Real Options: A Fuzzy Approach for Strategic Investments

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Abstract

This paper presents a proposal for evaluating real options. The current options-based models have provided new insights into capital-budgeting decisions. Unfortunately they are not widely used by corporate managers and practitioners as they are formally complex, rather difficult to understand and rest on strong implicit assumptions which considerably limit their scope of application. We propose a possible alternative by using a fuzzy expert system. We draw up a decision tree with multiple uncertain variables affecting the values of a compound option, consisting of a defer option, a growth option, an abandonment option. Some simulations test the economic soundness of the model as well as its consistency with the current models in the literature. The `vagueness' of the model enables us to raise the complexity in the situation treated while reducing at the same time the formal difficulties. A rather refined study can be accomplished by showing how inputs and outputs of the model interrelate one another.

Introduction

This paper presents a proposal of real option evaluations through fuzzy logic. We draw up a fuzzy expert system which automatically provides the value of an option to invest. In particular, we aim at showing that fuzzy logic seems to be a favourable framework for decision processes in capital budgeting. Further, many of the drawbacks implicit in the use of contingent claims methods and dynamic programming can be overwhelmed, to a certain extent, by a more `vague' approach.

In recent years a wide number of contributions have been published about real options. Since the eighties investment opportunities have been conceptually likened to financial options so that the use of Black-Scholes analysis seemed a better evaluation tool than the traditional NPV. The latter is not able to handle cases in which the decision maker has some flexibility on the project. The now-or-never investment the NPV rule subsumes is not always what the decision maker is dealing with. Sometimes decisions can be deferred, and even in a now-or-never context the investor may have the opportunity to abandon the investment or suspend operations for a while; she may sometimes contract or conversely expand the scale of the business or has some other kind of flexibility. Treating these cases as financial options induces the use of the Black-Scholes equation. This implies that the risk of the project can be spanned by existing assets in the economy. When this is not the case some problems arise, but then stochastic dynamic programming can provide as well analogous results (see [Dixit, Pindyck, 1994] for an exhaustive explanation and for references). Unfortunately, despite the formal elegance and the theoretical soundness of these methods, their applicability is rather difficult, due to formal complexity and to assumptions being not always realistic.

The success they have encountered in the literature in the last twenty years is not coupled by their practical use by corporate managers and practitioners. Using contingent claims analysis (henceforth CCA) or stochastic dynamic programming (henceforth SDP) requires advanced knowledge of mathematics, which most managers do not have. Further, CCA or SDP are not so appealing since they are not intuitive nor easily understood. We aim at constructing a model apt to overwhelm these difficulties. In particular, the model does not assume any particular stochastic process for the variable on which the project value and the option value are dependent. Neither we do any assumption about completeness of the market (CCA is bound to this strong assumption). We also show that, unlike CCA and SPD, the proposed expert system is able to handle a set of many random (as well as certain) variables if they are gathered in subsets playing different roles. Competition is treated directly, not indirectly through an expected decrease of payoffs, and the degree of exclusivity of the option is also considered via different inputs.

Another significant element is the capability of dealing with qualitative variables. In capital budgeting, especially in strategic investment decisions, many types of variables are considered in the decision process. Quantitative variables as the Net Present Value are very important, but they constitute only part of all variables considered. Some of them are inevitably linguistic variables, that is they cannot be translated in crisp values, unless sometimes indirectly. It is very difficult to quantify the ability of the management or the degree of differentiation or the intensity of rivalry in a sector or the level of entry and exit barriers. Even less natural is to think of them as stochastic processes following some standard path. The vagueness implicit in many variables is such that these variables are removed from CCA and SPD models, as they are impossible (or at least very difficult) to be treated. A first attempt to include qualitative variables in real options can be found in [Magni, 1998]. The latter's idea of coping with one qualitative variable is here generalized by means of the expert fuzzy system. The model should be rather appealing to managers as it is easily understandable:

- it relies on a general schema and on rules which are to be drawn up with the help of managers themselves;
- it does not require advanced knowledge of mathematics and it is intended to provide a tool which should, in principle, repeat the decision process accomplished by a panel of experts.

The evaluation is automatically determined once fixed the value of the inputs on the basis of the rules selected by the evaluator. Our model is then an algorithm which embodies a qualitative analysis in a quantitative framework. The choice of a fuzzy expert system to approach this problem is due to the fact that fuzzy expert systems are effective in representing explicit but ambiguous common-sense knowledge. Expert systems are knowledge-based systems that contain expert knowledge. One way to represent inexact data and knowledge closer to humanlike thinking is to use fuzzy rules instead of exact rules and fuzzy numbers instead of classic range of inputs variability. Fuzzy rules represent in a straightforward way "common-sense" knowledge and skills, or knowledge that is subjective, ambiguous, vague or contradictory. Fuzzy numbers helps translating the truth values for a fuzzy proposition, which is not TRUE or FALSE, as in Boolean logic, but includes all the greyness between the two extreme values. The secret of the success of fuzzy systems is that they are easy to implement, easy to maintain, easy to understand, robust and cheap [Kasabov, 1996].

1. Real Options: The Classical Approach.

Evaluation of investments under uncertainty is the main goal of capital budgeting. A widespread profitability index is the traditional Net Present Value, which measures how a project is able to increase wealth by emphasizing the role of the timing of projects' cash flows and of an alternative equivalent-risk asset. The discounted cash flow model is not suited for situations where the investor has the ability to defer decisions or abandon a project already undertaken or temporarily suspend production or expand business.

In such cases an investor is concerned with investment opportunities having some degree of flexibility: she is facing what is usually called a real option. A defer option is an option where the decision maker is allowed to postpone decision, a growth option consists of the option to enlarge the scale of the project, an abandonment option implies the opportunity to abandon the project when desired, a switch option consists of the ability to switch, and so on. One or more options often combine together in a single project: We have then a compound option. The value of a real option is function of one or more uncertain underlying variables that may considerably affect the project's cash flows. The analogy of these situations with financial options has been pointed out in the eighties [Kester, 1986] and this has led scholars and academics to apply Black-Scholes analysis for capital budgeting purposes [Trigeorgis, 1986, 1996]. The value of an investment option is then contingent to the value of some random variables and its exercise value is given by the expected net present value of the project. The implicit assumption is made that the project can be spanned by existing assets in capital markets. If this assumption is not encountered, then stochastic dynamic programming can be used and the decision process is shaped as an optimal stopping problem.

For the sake of clarity, we briefly remind here how the classical literature of real options copes with the case of one single underlying variable for a defer option (the exposition follows [Dixit and Pindyck, *op.cit.*]). We will make use of the stochastic dynamic programming, which can be applied even if the project cannot be spanned by existing assets (this is often the case for research and development investments).

Let V be the value of a project and assume that it follows a geometric brownian motion with drift α so that

$$dV = \alpha V dt + \sigma V dz \qquad \text{with} \qquad dz = \varepsilon\sqrt{dt} \tag{1.1}$$

where ε is a standardized normal variable (dz shows a Wiener process).

Thus, the current value of V can be observed but its future values are uncertain according to a lognormal distribution with a variance linearly increasing with respect to time. We assume that the investment opportunity never expires and that uncertainty is never resolved. An investor has the opportunity of investing in this project, say P, with an initial outlay of I. The investor must decide whether she has to invest straight away or wait for a better moment (financially speaking, this is an American call option). Waiting has a positive value, as it brings about further knowledge about the project profitability, but has also a negative value, in that the investor is renouncing to earn V. At a sufficiently high value of V, the decision maker is prone to invest in the project. But what does ``sufficiently high'' mean?

Let V^* be such that if $V>V^*$ the decision maker undertakes the investment, otherwise she waits for further information. V^* is then the threshold above which exercise is convenient. The threshold divides the two regions of waiting and investing. As for the former we have that the value of the option $F(V)$ is given by the Bellmann equation

$$F(V) = \frac{1}{1+\rho dt}\mathrm{E}(F(V+dV)) \tag{1.2}$$

where ρ is the discount rate (to be intended as the required rate of return), subjectively determined by the investor. (1.2) tells us that in the continuation region $V \le V^*$ the investor receives the expected continuation payoff *E(F(V+dV))* which is discounted for the length of time *dt*. Using Ito's Lemma we obtain

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \alpha V F'(V) - \rho F(V) = 0 \tag{1.3}$$

Conversely, in the optimal stopping region, that is $V>V^*$, exercise is required, so that

$$F(V) = V - I \tag{1.4}$$

The boundary conditions for this problem are

$$F(0)=0 \tag{1.5}$$

$$F(V^*)=V^*-I \tag{1.6}$$

$$F'(V^*)=1 \tag{1.7}$$

The solution is a function of the form

$$F(V) = A_1 V^{\kappa_1} + A_2 V^{\kappa_2} \tag{1.8}$$

where k_1 and k_2 are, respectively, the positive and negative solution of the following quadratic equation:

$$\frac{1}{2}\sigma^2 \kappa(\kappa - 1) + \alpha\kappa - \rho = 0 \tag{1.9}$$

(1.5) implies $A_2 = 0$ so that

$$F(V) = A_1 V^{\kappa_1}$$

whence we get the unique threshold

$$V^* = \frac{\kappa_1}{\kappa_1 - 1} I$$

Since $\frac{\kappa_1}{\kappa_1 - 1} > 1$ the value of waiting is such that the decision maker is not induced to invest if *V* reaches *I*, as the Net Present Value rule says, but only if it reaches a higher level. This is due to risk, which persuades investors to wait for a safer level of *V*. If the stochastic variations of *V* can be spanned by existing assets in the economy then we can use Black-Scholes assumptions: we will reach as well equation (1.3).

2. A New Model.

We aim at evaluating a compound real option consisting of a defer option, an abandonment option and a growth option. The latter two are contingent to the first one. The schema we draw up is general enough to be applied to a wide varieties of similar cases. It is also flexible as many variables can be easily substituted or added. In the classical literature of real options stochastic dynamic programming or contingent claims analysis is usually used with one single variable. This variable follows a particular random path such as geometric brownian motion or a Poisson jump process and influences the NPV of the project. Unfortunately real situations are more sophisticated and the idea of a diffusion process for a random variable is only sometimes acceptable. And then, which diffusion process should be preferred? Other difficulties arise when more than one random variable is involved in the evaluation process. For example, the option can be exclusive or shared, but the degree of exclusivity can vary with time depending on isolating devices owned by the investor and by the expected retaliation from competitors. Further, in strategic investments a qualitative analysis is often implemented: some factors are difficult to quantify but they may strongly influence the success of the investment. Moreover, our abandonment and growth options depend on random variables like exit barriers (the abandonment option) and the additional investment cost (the growth option). Obviously, only the second one is quantifiable with crisp values.

Traditionally, an investment analysis is based on the computation of the NPV. In strategic options, where so many and so qualitatively different variables are at work, it can be difficult to directly estimate the expected cash flows, sometimes it is even impossible to link qualitative variables to cash flows. Moreover, the decision maker has multiple objectives only some of which can be adequately captured in the computation of an NPV. So we propose a model where the NPV is only one of many indexes and cooperates with other drivers to determine the exercise value (*ExV*) of the option to invest. The expected cash flows are then to be intended just as those cash flows that the investor expects from the project regardless of any other consideration about competition and other qualitative factors. The investment value relies then on the NPV as well as on other drivers as we will see. In this complex situations, it is no surprise that traditional and modern capital budgeting techniques are forced to leave the ground. It is actually recognized [Lander, Pinches (1998)] that current real options models are not widely used in practice and an option-based analysis inevitably limits the scope of application simplifying for mathematical tractability. We intend to follow the suggestion of Lander and Pinches, who assert that ``the focus of the analysis should be on the initial decision to be made or the optimal strategy to implement, and *not the exact valuation obtained*'' (p.552, italics ours). They propose decision trees or influence diagrams to circumvent the difficulties of option-based models. This paper shows that fuzzy logic can be a valid alternative for modeling real options.

3. Fuzzy Set Theory

Fuzzy set theory was originally proposed as a means for representing indeterminacy and formalizing qualitative concepts that have no precise boundaries. In many situations, it is difficult to describe phenomena simply in terms of black and white distinctions. Language, our primary means of communication, is anything but precise. In fact, fuzzy set theory supports reasoning about these kinds of situations. It is based on gradation instead of sharp distinction. It is a method of reasoning that allows for partial or "fuzzy" description of reality.

Consider a classical (crisp) set A contained in a universe X. A fuzzy set A is defined by a set of ordered pairs,

$$A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$$

where $\mu_A(x)$ is called Membership function of the set *A*.
The **height** of a fuzzy set is the maximum value that its membership function realises.
A fuzzy set is called **normal** if its height is 1.
If the fuzzy set is not normal, it is always possible to **normalise** it, changing $\mu_A(x)$ with:

$$\tilde{\mu}_A(x) = \frac{\mu_A(x)}{\max_{x \in A} \mu_A(x)}$$

The **domain** of a fuzzy set A is the domain of. $\mu_A(x)$.
The **support** of a fuzzy set A is the subset of X in which the membership function is positive.
A fuzzy set A is a **Convex fuzzy set**, if
$\forall \lambda \in [0,1]$ and $\forall x_1, x_2 \in A$

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

A **fuzzy number** is a fuzzy set defined on the universe R, which is convex and normalised. A great variety of membership functions have been proposed in the scientific literature. The more common types of membership functions are the piece-linear or spline shapes. In this paper we use triangular, trapezoidal types called "Standard Membership Function".

3.1. A Fuzzy Expert System

An expert system is an intelligent machine that uses knowledge and inference procedures to solve problems that are difficult enough to require significant human expertise for their solutions. The knowledge of an expert system consists of facts and heuristics. The facts usually constitute a body of information that is widely shared, publicly available, and generally agreed upon by experts in the field. Heuristics concerns mostly private information and rules of good judgement that characterise expert-level decision making in the field. A fuzzy expert system is an expert system that utilises fuzzy sets and fuzzy logic to overcome some of the problems which occur when the data provided by the user are vague or incomplete. The power of fuzzy set theory comes from the ability to describe linguistically a particular phenomenon or process, and then to represent that description with a small number of very flexible rules. In a fuzzy system, the knowledge is contained both in its rules and in fuzzy sets, which hold general description of the properties of the phenomenon under consideration. One of the major differences between a fuzzy expert system and another expert system is that the first can infer multiple conclusions. In fact it provides all possible solutions whose truth is above a certain threshold, and the user or the application program can then choose the appropriate solution depending on the particular situation. This fact adds flexibility to the system and makes it more powerful. Fuzzy expert systems use fuzzy data, fuzzy rules, and fuzzy inference, in addition to the standard ones implemented in the ordinary expert systems.

Functionally a fuzzy system can be described as a function approximator. More specifically it aims at performing an approximate implementation of an unknown mapping

$\phi : A \subset R^n \rightarrow R^m$ where A is a compact of R^n. By means of variable knowledge relevant to the unknown mapping [Kosko, 1992] and [Wang, 1992] independently proved that fuzzy systems are dense in the space of continuous functions on a compact domain and therefore can approximate arbitrarily well any continuous function on a compact domain. The following are the main phases of a fuzzy system design:

1. Identification of the problem and choice of the type of fuzzy system which best suits the problem requirement. A modular system can be designed consisting of several fuzzy modules linked together. A modular approach, if applicable, may greatly simplify the design of the whole system, dramatically reducing its complexity and making it more comprehensible.
2. Definition of input and output variables, their linguistic attributes (fuzzy values) and their membership function (fuzzification of input and output).
3. Definition of the set of heuristic fuzzy rules. (IF-THEN rules).
4. Choice of the fuzzy inference method (selection of aggregation operators for precondition and conclusion).
5. Translation of the fuzzy output in a crisp value (defuzzification methods).
6. Test of the fuzzy system prototype, drawing of the goal function between input and output fuzzy variables, change of membership functions and fuzzy rules if necessary, tuning of the fuzzy system, validation of results.

In building fuzzy expert systems, the crucial steps are the fuzzification and the construction of blocks of fuzzy rules. These steps can be handled in two different ways. The first is accomplished by using information obtained through interviews to the experts of the problem. The second is accomplished by using methods of machine-learning, neural networks and genetic algorithms to learn membership functions and fuzzy rules. The two approaches are quite different. The first does not use the past history of the problem, but it relies on the experience of experts who have worked in the field for years. The second is based only on past data and project into the future the same structure of the past. The first approach seems preferable for our purpose, for our main goals are:

- constructing a general framework able to handle many infinite cases of investment opportunities;
- verifying the theoretical soundness and robustness of the model through sensitivity analysis, in order to assure that actual applicability is not biased;
- comparing our simulations with the traditional ones found in the literature of real options;
- presenting a completely new approach, which replace CCA and SDP with a fuzzy expert system, and showing how it can be immediately applied to real situations.

