] "Let's get real: a factor analytical approch to disaggregated business cycle dynamics" pp. 25

167. Marcello D'Amato e Barbara Pistoresi [1996] "So many Italies: Statistical Evidence on Regional Cohesion" pp. 31

168. Elena Bonfiglioli, Paolo Bosi, Stefano Toso [1996] "L'equità del contributo straordinario per l'Europa" pp. 20

169. Graziella Bertocchi, Michael Spagat [1996] "Il ruolo dei licei e delle scuole tecnico-professionali tra progresso tecnologico, conflitto sociale e sviluppo economico" pp. 37

170. Gianna Boero, Costanza Torricelli [1997] "The Expectations Hypothesis of the Term Structure of Interest Rates: Evidence for Germany" pp. 15

171. Mario Forni, Lucrezia Reichlin [1997] "National Policies and Local Economies: Europe and the US" pp. 22

172. Carlo Alberto Magni [1997] "La trappola del Roe e la tridimensionalità del Van in un approccio sistemico", pp. 16

173. Mauro Dell'Amico [1997] "A Linear Time Algorithm for Scheduling Outforests with Communication Delays on Two or Three Processor"pp. 18

174. Paolo Bosi [1997] "Aumentare l'età pensionabile fa diminuire la spesa pensionistica? Ancora sulle caratteristiche di lungo periodo della riforma Dini" pp. 13

175. Paolo Bosi e Massimo Matteuzzi [1997] "Nuovi strumenti per l'assistenza sociale" pp 31

176. Mauro Dell'Amico, Francesco Maffioli e Marco Trubian [1997] "New bounds for optium traffic assignment in satellite communication" pp. 21

177. Carlo Alberto Magni [1997] "Paradossi, inverosimiglianze e contraddizioni del Van: operazioni certe" pp. 9

178. Barbara Pistoresi e Marcello D'Amato [1997] "Persistence of relative unemployment rates across italian regions" pp. 25

179. Margherita Russo, Franco Cavedoni e Riccardo Pianesani [1997] "Le spese ambientali dei Comuni in provincia di Modena, 1993-1995" pp. 23

180. Gabriele Pastrello [1997] "Time and Equilibrium, Two Elisive Guests in the Keynes-Hawtrey-Robertson Debate in the Thirties" pp. 25

181. Luisa Malaguti e Costanza Torricelli [1997] "The Interaction Between Monetary Policy and the Expectation Hypothesis of the Term Structure of Interest rates in a N-Period Rational Expectation Model" pp. 27

182. Mauro Dell'Amico [1997] "On the Continuous Relaxation of Packing Problems – Technical Note" pp. 8

183. Stefano Bordoni [1997] "Prova di Idoneità di Informatica Dispensa Esercizi Excel 5" pp 49

184. Francesca Bergamini e Stefano Bordoni [1997] "Una verifica empirica di un nuovo metodo di selezione ottima di portafoglio" pp. 22

185. Gian Paolo Caselli e Maurizio Battini [1997] "Following the tracks of atkinson and micklewright the changing distribution of income and earnings in poland from 1989 to 1995".pp 21

186. Mauro Dell'Amico e Francesco Maffioli [1997] "Combining Linear and Non-Linear Objectives in Spanning Tree Problems" pp. 21

187. Gianni Ricci e Vanessa Debbia [1997] "Una soluzione evolutiva in un gioco differenziale di lotta di classe" pp.14

188. Fabio Canova e Eva Ortega [1997] "Testing Calibrated General Equilibrium Model" pp 34

189. Fabio Canova [1997] "Does Detrending Matter for the Determination of the Reference Cycle and the Selection of Turning Points?" pp. 35

190. Fabio Canova e Gianni De Nicolò [1997] "The Equity Premium and the Risk Free Rate: A Cross Country, Cross Maturity Examination" pp. 41

191. Fabio Canova e Angel J. Ubide [1997] "International Business Cycles, Financial Market and Household Production" pp. 32

192. Fabio Canova e Gianni De Nicolò [1997] "Stock Returns, Term Structure, Inflation and Real Activity: An International Perspective" pp. 33