We can formalize the steps in the following manner. For each linguistic variable, input x_i (i=1…m) and output y, we have to fix its own range of variability U_i and V.
$\forall$ *i*, (*i*=1…m), if n_i is the number of the linguistic attribute of the variable x_i and
$\hat{n} = \max_{i \in [1,m]} n_i$, we define the set

$$A^i = \{A_1^i, A_2^i, .., A_{j_i}^i, .., A_{n_i}^i\}$$

where $\forall\ j_i \in [1,\ n_i],\ \forall\ n_i \in [1, \hat{n}]$ $A_{j_i}^i$ are the fuzzy numbers describing the linguistic attributes of the input variable x_i. In the same way we define the set

$$B = \{B_1, B_2, ..., B_k, ..., B_r\}$$

where $\forall k \in [1, r]$ B_k are the fuzzy numbers describing the linguistic attributes of the output variable y.

At every elements of A^i and B a membership function is associated such that

$$\mu_{A^i_{j_i}}(x) : U_i \to [0,1] \qquad \text{and} \qquad \mu_{B_k} : V \to [0,1]$$

The elements of A^i and B overlap in some "grey" zone which cannot be characterised precisely. Many phenomena in the world do not fall clearly into one crisp category or another. Experts that use abstraction as a way of simplifying the problem can contribute to identify these "grey" zones.

The choice of the slopes of the elements of A^i and B is a mathematical translation of what the experts think about the single terms.

The second step is the block-rules construction.

We define the set of L fuzzy rules, where L$\leq \prod_1^m n_i$, $\forall$ $j_i \in [1,$ $n_i]$, $\forall$ n_i $\in [1, \hat{n}]$ $\forall k \in [1, r]$

IF (x_1 is $A^1_{j_1}$) $\otimes$ (x_2 is $A^2_{j_2}$) $\otimes \ldots \otimes$ (x_m is $A^m_{j_m}$) (3.1-1)

THEN (y is B_k), (3.1-2)

The relation (3.1-1) is called "precondition" and the symbol $\otimes$ represents one of the possible aggregation operators. In practical applications, the MIN and MAX operators, or a convex combination of them, are widely used and so a "negative" or "positive" compensation will occur for different values of γ.

$$\gamma MIN + (1-\gamma)MAX \quad \text{with } \gamma \in [0,1]$$

Instead of Min and Max, it is also possible to use other t-norms or conorms, which represent different ways of linking the "and" with the "or".

More generally, indicating with $\mu_{A\cap B}$ a general membership of the intersection and with $\mu_{A\cup B}$ a general membership of the union, we can define as membership of the aggregated set $A\Theta B$

$$\mu_{A\Theta B} = \mu_{A\cap B}^{1-\gamma} * \mu_{A\cup B}^{\gamma} \qquad \text{with } \gamma \in [0,1]$$

This is not, in general, a t-norm or a t-conorm. In particular, if we use the algebraic product and sum as intersection and union, we obtain the Gamma operator [Zimmerman, 1997].

$$\mu = \left(\prod_1^n \mu_i\right)^{(1-\gamma)} * \left(1 - \prod_1^n (1-\mu_i)\right)^{\gamma}$$

The parameter γ denotes the degree of compensation. As it is shown in some recent work, this aggregator concept can represent the human decision process more accurately than others [Zimmerman, 1980]. The relation (3.1-2) is called conclusion. The aggregation of precondition and conclusion can be made in several ways. The most used are the MAX and the BSUM methods. The choice depends on the type of application. The MAX has the meaning of keeping as "winner" the strongest rule, in the sense that if a rule is "firing" (activated) more than one time, the result is the maximum level of firing. In the BSUM case, all the firing degree is considered and the final result is the sum of the different level of activation (not over one). In any case, the two methods produce a fuzzy set, which has membership function $\mu_{agg}(y)$.

Now we have a result of the fuzzy inference system, which is a fuzzy replay. We need to return to a "crisp" value, and this step is called "defuzzification". This operation produces a "crisp" action $\overline{y}$ that adequately represents the membership function $\mu_{agg}(y)$. There is no unique way to perform this operation. To select the proper method, it is necessary to understand the linguistic meaning that underlies the defuzzification process. Two of these different linguistic meanings are of practical importance: the "*best compromise*" and the "*most plausible result*". A method of the first type is the Centre of Area (CoA) that produces the abscissa of the centre of gravity of the fuzzy output set

$$\overline{y} = \frac{\int_V y\mu_{agg}(y)dy}{\int_V \mu_{agg}(y)dy}$$

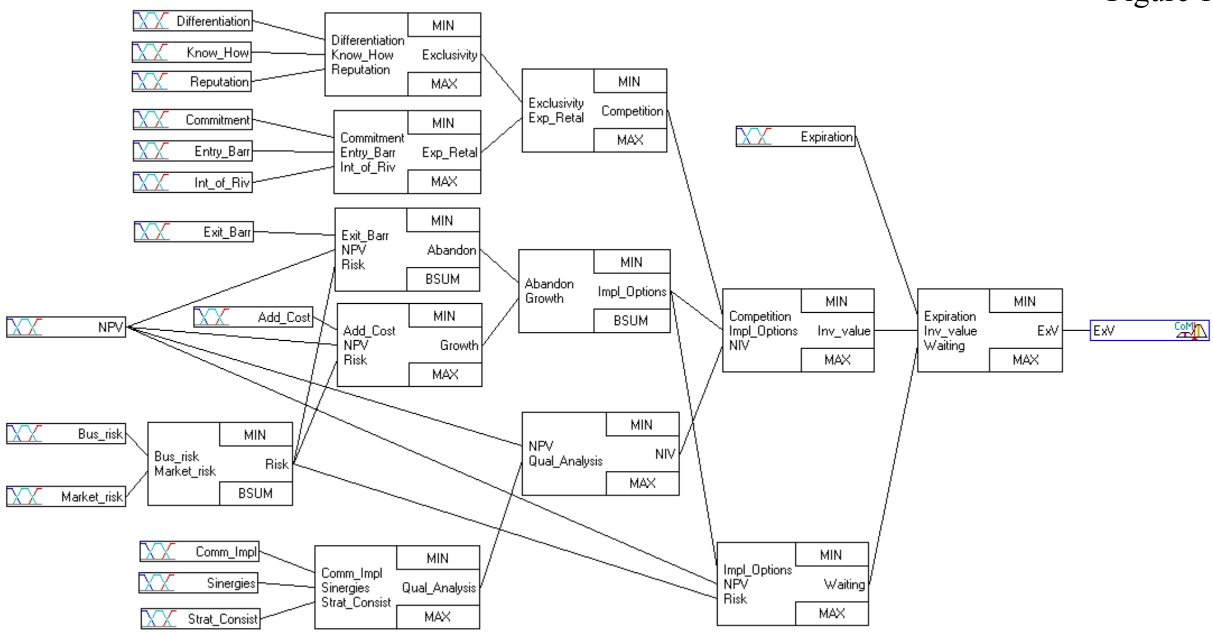
A method of the second type is the "Mean of Maximum" (MoM). Rather then balancing out the different inference results, this method selects the typical value of the terms that is most valid [Von Altrock, 1997].

4. The fuzzy model

Our model is a decision tree in which many variables (inputs), quantitative as well qualitative are taken into account. We call these variables *value drivers*, since they determine the exercise value of the option. The result, the exercise value *ExV* (output) is a number in [0,1]. The higher *ExV*, the higher the propensity to invest immediately rather than waiting. If *ExV*=0, then waiting is absolutely suggested, if *ExV*=1 then investing immediately is suggested. Any intermediate value gives us a degree of the investor's propensity to invest at once. On the basis of this degree, the investor herself will select which strategy to follow: for example if *ExV*=0.5 two investors can follow opposite courses of action. One of them could regard it as a suggestion for investing, the other as a suggestion for waiting.

The fuzzy model we carry out is an expert system. Conceptually, we can think of it as an index dependent on many variables. As you see in Figure 1, we deal with 15 value drivers, which are gathered in group of two or three giving rise to intermediate outputs by means of a rule block, as we will see later. The intermediate outputs are grouped as well and constitute in turn inputs giving rise to other intermediate outputs, and so on until the output *Exercise Value* is reached. The process of collecting the 15 value drivers in group of two or three and gradually aggregating facilitates our task of appraising the option. We can now describe the model by starting from the value drivers.

Figure 1



4.1. The Fifteen Value Drivers

In this chapter we specify the meaning and the role of each value driver.

- *(Differentiation) Differentiation.* It refers to brand identification and customer loyalties, due to past customer service, advertising: a product or service is differentiated when the customer perceives it as being unique. This driver must be intended as the degree of differentiation the investor is likely to obtain with the undertaking of the investment.

- *(KnowHow) Know-How.* Exclusive knowledge and/or specialized skills, proprietary technology or patents etc. This driver is to be intended as an observable variable but is subjectively determined by the evaluator.

- *(Reputation) Reputation.* A high reputation means that customers perceive the firm as highly reliable in product or service quality. It is strictly connected with the image of the firm. This driver is observable.

- *(Commitment) Commitment.* If the firm is able to convince its competitors that it is committed to enter the business, this increases the chance that they will not retaliate. This driver is certain as the decision maker herself defines her own degree of commitment in the project.

- *(EntryBar) Entry Barriers.* They are economic and strategic factors that deter entry in a business (among others, economies of scale, capital requirements, access to distributions channels, favorable access to raw materials, favorable location). This driver is observable.

- *(IntOfRiv) Intensity of rivalry.* Rivalry is intense in connection with strong price competition, advertising battles, increased customer service, frequent retaliations etc. This driver is observable.

- (*ExitBar*) *Exit Barriers*. They are economic, strategic, emotional factors that deter exit from a business even when the returns are low or negative (some of these include specialized assets, fixed costs of exit, strategic interrelationships, emotional barriers, legal restrictions). This driver is to be intended as the expected barriers in case of abandonment of the project.

- *(AddCost) Additional Cost.* It refers to the additional investment required in case the investor decides to enlarge the scale of the investment previously undertaken. It must be intended as the expected cost to be added if the investment is to be enlarged in scale.

- *(BusRisk) Business risk.* It refers to the volatility of the project's cash flows. This driver is to be intended as a degree of uncertainty. The same for the next driver.

- (*MarketRisk) Market Risk.* It refers to the trend of the entire market, obviously independent from the single project.

- *(NPV) Net Present Value.* The Expected Net Present Value is given by the algebraic sum of the prospective cash flows discounted at the required rate of return. The latter is obtained by the sum of the risk-free rate R_f and a premium for the risk. To compute the NPV the investor determines a prospective sequence of cash flows a_s, $s=0,1,2,\dots,N,$ where N is a fixed terminal horizon. At time N the investment is liquidated and b is the liquidation value. The discount rate ρ is thought to be dependent on the risk-free rate R_f and the risk in such a way that

$$\rho = R_f + \beta(BusRisk + MarketRisk) \qquad (4.1\text{-}1)$$

where β is subjectively determined by the investor. The NPV is then given by the sum

$$NPV = \sum_{s=0}^{N} \frac{a_s}{(1+\rho)^s} + \frac{b}{(1+\rho)^N} \qquad (4.1\text{-}2)$$

(for our simulations we conventionally set $N=10,$ with no loss of generality). This driver is to be intended as an expected value.

- *(CommImpl) Communication and implementation.* Communication has to do with the ability of managers to involve and stimulate employees, to communicate enthusiasm for the achievement of a shared goal, and pride in their own abilities, as well as to provide emotional support and encouragement. Implementation refers to potential strength of the entire firm to succeed in the business, depending, among others, on managers' and employees' skills and stubbornness. It is intended as an expected value.

- *(Synergies) Synergies*. Economies of scale, joint costs, which occur when a firm producing product or service A is inherently able to provide product or service B. Synergies occur when positive by-products stem form entering the business; this means, among others, the creation of intangible assets such as brand names, know-how, suppliers and distributors shared for more than one product or service etc. This driver is to be intended as the expected synergies from the project.

- *(StratConsist) Strategic consistency.* It refers to the coherence of the investment in hand with the investor's goals, as well as the impact of the project on investor's identity or image. This driver is observable.

- *(Expiration) Expiration Risk.* This factor takes into consideration that the option, if shared with competitors, can be *una tantum* in the sense that it vanishes once it has been exercised. If no other investor exercises the option, it is assumed to last forever. This risk has nothing to do with *Exclusivity.* In fact, a real option can be a shared option in two different meanings:
  (a) other competitors hold similar or equal options. They have the opportunity to exercise their option regardless of exercise by our decision-maker. This implies that rivals can retaliate with analogous options once the investor exercises her own;
  (b) many competitors hold one single option, such that if one of them exercises it, it expires and cannot be exercised by any other competitor.

  Type (a) refers to *Exclusivity*, type (b) refers to *Expiration.* A high degree of exclusivity does not mean, in our context, that the option is not shared, it only means that, *ceteris paribus*, the investor can effectively defend against competitors the payoff derived by the investment undertaken (using know-how, product or service differentiation, reputation. [Magni, 1996] introduces the distinction, for shared options, between repeatable options (case (a)) and *una tantum* options (case (b)), while studying some features of them in a dynamic programming framework.

  As an example for *una tantum* option the reader can think of a firm for sale: all competitors can buy it (the option is shared), but once it has been bought by one of them the option vanishes. This driver is to be intended as a degree of uncertainty.

4.2. The Application.

The fifteen aforementioned variables can be observed or forecasted periodically by the decision maker, so that a value can be fixed for all of them. Automatically, our expert system computes the Exercise Value of the option, *ExV*, and the decision maker can therefore select her preferred courses of action: investing or waiting for the next period. The decision maker herself fixes the value ExV^* above which investment should be undertaken. Consequently, she will invest immediately if $ExV>ExV^*$, wait otherwise.

The model is an evaluation tree, which we run along from branches to trunk. We start then from the drivers which are gradually gathered by means of rule blocks giving rise to intermediate outputs. We must also specify how *NPV* has to be considered. In strategic investments the Net Present Value is only one of many other factors that influence the evaluation process. We assume that the investor forecasts the cash flows as a mere starting point, without any consideration of the other factors. The latter are related to competitor's behavior, exclusivity of the option, strategic synergies and management abilities. All these drivers may influence the final payoff in a way that is difficult to forecast in crisp values. Therefore, the cash flows a_s are not all the investor can get from the project. This will turn to be important when we shall describe some simulations we have made.

Looking at Figure 1 you note at top left *Differentiation, KnowHow* and *Reputation:* They are isolating devices influencing the degree of *exclusivity* of the investment option. The higher is one of them, the higher is the degree of exclusivity (*ceteris paribus*). In other terms, the latter is a function increasing with respect to the isolating devices. The shape of this function depends on the rules chosen by the decision maker herself. In Table 1 you can see how the rule block *Exclusivity* works.

Table 1

IF			THEN	
Differentiation	**Know_How**	**Reputation**	**DoS**	**Exclusivity**
low	low	low	1.00	low
low	low	medium	1.00	low
low	low	high	1.00	medium_low
low	medium	low	1.00	low
low	medium	medium	1.00	medium_low
low	medium	high	1.00	medium_low
low	high	low	1.00	medium_low
low	high	medium	1.00	medium_low
low	high	high	1.00	medium_high
medium	low	low	1.00	low
medium	low	medium	1.00	medium_low
medium	low	high	1.00	medium_low
medium	medium	low	1.00	medium_low
medium	medium	medium	1.00	medium_low
medium	medium	high	1.00	medium_high
medium	high	low	1.00	medium_low
medium	high	medium	1.00	medium_high
medium	high	high	1.00	high
high	low	low	1.00	medium_low
high	low	medium	1.00	medium_low
high	low	high	1.00	medium_high
high	medium	low	1.00	medium_low
high	medium	medium	1.00	medium_high
high	medium	high	1.00	high
high	high	low	1.00	medium_high
high	high	medium	1.00	high
high	high	high	1.00	high

Commitment, EntryBar and *IntofRiv* have impact on what we call *Expected Retaliation (ExpRetal)*. A high commitment will deter high retaliation from rivals, high entry barriers are set by competitors if they intend to preempt entry. The intensity of rivalry is another factor which directly indicates the chance the investor has to be opposed with retaliation. Analogous rules are selected by the decision maker to infer the value of *ExpRetal.*

The risk is the result of the *BusRisk* and the *MarketRisk.* These are subjectively determined by the decision maker and define the investment's *Risk.* The rules connecting the two kind of risk can be seen in Table 2. From the inspection of the table is obvious that the evaluator aggregates the two risks with a stronger consideration for the *BusRisk.* In fact, we have the rules

IF (Low AND High) THEN (MediumLow)

whereas we have

IF (High AND Low) THEN (MediumHigh)

Table 2

IF		THEN	
Bus_risk	**Market_risk**	**DoS**	**Risk**
low	Low	1.00	low
low	Medium	1.00	medium_low
low	high	1.00	medium_low
medium	low	1.00	medium_low
medium	medium	1.00	medium_high
medium	high	1.00	medium_high
high	low	1.00	medium_high
high	medium	1.00	medium_high
high	high	1.00	high

CommImpl, Sinergies and *StratConsist* constitute what we call the *Qualitative analysis* (*QualAnalysis*) and the rules define it as an increasing function with respect to all three drivers.

NPV is an input for more than one intermediate output. To begin with, it determines, along with *Risk* and *ExitBar*, the value of the abandonment option (*Abandon*). The latter is made to increase with respect to *Risk* and decreasing with respect to *NPV* and *ExitBar*. The reason is obvious: if *Risk* increases then the investor could be involved in a great loss if things go bad. The opportunity of abandoning the project has a higher value. Further, the higher the *NPV* the smaller the value of this option since it seems less probable that the investor will leave the business. Finally, higher *ExitBar* make exit more difficult so that *Abandon* is small.

Likewise, *Risk*, *NPV* and additional cost influence the growth option value (*Growth*): the latter is increasing with respect to *Risk* and *NPV* and decreasing with *AddCost*. In fact, a higher *Risk* means a higher volatility of cash flows and then the opportunity to higher increase of earnings from the project if things go well, while maintaining the opportunity of resign from expanding the scale of the business if things go bad. If the *NPV* increases, this indicates that things are supposed to go well so that an expansion opportunity could be convenient. An increased *AddCost* diminishes the value of *Growth*.

The intermediate outputs *Exclusivity* and *ExpRetal* give rise to *Competition,* which informs the evaluator about the role of competitors if investment is undertaken. *Competition* increases with *ExpRetal* while decreases with *Exclusivity.*

Abandon and *Growth* determine the value of what we call *Implicit Options (ImplOptions),* i.e. the right to give up the project or to expand it once the investment is undertaken.

QualAnalysis and *NPV* join in the *Net Investment Value (NIV)*, which gives us the value of the opportunity of investing now, net of the value of other implicit options and the degree of competition.

NIV, ImplOptions and *Competition* affect the overall *Investment Value* (*InvValue*). The higher is *InvValue* the more inclined to investing now is the decision maker.

The rules for *InvValue* show some interesting features. An excerpt of this rule block is shown in Table 3. The last two rows (with blank spaces in the first column) tell us that whatever the degree of *Competition*, if *ImplOptions* is Low or Medium and *NIV* is VeryLow, then *InvValue* is VeryLow or Low. The two rows preceding the latter two say that whatever the value of *Competition,* if *ImplOptions* is VeryLow and *NIV* is VeryLow or Low, then *InvValue* is also VeryLow or Low. These four rules show that when *NIV* has a small value and *ImplOptions* is not more than Medium, then *Competition* does not play any role in determining *InvValue*.

We can also note that *ImplOptions* and *NIV* are able to more than compensate an extremely high *Competition* if one of the former two is at least Medium and the other

VeryHigh. In fact, we have

IF (VeryHigh AND Medium AND VeryHigh) THEN (MediumHigh)

as well as

IF (VeryHigh AND VeryHigh AND Medium) THEN (MediumHigh)

Table 3

IF			THEN	
Competition	**Impl_Options**	**NIV**	**DoS**	**Inv_value**
very_high	medium	low	1.00	Low
very_high	medium	medium	1.00	Low
very_high	medium	high	1.00	medium_low
very_high	medium	very_high	1.00	medium_high
very_high	high	very_low	1.00	low
very_high	high	low	1.00	low
very_high	high	medium	1.00	medium_low
very_high	high	high	1.00	medium_high
very_high	high	very_high	1.00	medium_high
very_high	very_high	very_low	1.00	low
very_high	very_high	low	1.00	low
very_high	very_high	medium	1.00	medium_high
very_high	very_high	high	1.00	medium_high
very_high	very_high	very_high	1.00	high
	very_low	very_low	1.00	very_low
	very_low	low	1.00	low
	low	very_low	1.00	very_low
	medium	very_low	1.00	low

Opposed to *InvValue* is the value of waiting (*Waiting)*. It is determined by *ImplOptions*, *NPV*, and *Risk*. The *Risk* is such as to suggest waiting in an increasing relation, whereas with *ImplOptions* and *NPV* the relation is reversed. In fact if the latter two increase than the proclivity to waiting decreases. An excerpt of this rule block is shown in Table 4 and Table 5. Looking at Table 4 the value of waiting is rather high or even very high if *NPV* is NoProfit unless *ImplOptions* is VeryHigh (see Table 4). Then *InvValue* is not very influenced by *ImplOptions* unless the latter has a very great value. The NPV is then more important, which certainly makes sense (but the rules are subjective so the locution `makes sense' should be considered improper in this context).

Further our decision maker is quite averse to risk, since *Waiting* is no less than MediumHigh whenever the *Risk* is High, except for the two cases where *NPV* is Profit and *ImplOptions* is High or VeryHigh (see Table 5).

Moreover we have constructed these rules so that whenever the risk is Low or Medium-Low the value of waiting is determined by the other two inputs. If *Risk* is Low or Medium-Low, *NPV* is NoProfit and *ImplOptions* is not VeryHigh, then the decision maker assigns a great value to *Waiting.* In fact, in these cases, *Waiting* is High or MediumHigh (see Table 4). The propensity to waiting is not caused by *Risk* (which is quite low), but by an investment value which is not so high, as well as by a value of the other options which is not so high.

Table 4

IF			THEN	
Impl_Options	**NPV**	**Risk**	**DoS**	**Waiting**
very_high	no_profit	low	1.00	medium_low
very_high	no_profit	medium_low	1.00	medium_low
high	no_profit	low	1.00	medium_high
high	no_profit	medium_low	1.00	medium_high
high	no_profit	medium_high	1.00	medium_high
very_high	no_profit	medium_high	1.00	medium_high
very_low	no_profit	low	1.00	high
very_low	no_profit	medium_low	1.00	high
low	no_profit	low	1.00	high
low	no_profit	medium_low	1.00	high
low	no_profit	medium_high	1.00	high
medium	no_profit	low	1.00	high
medium	no_profit	medium_low	1.00	high
medium	no_profit	medium_high	1.00	high
high	no_profit	high	1.00	high
very_high	no_profit	high	1.00	high
very_low	no_profit	medium_high	1.00	very_high
very_low	no_profit	high	1.00	very_high
low	no_profit	high	1.00	very_high
medium	no_profit	high	1.00	very_high

Table 5

IF			THEN	
Impl_Options	**NPV**	**Risk**	**DoS**	**Waiting**
very_low	no_profit	high	1.00	very_high
very_low	indifferent	high	1.00	very_high
very_low	profit	high	1.00	high
low	no_profit	high	1.00	very_high
low	indifferent	high	1.00	high
low	profit	high	1.00	medium_high
medium	no_profit	high	1.00	very_high
medium	indifferent	high	1.00	high
medium	profit	high	1.00	medium_high
high	no_profit	high	1.00	high
high	indifferent	high	1.00	medium_high
high	profit	high	1.00	medium_low
very_high	no_profit	high	1.00	high
very_high	indifferent	high	1.00	medium_high
very_high	profit	high	1.00	medium_low

One more step and we reach the tree-trunk. *Waiting* and *InvValue* compete for the *Exercise Value* (*ExV*), which says whether the options must be exercised or not. The higher *ExV* the higher the propensity to invest immediately.

This final rule block is analogous to the comparison accomplished by stochastic dynamic programming: the continuation region is countered by the stopping region. *Waiting* is our fuzzy continuation region, *InvValue* is our fuzzy stopping region. But we have also added another important driver: *Expiration.* That is, if *Waiting* increases and *InvValue* decreases then *ExV* increases, but, *ceteris paribus*, the higher is the risk that the option fades out the higher is the propensity to exercise the option. Note that *Expiration* does not play any role if *InvValue* is less than MediumHigh (i.e. VeryLow, Low or MediumLow): *ExV* is in these cases VeryLow, Low, MediumLow (except one case where it is Medium). This is obvious: To exercise the option *InvValue* must be sufficiently high regardless of the value of *Expiration.* The latter indicates the risk that the investment opportunity is taken away by a competitor. But if this opportunity is not so good, then the investor does not care so much if someone else exercises it (see Table 6).

Table 6

IF			THEN	
Expiration	**Inv_value**	**Waiting**	**DoS**	**ExV**
	very_low		1.00	very_low
Low	low	high	1.00	very_low
Low	low	very_high	1.00	very_low
Medium	low	very_high	1.00	very_low
High	low	very_high	1.00	very_low
Low	low	very_low	1.00	low
Low	low	low	1.00	low
low	low	medium_low	1.00	low
low	low	medium_high	1.00	low
medium	low	very_low	1.00	low
medium	low	low	1.00	low
medium	low	medium_low	1.00	low
medium	low	medium_high	1.00	low
medium	low	high	1.00	low
high	low	very_low	1.00	low
high	low	low	1.00	low
high	low	medium_low	1.00	low
high	low	medium_high	1.00	low
high	low	high	1.00	low
low	medium_low	very_high	1.00	very_low
low	medium_low	medium_high	1.00	low
low	medium_low	high	1.00	low
medium	medium_low	medium_high	1.00	low
medium	medium_low	high	1.00	low
medium	medium_low	very_high	1.00	low
high	medium_low	very_high	1.00	low
low	medium_low	very_low	1.00	medium_low
low	medium_low	low	1.00	medium_low
low	medium_low	medium_low	1.00	medium_low
medium	medium_low	very_low	1.00	medium_low
medium	medium_low	low	1.00	medium_low
medium	medium_low	medium_low	1.00	medium_low
high	medium_low	low	1.00	medium_low
high	medium_low	medium_low	1.00	medium_low
high	medium_low	medium_high	1.00	medium_low
high	medium_low	high	1.00	medium_low
high	medium_low	very_low	1.00	medium

5. Sensitivity Analysis

We have accomplished some simulations which are essential for testing the robustness and theoretical soundness of the model. Some of these are here thoroughly investigated. As for the simulations of Figg.2-6 and Fig.11 we have three Tables, a large graph and twelve smaller graphs. The first Table shows the values selected of the inputs which determine the NPV (in the last row we have ρ, which is calculated as in (4.1-1)). The second Table shows the values selected for the Fuzzy Logic System (FLS) inputs, that is the inputs of our fuzzy expert system (one of them is the NPV, which is determined on the basis of the preceding Table). The third Table automatically shows the value of the intermediate outputs as well as the value of the final output *ExV* (whose row is always shaded). In each simulation we consider eleven cases, where some of the inputs (NPV inputs and/or FLS inputs) are made to vary, while the others are maintained constant. The rows corresponding to the varying inputs are shaded. The values of the outputs recorded in the third Table are graphically represented in the subsequent figures. They enable the reader to directly understand how the value of *ExV* is determined (while looking, at the same time, at the tree in Fig.1) and therefore allow for a correct justification of the final variation. The abscissa of each graph marks the different cases involved (case 1,2,…,11). As for Figg.7-10 only *ExV* is graphically represented.

Figure 2 is rather interesting as it shows how *ExV* changes as both *BusRisk* and M*arketRisk* rises and *AddCost* decreases*,* while all other inputs are held constant. In the horizontal axis we have case 1,2,…,11 corresponding to increasing values of the inputs (see Figure 2.2). Inspecting Figg.2.3-2.4 we see that *ExV* is 0.87 and it is a steady line until case 6; then it sharply decreases to 0.29 in case 11 where both risks are at a maximum level of 1.00. The reason can be inferred by the analysis of Figg. 2.5-2.16, which show different effects combining together. As both types of risk increase *Risk* increase monotonically from 0.00 to 1.00, as we expect. With *Risk* increasing the NPV decreases monotonically, due to (4.1-1) and (4.1-2). *QualAnalysis* is constant and *NIV* is constant at a high level until case 6, then it sharply declines. This is due to the fact that when the NPV is positive, i.e. *NPV* is Profit, and *QualAnalysis* is at least MediumLow*,* then *NIV* is no less than High.The value of *ImplOptions* also rises as both *Abandon* and *Growth* increase (as for *Growth,* its increase is a little smaller than *Abandon*, because the higher *Risk* and the lower *AddCost* are partly offset by a decreasing NPV). Further, *Competition* is constant. So we have a very high constant *NIV,* an increasing *ImplOptions* and a low constant *Competition.* The result is that *InvValue* is even higher than *NIV* until case 6*.* Also, *Waiting* does not rise so much, since even though *Risk* sharply increases, this is partly compensated by higher and higher *ImplOptions* and a Net Present Value that is always positive. So, with a very high value of *NIV* the decision maker is inclined to invest immediately. *Waiting* is then so low that the propensity to invest now is reinforced. Further *Expiration* is not so high to limit the high value of *ExV,* so that, at last, *ExV* is 0.87. From case 7 on things change a lot. Even if the value of *ImplOptions* keeps on rising, *InvValue* is increasing and *Waiting* tends to sharply decrease. Actually, the risk is now too high be overlooked and the decrease in *AddCost* is not so important, so that the fall of the NPV suggests caution. Then, *ExV* decreases until reaching 0.29.

It is also worthwhile noting that the high value of *ExV* until case 6 is to be connected to an extremely high value of the Net Present Value[1], which compensates for increasing risk (at least for a reasonable level of risk). When the NPV is not so high things change.

[1] The range of the NPV has been fixed in such a way that below –75.000 the NPV is VeryLow with membership degree 1.00 and above 75.000 it is VeryHigh with membership degree 1.00.

Figure 2.1

NPV Inputs	1	2	3	4	5	6	7	8	9	10	11
a0	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000
a1	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a2	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a3	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a4	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a5	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a6	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a7	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a8	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a9	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
a10	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000	90.000
b	80.000	80.000	80.000	80.000	80.000	80.000	80.000	80.000	80.000	80.000	80.000
Rf	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08
β	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
ρ	0,08	0,16	0,24	0,32	0,40	0,48	0,56	0,64	0,72	0,80	0,88

Figure 2.2

FLS Inputs	1	2	3	4	5	6	7	8	9	10	11
Add Cost	50.000	47.000	44.000	41.000	38.000	35.000	32.000	29.000	26.000	23.000	20.000
Bus risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Comm Impl	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Commitment	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Differentiation	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Entry Barr	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Exit Barr	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Know How	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Market risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
NPV	540963	353125	240675	168719	119987	85368	59769	40194	24801	12409	2232
Reputation	1	1	1	1	1	1	1	1	1	1	1
Sinergies	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Strat Consist	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6

Figure 2.3

Outputs	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,27	0,33	0,37	0,41	0,43	0,60	0,67	0,71	0,74	0,77	0,92
Growth	0,33	0,47	0,56	0,61	0,68	0,77	0,79	0,81	0,83	0,83	0,87
Impl_Options	0,25	0,42	0,47	0,52	0,56	0,68	0,72	0,75	0,76	0,78	0,85
Exclusivity	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73
Exp_Retal	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25
Competition	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19
Qual_Analysis	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
NIV	0,80	0,80	0,80	0,80	0,80	0,80	0,68	0,58	0,51	0,44	0,33
Risk	0,00	0,19	0,30	0,37	0,48	0,67	0,72	0,76	0,79	0,86	1,00
Inv_Value	0,85	0,85	0,85	0,85	0,85	0,85	0,66	0,61	0,56	0,50	0,48
Waiting	0,20	0,22	0,23	0,18	0,24	0,26	0,38	0,47	0,51	0,54	0,59
ExV	**0,87**	**0,87**	**0,87**	**0,87**	**0,87**	**0,87**	**0,64**	**0,59**	**0,51**	**0,43**	**0,29**