193. Fabio Canova e Morten Ravn [1997] "The Macroeconomic Effects of German Unification: Real Adjustmentsand the Welfare State" pp. 34

194. Fabio Canova [1997] "Detrending and Business Cycle Facts" pp. 40

195. Fabio Canova e Morten O. Ravn [1997] "Crossing the Rio Grande: Migrations, Business Cycle and the Welfare State" pp. 37

196. Fabio Canova e Jane Marrinan [1997] "Sources and Propagation of International Output Cycles: Common Shocks or Transmission?" pp. 41

197. Fabio Canova e Albert Marcet [1997] "The Poor Stay Poor: Non-Convergence Across Countries and Regions" pp. 44

198. Carlo Alberto Magni [1997] "Un Criterio Strutturalista per la Valutazione di Investimenti" pp. 17

199. Stefano Bordoni [1997] "Elaborazione Automatica dei Dati" pp. 60

200. Paolo Bertella Farnetti [1997] "The United States and the Origins of European Integration" pp. 19

201. Paolo Bosi [1997] "Sul Controllo Dinamico di un Sistema Pensionistico a Ripartizione di Tipo Contributivo" pp 17

202. Paola Bertolini [1997] "European Union Agricultural Policy: Problems and Perspectives" pp18

203. Stefano Bordoni [1997] "Supporti Informatici per la Ricerca delle soluzioni di Problemi Decisionali" pp30

204. Carlo Alberto Magni [1997] "Paradossi, Inverosimiglianze e Contraddizioni del Van: Operazioni Aleatorie" pp10

205. Carlo Alberto Magni [1997] "Tir, Roe e Van: Distorsioni linguistiche e Cognitive nella Valutazione degli Investimenti" pp 17

206. Gisella Facchinetti, Roberto Ghiselli Ricci e Silvia Muzzioli [1997] "New Methods For Ranking Triangular Fuzzy Numbers: An Investment Choice" pp 9

207. Mauro Dell'Amico e Silvano Martello [1997] "Reduction of the Three-Partition Problem" pp16

208. Carlo Alberto Magni [1997] "IRR, ROE and NPV: a Systemic Approach" pp. 20

209. Mauro Dell'Amico, Andrea Lodi e Francesco Maffioli [1997] "Solution of the cumulative assignment problem with a well-structured tabu search method" pp. 25

210. Carlo Alberto Magni [1997] "La definizione di investimento e criterio del Tir ovvero: la realtà inventata" pp.16

211. Carlo Albero Magni [1997] "Critica alla definizione classica di investimento: un approccio sistematico" pp17

212. Alberto Roverato [1997] "Asymptotic prior to posterior analysis for graphical gaussian models" pp.8

213. Tindara Addabbo [1997] "Povertà nel 1995 analisi statica e dinamica sui redditi familiari" pp 64

214. Gian Paolo Caselli e Franca Manghi [1997] "La transizione da piano a mercato e il modello di Ising" pp15

215. Tindara Addabbo [1998] "Lavoro non pagato e reddito esteso: un'applicazione alle famiglie italiane in cui entrambi i coniugi sono lavoratori dipendenti" pp 54

216. Tindara Addabbo [1998] "Probabilità di occupazione e aspettative individuali" pp 36

217. Lara Magnani [1998] "Transazioni, contratti e organizzazioni: una chiave di lettura della teoria economica dell'organizzazione pp 39

218. Michele Lalla, Rosella Molinari e Maria Grazia Modena [1998] "La progressione delle carriere: i percorsi in cardiologia" pp 46

219. Lara Magnani [1998] "L'organizzazione delle transazioni di subfornitura nel distretto industriale" pp 40

220. Antonio Ribba [1998] "Recursive VAR orderings and identification of permanent and transitory shocks" pp12

221. Antonio Ribba [1998] "Granger-causality and exogeneity in cointegrated Var models" pp 5

222. Luigi Brighi e Marcello D'Amato [1998] "Optimal Procurement in Multiproduct Monopoly" pp 25

223. Paolo Bosi, Maria Cecilia Guerra e Paolo Silvestri [1998] "La spesa sociale nel comune Modena" Rapporto intermedio pp 37