Figure 2.4

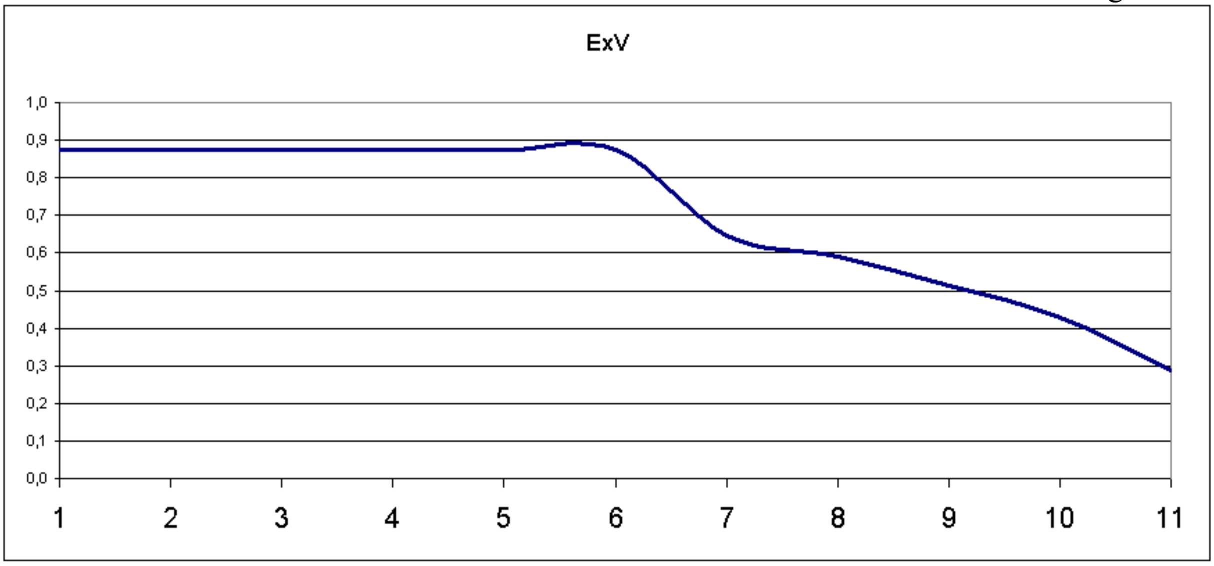


Figure 2.5

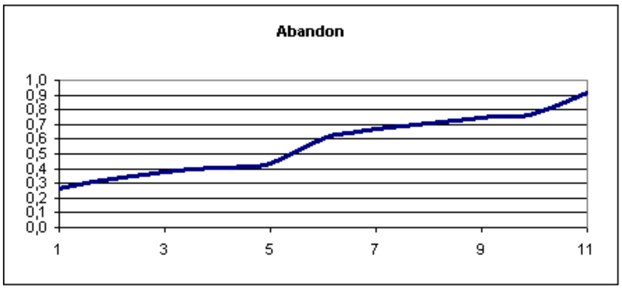


Figure 2.6

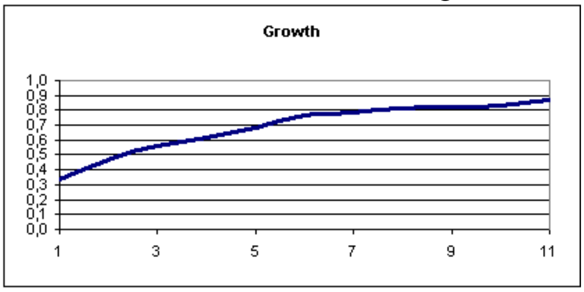


Figure 2.7

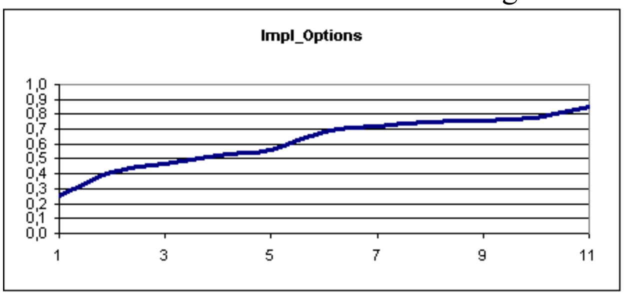


Figure 2.8

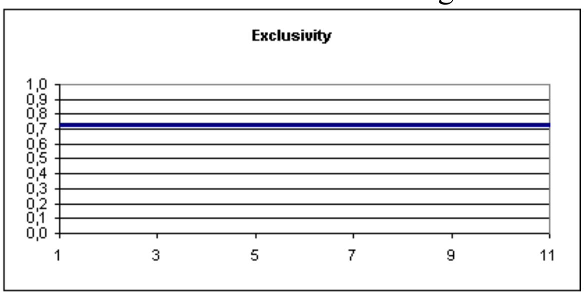


Figure 2.9

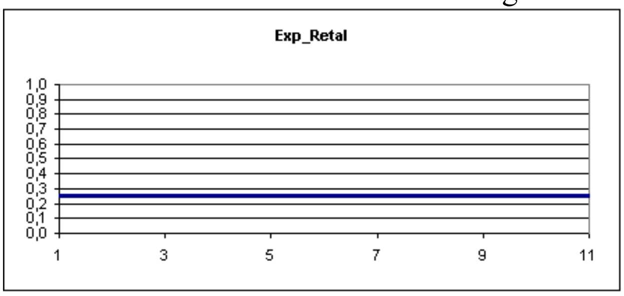


Figure 2.10

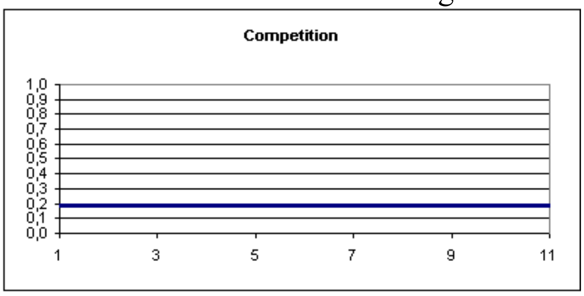


Figure 2.11

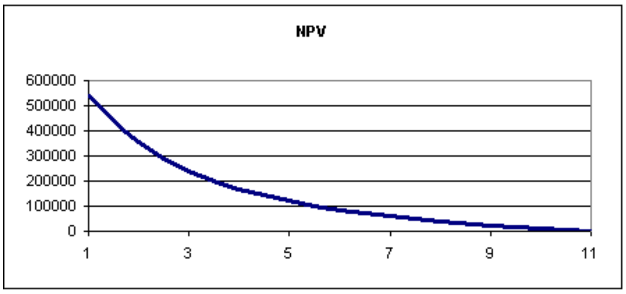


Figure 2.12

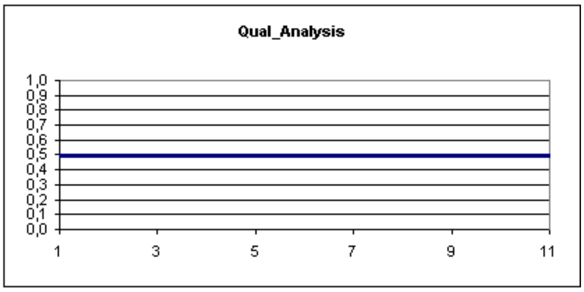


Figure 2.13

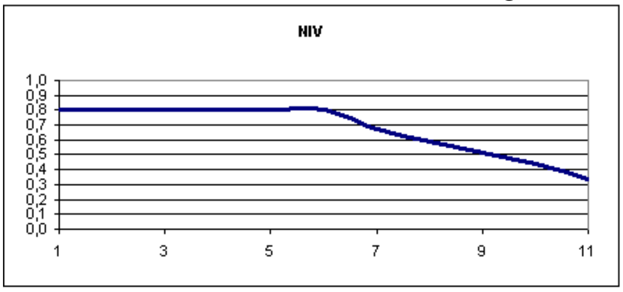


Figure 2.14

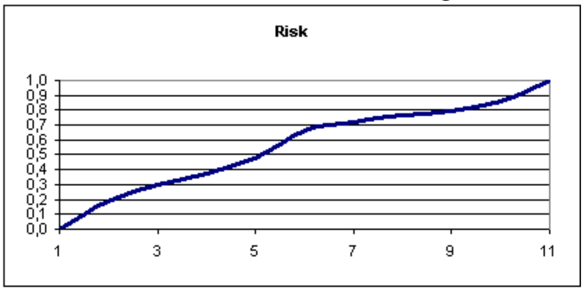


Figure 2.15

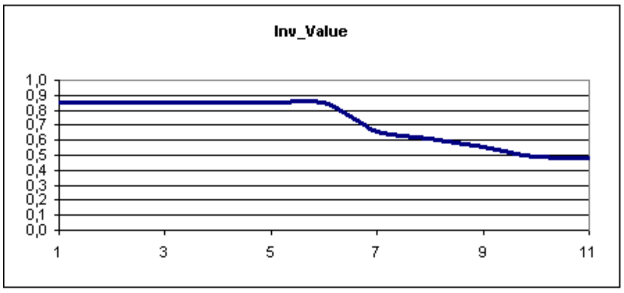
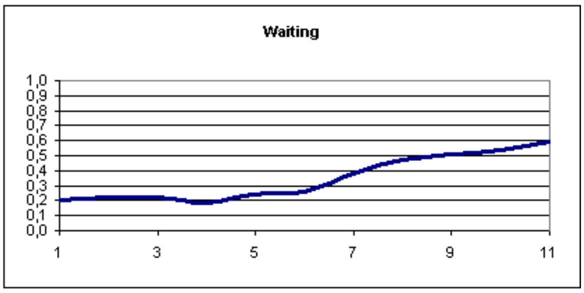


Figure 2.16



A similar situation is illustrated in Fig.3, where the expected cash flows are equal to 65.000 from time 1 to time 10 and *b* is 10.000. You can see a different path for *ExV*. In fact, the NPV is very high for the first three cases, but from case 4 it sharply declines becoming negative in case 9. In such a situation *Risk* plays an important role even when its value is not high.

Figure 3.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000	- 100.000
a1	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a2	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a3	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a4	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a5	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a6	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a7	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a8	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a9	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
a10	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000
b	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Rf	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08
β	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
ρ	0,08	0,16	0,24	0,32	0,40	0,48	0,56	0,64	0,72	0,80	0,88

Figure 3.2

FLS Inputs	1	2	3	4	5	6	7	8	9	10	11
Add Cost	50.000	47.000	44.000	41.000	38.000	35.000	32.000	29.000	26.000	23.000	20.000
Bus risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Comm Impl	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Commitment	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Differentiation	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Entry Barr	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Exit Barr	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Know How	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Market risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
NPV	340787	216427	140484	91100	57228	32929	14829	912	-10076	-18950	-26252
Reputation	1	1	1	1	1	1	1	1	1	1	1
Sinergies	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Strat Consist	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6

Figure 3.3

Outputs	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,27	0,33	0,37	0,41	0,45	0,61	0,67	0,76	0,83	0,83	0,94
Growth	0,33	0,47	0,56	0,61	0,67	0,78	0,75	0,80	0,77	0,74	0,87
Impl_Options	0,25	0,42	0,47	0,52	0,56	0,67	0,72	0,82	0,78	0,75	0,86
Exclusivity	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	0,73
Exp_Retal	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25
Competition	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19
Qual_Analysis	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
NIV	0,80	0,80	0,80	0,80	0,67	0,55	0,46	0,31	0,26	0,24	0,22
Risk	0,00	0,19	0,30	0,37	0,48	0,67	0,72	0,76	0,79	0,86	1,00
Inv_Value	0,85	0,85	0,85	0,85	0,64	0,54	0,50	0,46	0,39	0,35	0,37
Waiting	0,20	0,22	0,23	0,18	0,28	0,39	0,47	0,46	0,56	0,64	0,67
ExV	0,87	0,87	0,87	0,87	0,63	0,52	0,48	0,32	0,26	0,24	0,25

Figure 3.4

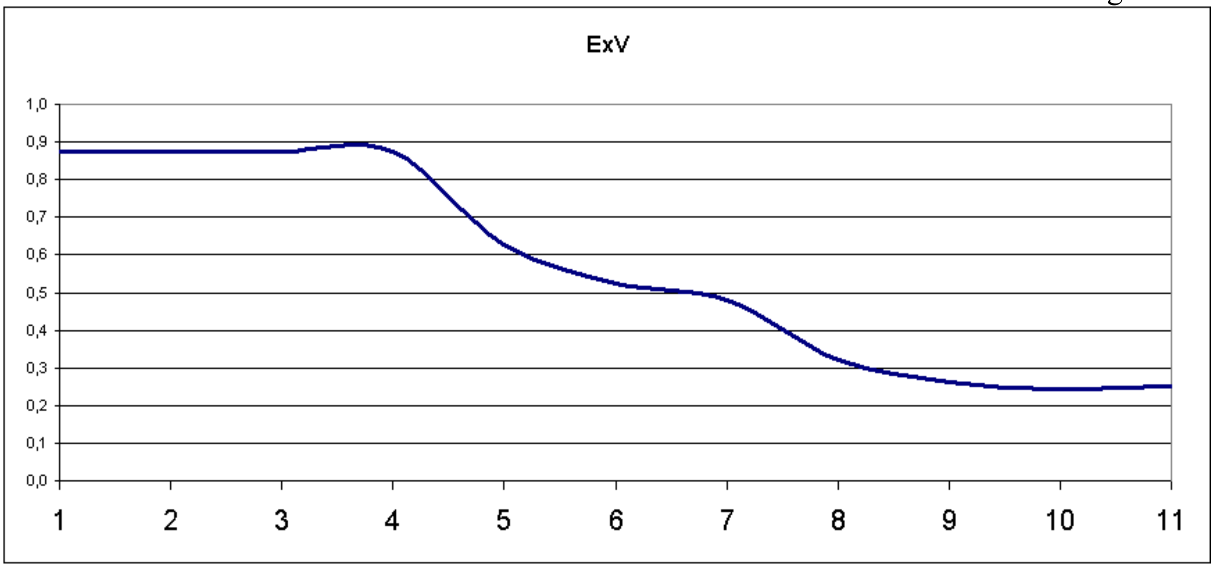


Figure 3.5

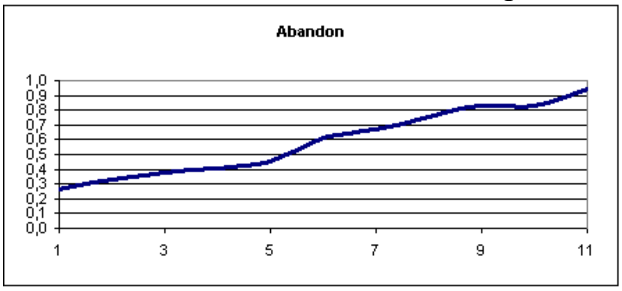


Figure 3.6

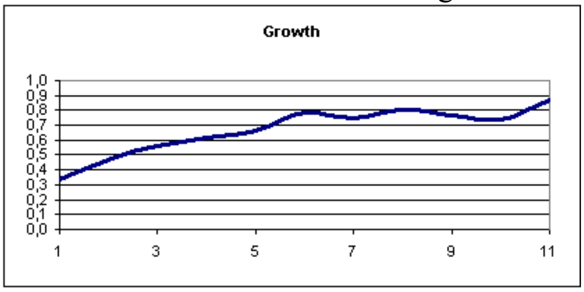


Figure 3.7

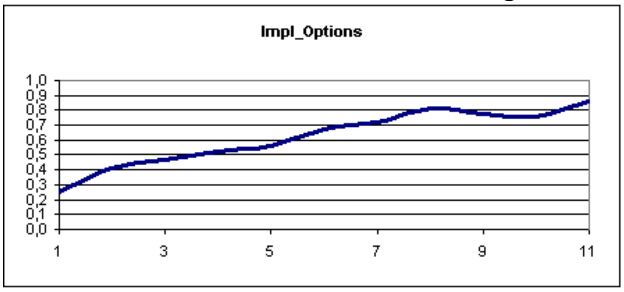


Figure 3.8

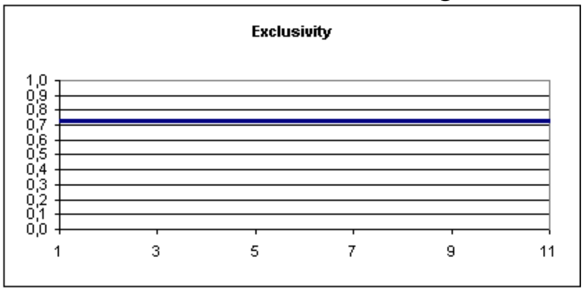


Figure 3.9

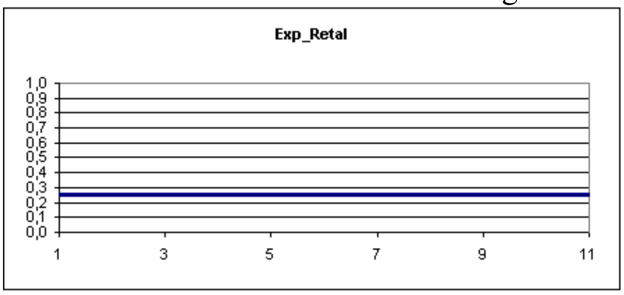


Figure 3.10

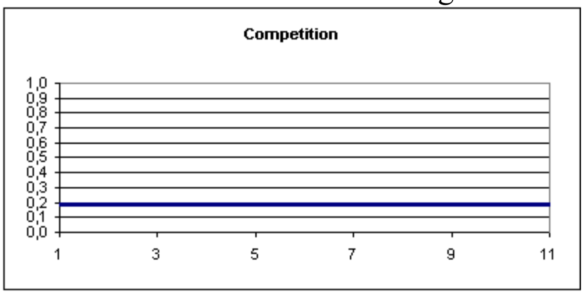


Figure 3.11

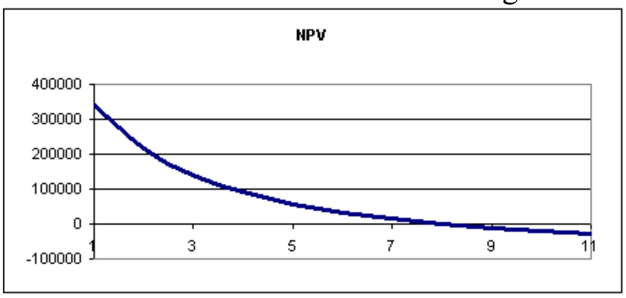


Figure 3.12

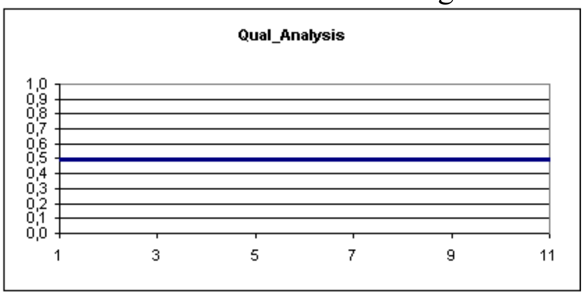


Figure 3.13

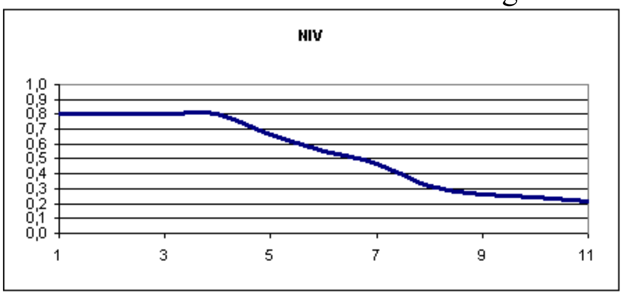


Figure 3.14

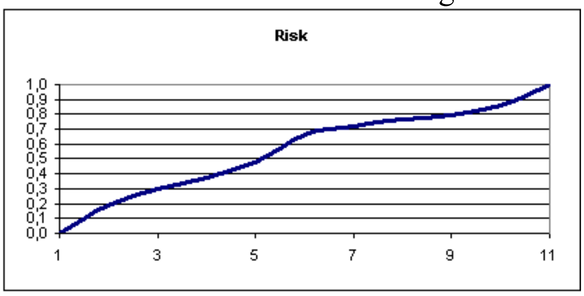


Figure 3.15

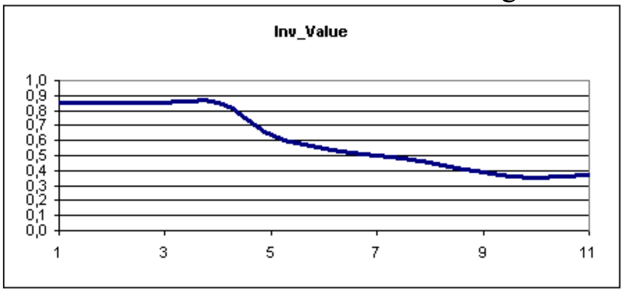
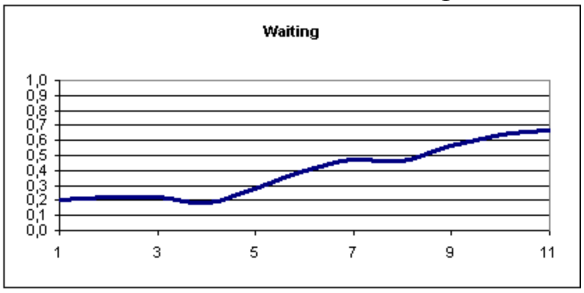


Figure 3.16



Interesting relations are depicted in Fig.4. Now we have that only *BusRisk* is made to rise. *ExV* does not show a monotonic trend. The reason is that the model is such that the relations among inputs and intermediate outputs are not simple. Note that *ExV* falls from 0.62 (case 1) to 0.43 (case 4). This is due to a slightly decreasing *InvValue* and an increasing *Waiting* (Fig.4.3). The latter increases for *NPV* is decreasing and *Risk* is increasing, while *ImplOptions* does not change much (this is due in turn to the fact that *Growth* does not rise sufficiently, since the increase in *Risk* is almost compensated by the reduction of *NPV*). The former decreases as *Competition* is constant and the decrease in *NIV* is not compensated by *ImplOptions*, which remains almost constant. From case 5 things change. Even if *NIV* keeps on falling monotonically, the level of *Risk* is now such that the rise of *ImplOptions* compensates the fall so that *InvValue* begins to invert the trend. At the same time, *Waiting* reduces.

From case 7 the function decreases again until 10 where a final increases can be noted. The reader can justifies herself/himself these changes by inspection of Figg.4.3-4.16.

Figure 4.1

NPV Inputs	1	2	3	4	5	6	7	8	9	10	11
a0	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000	- 3.000
a1	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a2	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a3	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a4	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a5	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a6	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a7	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a8	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a9	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
a10	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
b	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Rf	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08
β	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80
ρ	0,09	0,17	0,25	0,33	0,41	0,49	0,57	0,65	0,73	0,81	0,89

Figure 4.2

FLS Inputs	1	2	3	4	5	6	7	8	9	10	11
Add Cost	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
Bus risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Comm Impl	1	1	1	1	1	1	1	1	1	1	1
Commitment	1	1	1	1	1	1	1	1	1	1	1
Differentiation	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Market risk	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
NPV	46624	31965	23237	17677	13923	11263	9298	7797	6617	5667	4887
Reputation	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
Sinergies	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
Strat Consist	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7

Figure 4.3

Outputs	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,13	0,24	0,26	0,28	0,29	0,34	0,40	0,45	0,51	0,57	0,64
Growth	0,55	0,57	0,59	0,62	0,65	0,64	0,63	0,62	0,61	0,63	0,69
Impl_Options	0,39	0,38	0,39	0,41	0,43	0,46	0,52	0,54	0,57	0,60	0,72
Exclusivity	0,47	0,47	0,47	0,47	0,47	0,47	0,47	0,47	0,47	0,47	0,47
Exp_Retal	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
Competition	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Qual_Analysis	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80
NIV	0,81	0,71	0,67	0,64	0,62	0,60	0,59	0,57	0,56	0,56	0,55
Risk	0,01	0,08	0,15	0,21	0,28	0,34	0,41	0,47	0,54	0,60	0,67
Inv_Value	0,63	0,59	0,57	0,56	0,57	0,60	0,62	0,61	0,60	0,61	0,65
Waiting	0,28	0,35	0,40	0,44	0,42	0,40	0,38	0,42	0,45	0,45	0,41
ExV	0,62	0,53	0,47	0,43	0,45	0,48	0,50	0,49	0,48	0,48	0,58

Figure 4.4

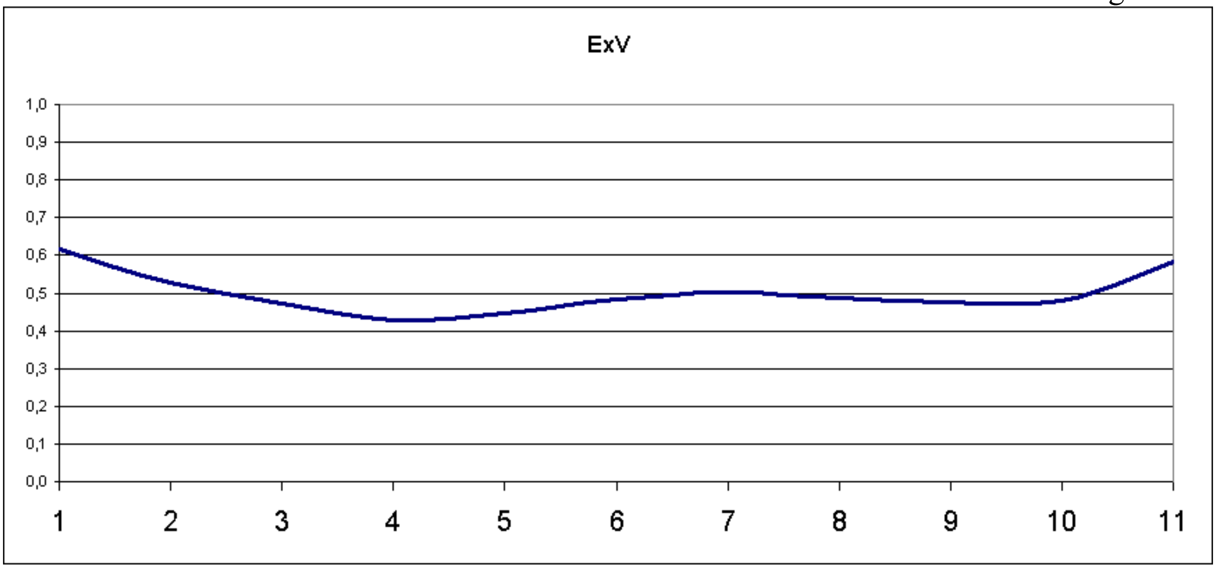


Figure 4.5

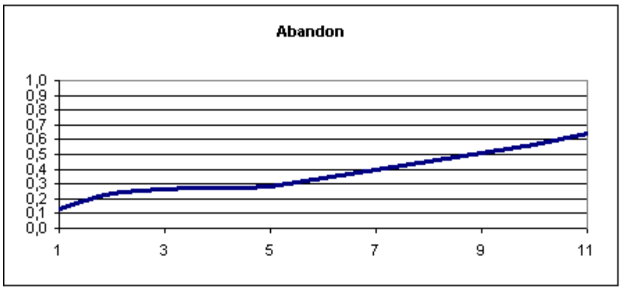


Figure 4.6

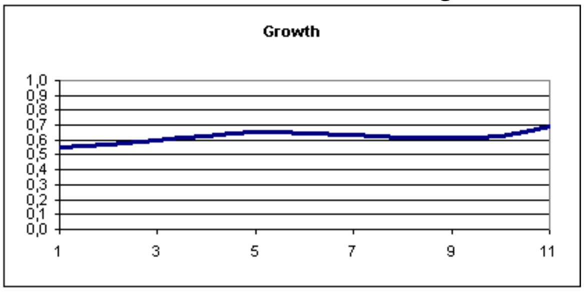


Figure 4.7

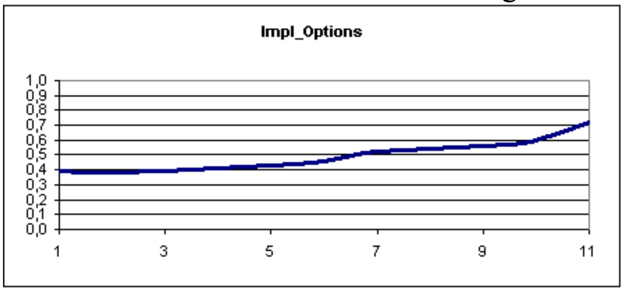


Figure 4.8

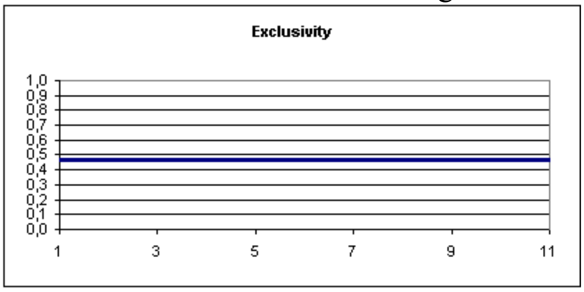


Figure 4.9

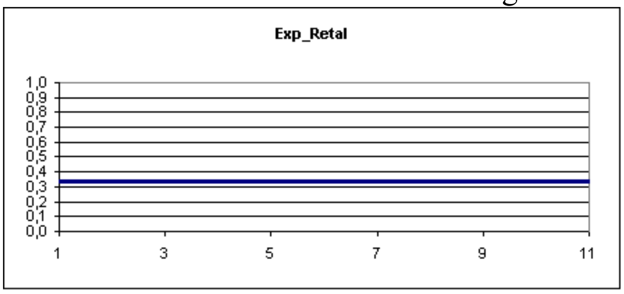


Figure 4.10

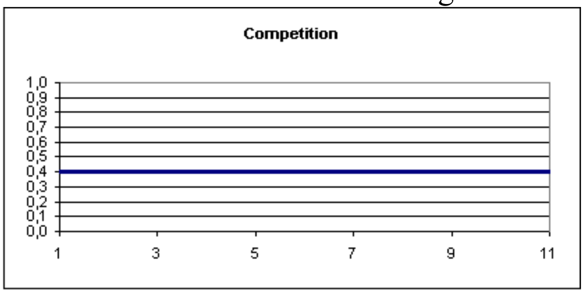


Figure 4.11

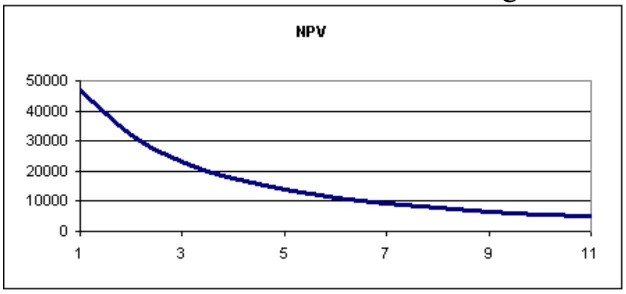


Figure 4.12

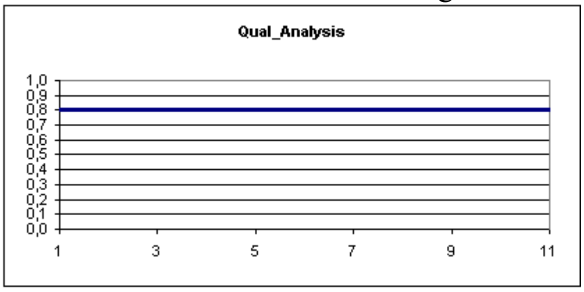


Figure 4.13

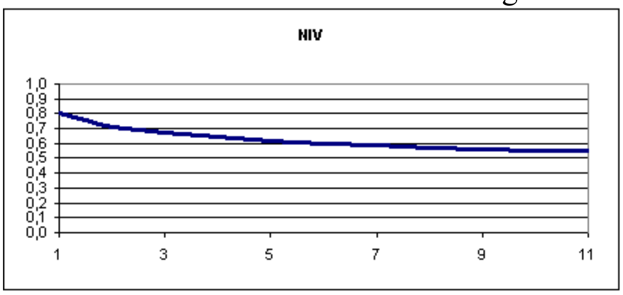


Figure 4.14

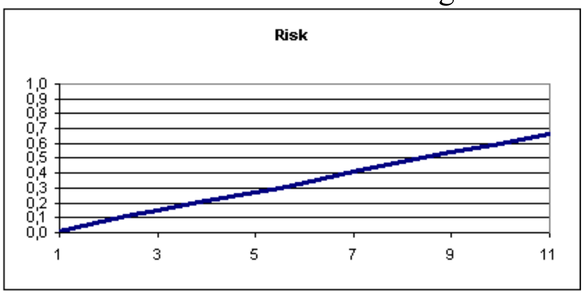
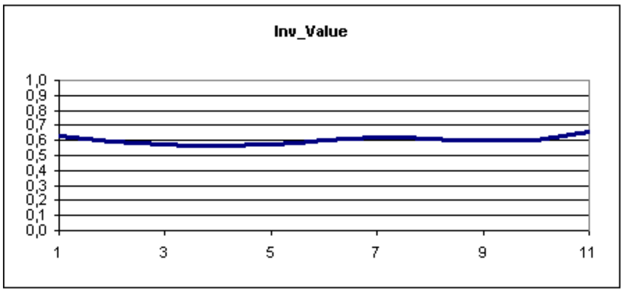


Figure 4.15 Figure 4.16



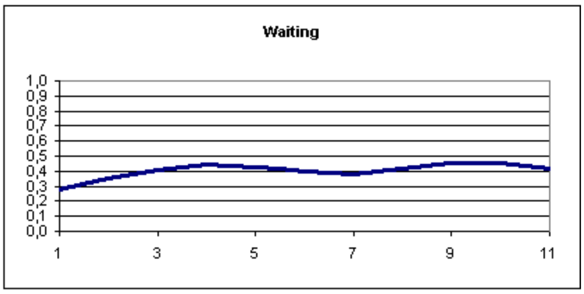


Fig.5 shows the role played by the risk-free rate in determining *ExV*. The risk-free rate has a remarkable impact on the Net Present Value and through it on the other outputs. Note that *ImplOptions* does not change so much (Fig.5.7). This derives from the behavior of *Abandon,* which slightly increases, and *Growth*, which decreases. The two compensate, more or less, so *InvValue* is determined almost completely by *NIV*. The latter decreases as the NPV decreases (even though in a smoother way since *Competition* is low and the value of *ImplOptions* is medium). Consequently, *InvValue* decreases. At the same time, *Waiting* increases from an initial 0.29 to the final 0.51 and this reinforces the reduction of *ExV.* Actually, if the risk-free rate is 50 percent, the NPV of the project is negative, the propensity to invest immediately should be rather small. The reader could expect an even lower level of *ExV*, since anyone would prefer to invest at such a high risk-free rate rather than invest to a medium-risk project with negative NPV. The fact that *ExV* is 0.24 in case 11 is due to a rather high degree of *Exclusivity* of the option, a low degree of *Competition*, and a value of the implicit options which is not so small. If these three factors were lower, then *ExV* would be much lower.

Figure 5.1

NPV Inputs	1	2	3	4	5	6	7	8	9	10	11
a0	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000	- 40.000
a1	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a2	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a3	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a4	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a5	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a6	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a7	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a8	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a9	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a10	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
b	15.000	15.000	15.000	15.000	15.000	15.000	15.000	15.000	15.000	15.000	15.000
Rf	0,00	0,05	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50
β	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10
ρ	0,09	0,14	0,19	0,24	0,29	0,34	0,39	0,44	0,49	0,54	0,59

Figure 5.2

FLS Inputs											
	1	2	3	4	5	6	7	8	9	10	11
Add Cost	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
Bus risk	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Comm Impl	0	0	0	0	0	0	0	0	0	0	0
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Market risk	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
NPV	94689	68368	49413	35382	24737	16476	9935	4660	338	-3257	-6285
Reputation	1	1	1	1	1	1	1	1	1	1	1
Sinergies	1	1	1	1	1	1	1	1	1	1	1
Strat Consist	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Figure 5.3

Outputs											
	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,33	0,36	0,42	0,45	0,47	0,48	0,48	0,48	0,48	0,49	0,50
Growth	0,92	0,87	0,78	0,73	0,69	0,66	0,64	0,61	0,59	0,59	0,59
Impl_Options	0,50	0,52	0,56	0,58	0,58	0,58	0,58	0,57	0,57	0,57	0,57
Exclusivity	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30
Qual_Analysis	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
NIV	0,75	0,70	0,58	0,49	0,41	0,36	0,32	0,29	0,25	0,24	0,23
Risk	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48
Inv_Value	0,76	0,69	0,55	0,47	0,42	0,39	0,36	0,33	0,30	0,29	0,28
Waiting	0,29	0,28	0,26	0,31	0,37	0,41	0,44	0,47	0,49	0,50	0,51
ExV	**0,75**	**0,69**	**0,54**	**0,47**	**0,41**	**0,37**	**0,33**	**0,29**	**0,25**	**0,24**	**0,24**

Figure 5.4

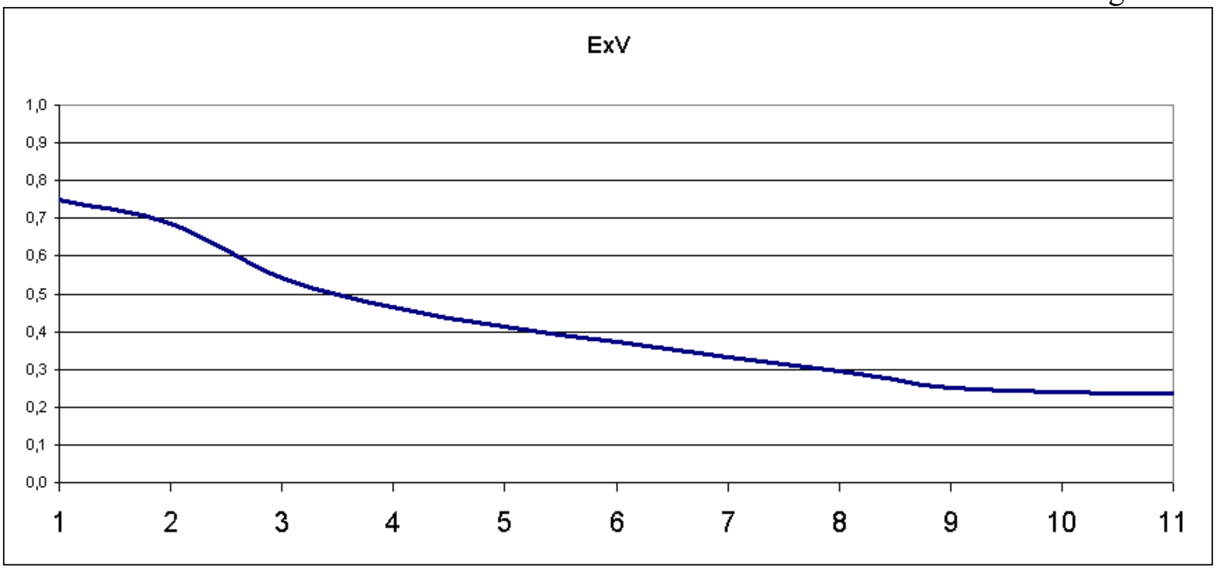


Figure 5.5

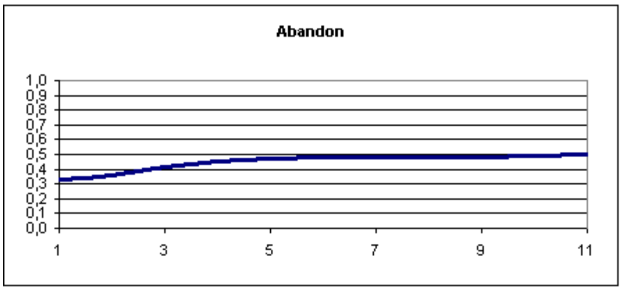


Figure 5.6

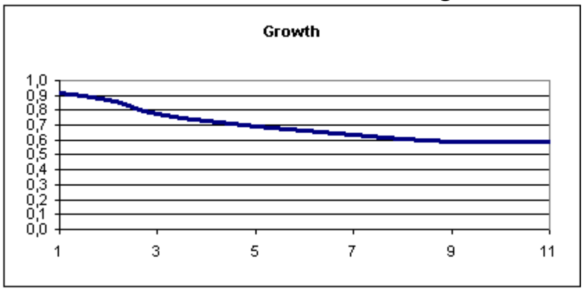


Figure 5.7

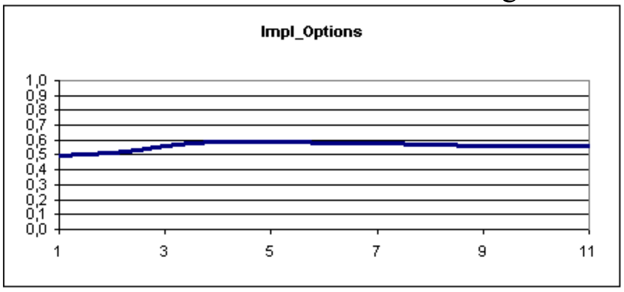


Figure 5.8

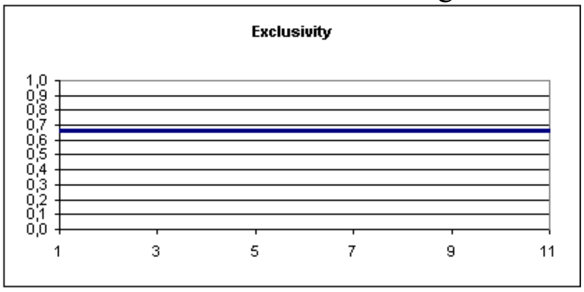


Figure 5.9

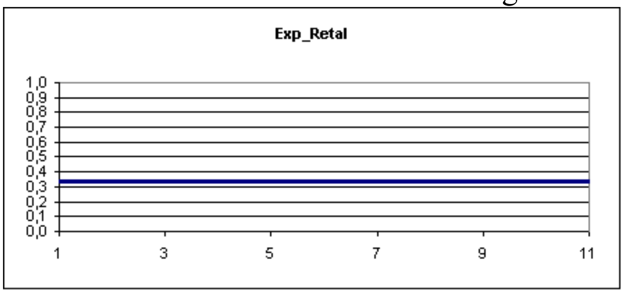


Figure 5.10

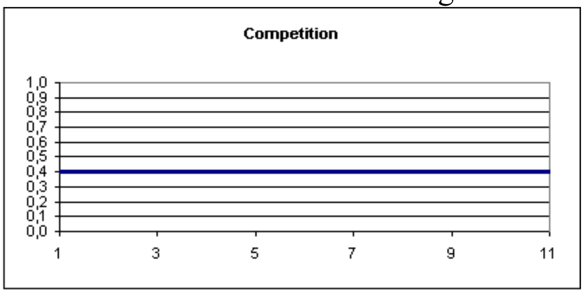


Figure 5.11

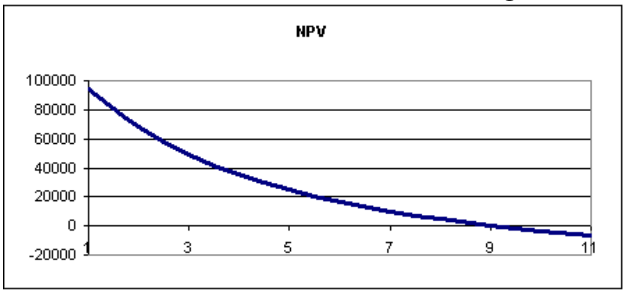


Figure 5.12

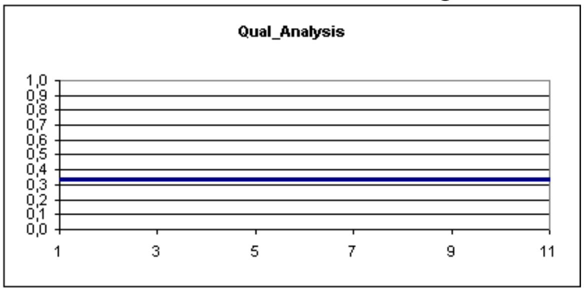


Figure 5.13

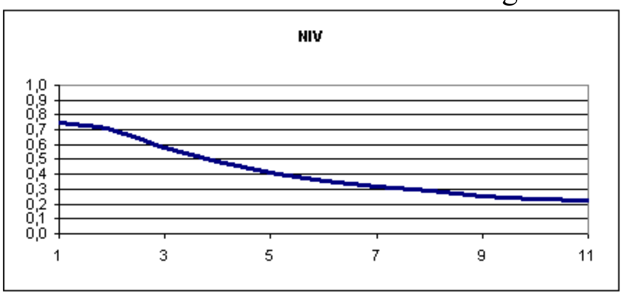


Figure 5.14

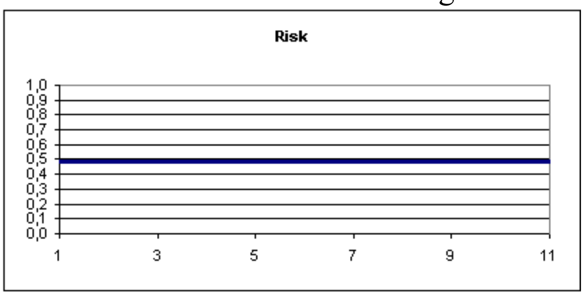


Figure 5.15

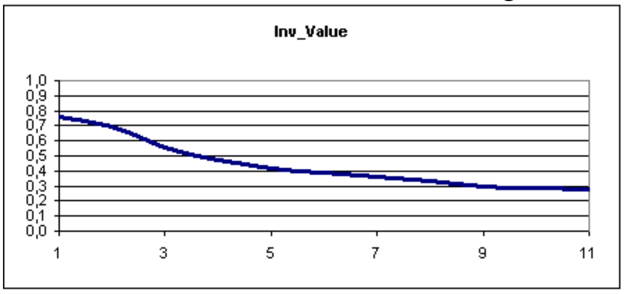


Figure 5.16

Fig. 6 show how sensitive is *ExV* to a change in *b*, the residual value of the project. We can note that the situation does not change much from *b=0* to *b=50000* (Fig.6.1). This is quite consistent with the idea that the final flow should not be very important in determining the decision (unless it is abnormally high).

Figure 6.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000
a1	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a2	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a3	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a4	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a5	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a6	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a7	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a8	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a9	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
a10	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
b	-	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000
Rf	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23

Figure 6.2

FLS Inputs											
	1	2	3	4	5	6	7	8	9	10	11
Add Cost	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
Bus risk	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Comm Impl	0	0	0	0	0	0	0	0	0	0	0
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Market risk	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
NPV	15985	16616	17247	17878	18509	19140	19770	20401	21032	21663	22294
Reputation	1	1	1	1	1	1	1	1	1	1	1
Sinergies	1	1	1	1	1	1	1	1	1	1	1
Strat Consist	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Figure 6.3

Outputs											
	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,48	0,48	0,48	0,48	0,48	0,48	0,47	0,47	0,47	0,47	0,47
Growth	0,66	0,66	0,66	0,67	0,67	0,67	0,67	0,68	0,68	0,68	0,68
Impl_Options	0,58	0,58	0,58	0,58	0,58	0,58	0,58	0,58	0,58	0,58	0,58
Exclusivity	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67	0,67
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30
Qual_Analysis	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
NIV	0,36	0,36	0,36	0,37	0,37	0,38	0,38	0,39	0,39	0,39	0,40
Risk	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48
Inv_Value	0,38	0,39	0,39	0,39	0,39	0,40	0,40	0,40	0,40	0,41	0,41
Waiting	0,41	0,41	0,41	0,40	0,40	0,40	0,39	0,39	0,39	0,38	0,38
ExV	0,37	0,37	0,37	0,38	0,38	0,38	0,39	0,39	0,39	0,40	0,40

Figure 6.4

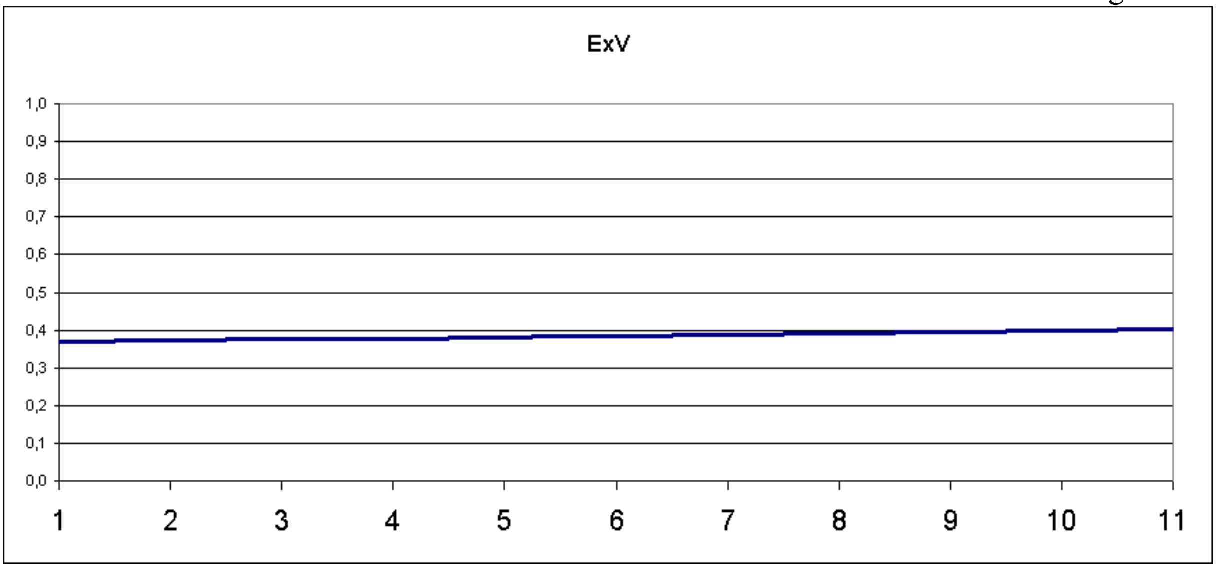


Figure 6.5

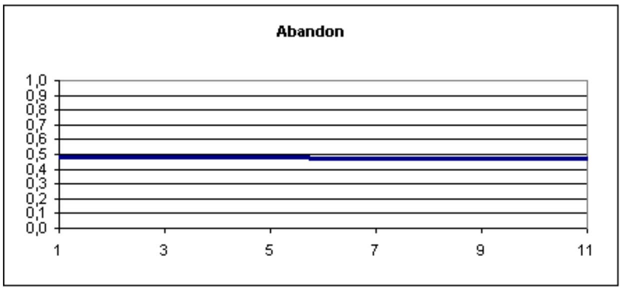


Figure 6.6

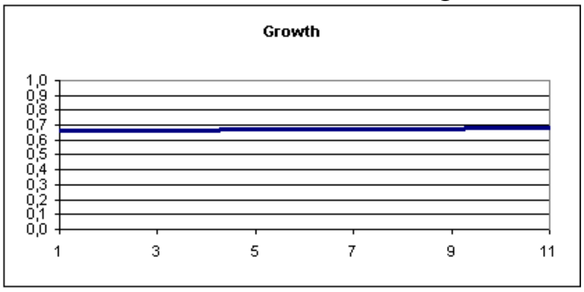


Figure 6.7

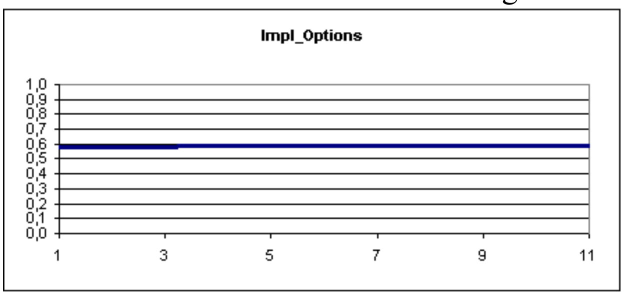


Figure 6.8

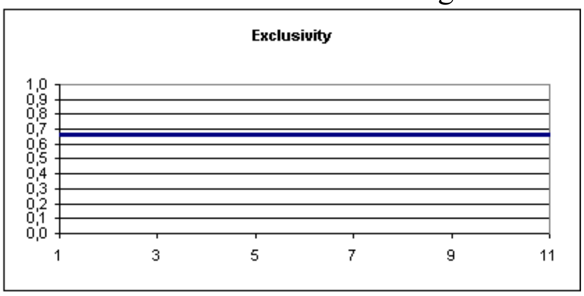


Figure 6.9

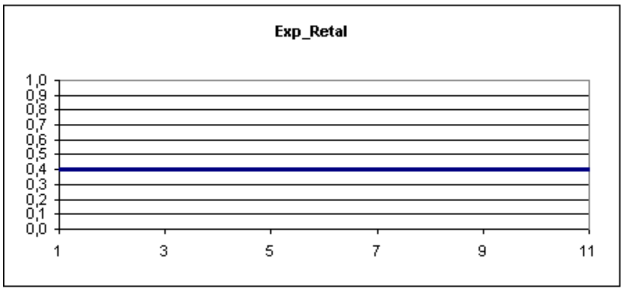


Figure 6.10

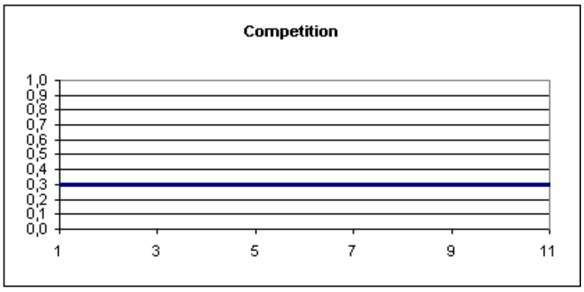


Figure 6.11

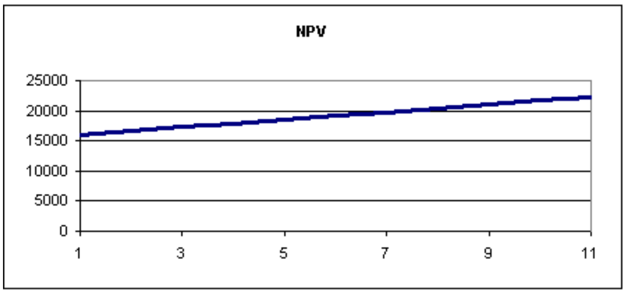


Figure 6.12

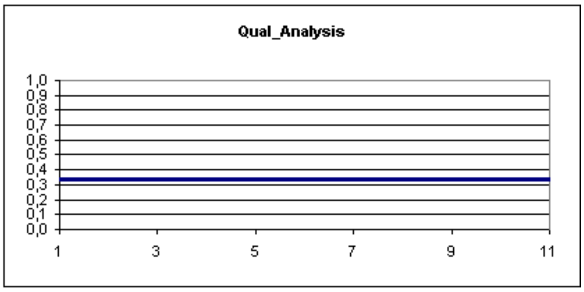


Figure 6.13

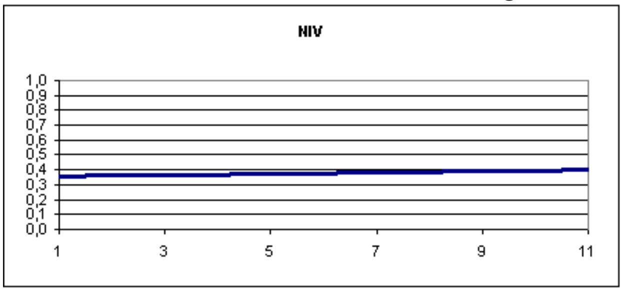


Figure 6.14

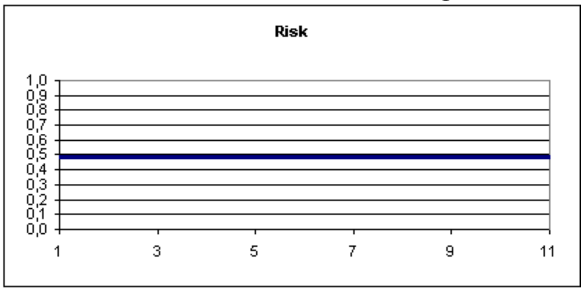


Figure 6.15

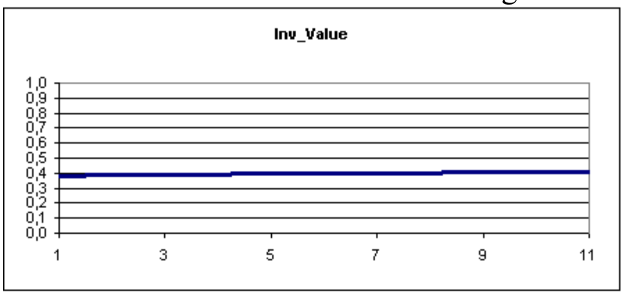


Figure 6.16

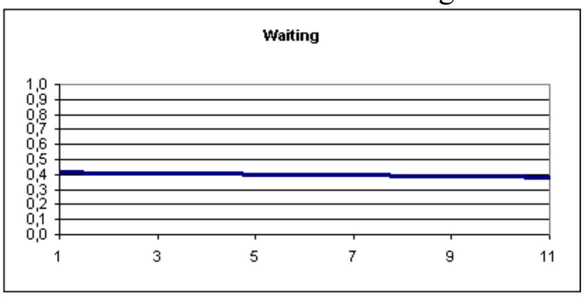


Fig. 7 shows *ExV* as a function of 6 value drivers for different values of *BusRisk*, other things equal. The value drivers under considerations are: *Differentiation*, *KnowHow*, *Reputation*, *CommImpl*, *Sinergies*, *StratConsist*. We set *BusRisk=0, 0.2, 0.4, 0.6, 0.8, 1.00* and draw up the corresponding curves in Fig.7.4. Fig 7.2 shows only the case *BusRisk=0.* For any fixed value of *BusRisk* we have an increasing function. Note that *ExV* rises from very low values (case 1) to very high values (case 2), while the NPV is fixed. This means that qualitative drivers may have a large impact on the exercise value of the option to invest; in particular, even if the NPV is held fixed to the rather high value of 48.401. *ExV* is very low if the six drivers are not favorable.

The curve shifts down as *BusRisk* is being augmented and for fixed values of the 6 drivers the propensity to invest immediately is being diminished (Fig.7.4).

Figure 7.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000
a1	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a2	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a3	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a4	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a5	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a6	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a7	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a8	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a9	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a10	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
b	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Rf	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,09	0,09	0,09	0,09	0,09	0,09	0,09	0,09	0,09	0,09	0,09

Figure 7.2

FLS Inputs											
	1	2	3	4	5	6	7	8	9	10	11
Add Cost	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Bus risk	0	0	0	0	0	0	0	0	0	0	0
Comm Impl	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Market risk	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
NPV	48401	48401	48401	48401	48401	48401	48401	48401	48401	48401	48401
Reputation	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Sinergies	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Strat Consist	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

Figure 7.3

Outputs											
	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,37	0,37	0,37	0,37	0,37	0,37	0,37	0,37	0,37	0,37	0,37
Growth	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68
Impl_Options	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52
Exclusivity	0,00	0,07	0,13	0,20	0,27	0,33	0,50	0,62	0,71	0,83	1,00
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,80	0,75	0,71	0,67	0,62	0,55	0,43	0,34	0,28	0,21	0,05
Qual_Analysis	0,00	0,07	0,13	0,20	0,27	0,33	0,50	0,62	0,71	0,83	1,00
NIV	0,41	0,47	0,51	0,55	0,56	0,57	0,62	0,65	0,68	0,73	0,82
Risk	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,13
Inv_Value	0,33	0,44	0,50	0,55	0,57	0,57	0,61	0,63	0,65	0,71	0,85
Waiting	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25
ExV	0,27	0,42	0,49	0,55	0,57	0,57	0,61	0,62	0,63	0,69	0,79

Figure 7.4

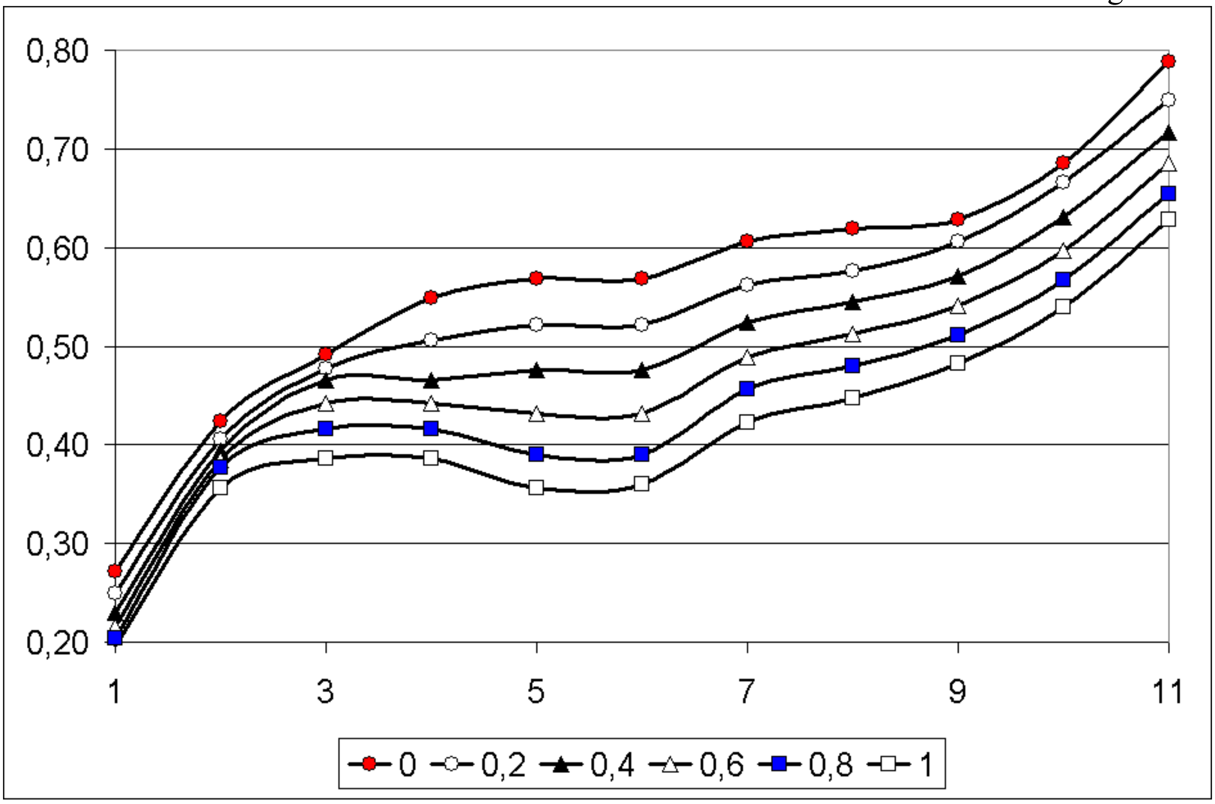


Fig.8 shows *ExV* as a function of the aforementioned value drivers but now for different values of the risk-free rate. We set $R_f=0,\ 0.05,\ 0.1,\ 0.15,\ 0.20$ and 0.25 (Fig.8.1 shows only the case $R_f=0$). In this case as risk-free rate rises the curve shifts down.

Figure 8.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000	- 20.000
a1	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a2	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a3	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a4	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a5	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a6	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a7	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a8	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a9	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a10	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
b	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Rf	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08

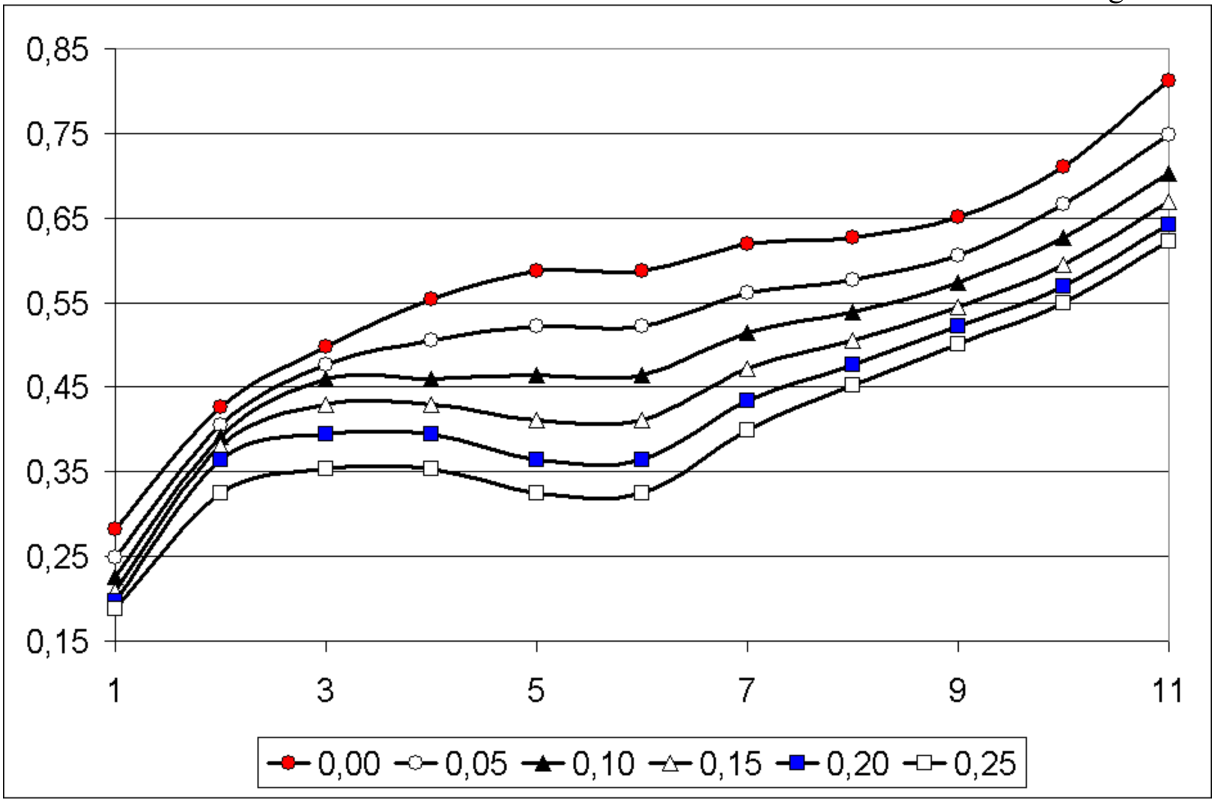
Figure 8.2

FLS Inputs	1	2	3	4	5	6	7	8	9	10	11
Add Cost	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Bus risk	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Comm Impl	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Market risk	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
NPV	51733	51733	51733	51733	51733	51733	51733	51733	51733	51733	51733
Reputation	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Sinergies	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Strat Consist	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1

Figure 8.3

Outputs	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45
Growth	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75
Impl_Options	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55
Exclusivity	0,00	0,07	0,13	0,20	0,27	0,33	0,50	0,62	0,71	0,83	1,00
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,80	0,75	0,71	0,67	0,62	0,55	0,43	0,34	0,28	0,21	0,05
Qual_Analysis	0,00	0,07	0,13	0,20	0,27	0,33	0,50	0,62	0,71	0,83	1,00
NIV	0,42	0,48	0,52	0,56	0,58	0,59	0,64	0,67	0,70	0,75	0,84
Risk	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30
Inv_Value	0,34	0,44	0,50	0,56	0,59	0,59	0,63	0,64	0,67	0,72	0,86
Waiting	0,24	0,24	0,24	0,24	0,24	0,24	0,24	0,24	0,24	0,24	0,24
ExV	**0,28**	**0,43**	**0,50**	**0,55**	**0,59**	**0,59**	**0,62**	**0,63**	**0,65**	**0,71**	**0,81**

Figure 8.4



Note that both in Fig.7.4 and Fig.8.4 the distance among the 6 curves is at its maximum in cases 5 and 6, where the value drivers considered are neither low nor high. In these cases *ExV* is more sensible to change in *BusRisk* and R_f respectively. This may be justified in the following way: if the 6 drivers are very low or low, then it does not matter so much whether the risk is low or high. If *Risk* is low, the investor is quite sure of her own NPV, but as we know NPV is calculated regardless of any consideration about all others qualitative factors. If these factors are not favorable at all, *Risk* does not play a major role (the latter is played by the six qualitative drivers). The same is true for very high values of *Risk* though the distance among curves is a little greater. Conversely, when the 6 drivers at hand are neither low nor high (case 5 and case 6) , there is a high degree of indeterminacy so that *Risk* makes things change a lot. As for Fig.8.4 when the 6 drivers are not favorable, it does not matter much whether R_f is small or not, and the same is true for high values of the six drivers. When indeterminacy prevails in the latter, then R_f plays a major role.

Fig.9 shows *ExV* as a function of both *BusRisk* and *MarketRisk* for different values of a_0, the initial cost. We set a_0= *-5000,-10000,-15000,-20000,* and *–25000* (Fig.9.1 shows only the case a_0 = *-5000*). The curve is decreasing for higher values of risk, as we expect, and it shifts down as the initial cost a_0 rises (Fig.9.4). Note that while *ExV* is decreasing monotonically, the reduction is not so sharp[2]: for a_0=*-5000* *NIV* decreases from 0.65 to 0.41 but *ImplOptions* increases from 0.44 to 0.75. The effect of *NIV* prevails, so that *InvValue* does not reduce so much (from 0.66 to 0.44) and *Waiting* increases from 0.2 to 0.55, which leads to the reduction in *ExV* from 0.68 to 0.41. For the other value of a_0 the behavior is analogous.

Figure 9.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000	- 5.000
a1	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a2	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a3	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a4	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a5	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a6	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a7	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a8	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a9	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
a10	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
b	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Rf	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,05	0,09	0,13	0,17	0,21	0,25	0,29	0,33	0,37	0,41	0,45

[2] Again, this is our judgment, maybe another evaluator could interpret the reduction differently.

Figure 9.2

FLS Inputs											
	1	2	3	4	5	6	7	8	9	10	11
Add Cost	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
Bus risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Comm Impl	0	0	0	0	0	0	0	0	0	0	0
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Market risk	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
NPV	78356	63401	52208	43666	37027	31779	27564	24131	21296	18927	16925
Reputation	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Sinergies	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Strat Consist	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Figure 9.3

Outputs											
	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,00	0,22	0,28	0,34	0,39	0,53	0,58	0,59	0,59	0,60	0,67
Growth	0,59	0,70	0,75	0,71	0,73	0,81	0,79	0,79	0,82	0,85	0,92
Impl_Options	0,44	0,48	0,50	0,50	0,56	0,64	0,64	0,65	0,66	0,67	0,75
Exclusivity	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42
Qual_Analysis	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
NIV	0,65	0,60	0,56	0,53	0,50	0,47	0,46	0,44	0,43	0,42	0,41
Risk	0,00	0,19	0,30	0,37	0,48	0,67	0,72	0,76	0,79	0,86	1,00
Inv_Value	0,66	0,59	0,55	0,53	0,49	0,47	0,45	0,44	0,43	0,43	0,44
Waiting	0,20	0,25	0,25	0,28	0,32	0,42	0,47	0,51	0,53	0,54	0,55
ExV	0,68	0,58	0,55	0,52	0,49	0,47	0,45	0,44	0,42	0,41	0,37

Figure 9.4

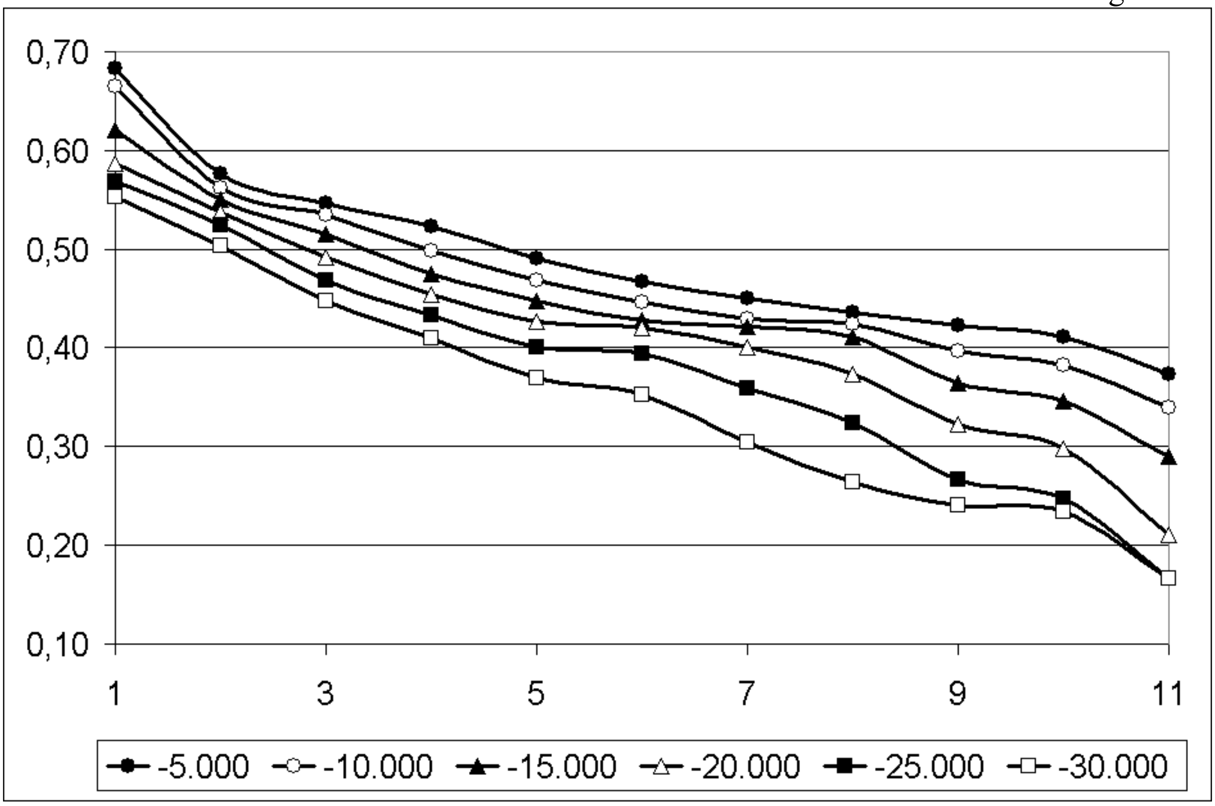


Fig.10 shows the case where cash flows released by the project increase by α percent each period (included the liquidation value *b*). *ExV* is now seen as a function of α for different values of *BusRisk* (*=0, 0.2, 0.4 ,0.6, 0.8, 1)*. Fig.10.2 shows only the zero case. We have now

$$a_s = a_{s-1}(1+\alpha) \qquad s = 2,\ldots,10 \qquad \text{and} \qquad b = a_{10}(1+\alpha)$$

For any fixed *BusRisk* the function is increasing, as it is obvious (the NPV increases as α increases, which causes *NIV* and *ImplOptions* to rise, which in turn cuses *InvValue* to rise, being other things equal). But each curve keeps almost constant until case 6, then begins to increase faster. The reason is that that the values we have selected lead to such a small NPV until case 6 that *InvValue* remains almost unvaried. However, for *BusRisk=1* there is no great variation in *ExV* from case 6 to case 11 (the risk is too high, it is better to wait, even if *α=0.43*), for *BusRisk=0* the variation is much more remarkable, going from 0.17 to 0.69. Further, the distance among the curves is small until 6. From case 7 the distance grows larger.

Figure 10.1

NPV Inputs											
	1	2	3	4	5	6	7	8	9	10	11
a0	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000	- 60.000
a1	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
a2	5.150	5.350	5.550	5.750	5.950	6.150	6.350	6.550	6.750	6.950	7.150
a3	5.305	5.725	6.161	6.613	7.081	7.565	8.065	8.581	9.113	9.661	10.225
a4	5.464	6.125	6.838	7.604	8.426	9.304	10.242	11.240	12.302	13.428	14.621
a5	5.628	6.554	7.590	8.745	10.027	11.444	13.007	14.725	16.608	18.665	20.908
a6	5.796	7.013	8.425	10.057	11.932	14.077	16.519	19.290	22.420	25.944	29.899
a7	5.970	7.504	9.352	11.565	14.199	17.314	20.979	25.270	30.267	36.063	42.755
a8	6.149	8.029	10.381	13.300	16.897	21.296	26.644	33.103	40.861	50.127	61.140
a9	6.334	8.591	11.523	15.295	20.107	26.195	33.838	43.365	55.162	69.677	87.430
a10	6.524	9.192	12.790	17.589	23.927	32.219	42.974	56.808	74.469	96.851	125.024
b	6.720	9.836	14.197	20.228	28.473	39.630	54.577	74.419	100.533	134.623	178.785
Rf	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19
α	0,03	0,07	0,11	0,15	0,19	0,23	0,27	0,31	0,35	0,39	0,43

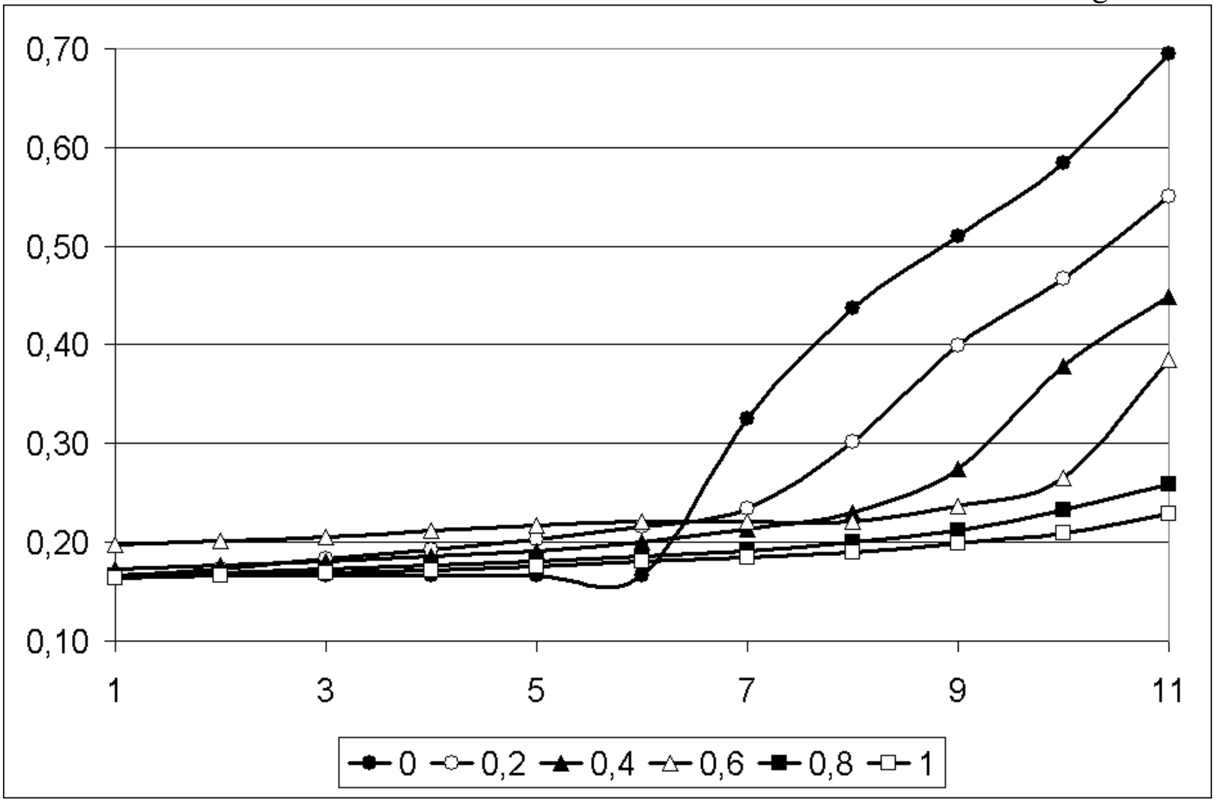
Figure 10.2

FLS Inputs											
	1	2	3	4	5	6	7	8	9	10	11
Add Cost	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
Bus risk	0	0	0	0	0	0	0	0	0	0	0
Comm Impl	0	0	0	0	0	0	0	0	0	0	0
Commitment	0	0	0	0	0	0	0	0	0	0	0
Differentiation	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Expiration	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Market risk	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
NPV	-34945	-30999	-26170	-20249	-12983	-4064	6881	20303	36740	56840	81374
Reputation	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Sinergies	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Strat Consist	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Figure 10.3

Outputs											
	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,49	0,47	0,45	0,42	0,39	0,35	0,33	0,33	0,33	0,33	0,33
Growth	0,51	0,53	0,55	0,58	0,59	0,59	0,62	0,67	0,73	0,81	0,92
Impl_Options	0,50	0,50	0,50	0,50	0,49	0,46	0,45	0,45	0,45	0,45	0,50
Exclusivity	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53	0,53
Exp_Retal	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Competition	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42
Qual_Analysis	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
NIV	0,13	0,15	0,16	0,17	0,19	0,23	0,34	0,43	0,50	0,57	0,65
Risk	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
Inv_Value	0,20	0,20	0,20	0,20	0,20	0,20	0,33	0,43	0,51	0,58	0,67
Waiting	0,59	0,58	0,56	0,53	0,49	0,46	0,43	0,40	0,36	0,32	0,20
ExV	**0,17**	**0,17**	**0,17**	**0,17**	**0,17**	**0,17**	**0,32**	**0,44**	**0,51**	**0,58**	**0,69**

Figure 10.4



Finally, Fig.11 points out the dependance of *ExV* on *Expiration.* The latter influences the former directly. As you note, *ExV* starts from a medium level, despite an extremely high NPV: the high levels of *Competition* and *Risk* and the low level of *QualAnalysis* compensate for the high levels of NPV and *ImplOptions*, so that *InvValue* is not so high and *Waiting* is not so low as to suggest a high propensity to exercise the option (Figg.11.5-11.16). With *Expiration* increasing from 0 to 0.5 the situation remains unvaried. Only when the risk of expiration becomes relevant, the exercise value gradually increases. When *Expiration* reaches level 1.00 the decision maker is incurring a very high risk of losing the right to exercise the option, as another competitor is likely to exercise it. This leads to the higher value *ExV=0.63*. Such a degree could be reasonably regarded by the evaluator as sufficiently high and the investment could be undertaken even if, were it not for the high risk of expiration, the project would not be completely attractive.

Figure 11.1

NPV Inputs	1	2	3	4	5	6	7	8	9	10	11
a0	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000	-30.000
a1	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a2	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a3	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a4	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a5	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a6	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a7	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a8	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a9	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
a10	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
b	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000	50.000
Rf	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03
β	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
ρ	0,31	0,31	0,31	0,31	0,31	0,31	0,31	0,31	0,31	0,31	0,31

Figure 11.2

FLS Inputs	1	2	3	4	5	6	7	8	9	10	11
Add Cost	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
Bus risk	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
Comm Impl	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Commitment	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3
Differentiation	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Entry Barr	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Exit Barr	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Expiration	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
Int of Riv	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Know How	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Market risk	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7	0,7
NPV	123813	123813	123813	123813	123813	123813	123813	123813	123813	123813	123813
Reputation	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Sinergies	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Strat Consist	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

Figure 11.3

Outputs	1	2	3	4	5	6	7	8	9	10	11
Abandon	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61
Growth	0,93	0,93	0,93	0,93	0,93	0,93	0,93	0,93	0,93	0,93	0,93
Impl_Options	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68	0,68
Exclusivity	0,28	0,28	0,28	0,28	0,28	0,28	0,28	0,28	0,28	0,28	0,28
Exp_Retal	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42	0,42
Competition	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61	0,61
Qual_Analysis	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07
NIV	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55	0,55
Risk	0,76	0,76	0,76	0,76	0,76	0,76	0,76	0,76	0,76	0,76	0,76
Inv_Value	0,51	0,51	0,51	0,51	0,51	0,51	0,51	0,51	0,51	0,51	0,51
Waiting	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
ExV	**0,48**	**0,48**	**0,48**	**0,48**	**0,48**	**0,48**	**0,53**	**0,56**	**0,57**	**0,62**	**0,63**

Figure 11.4

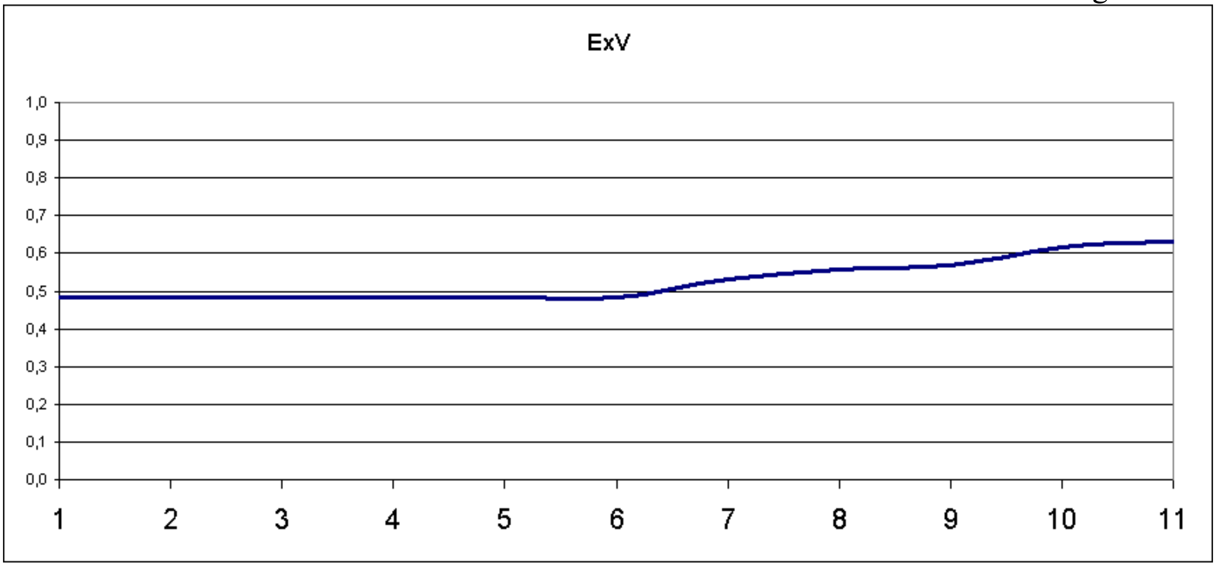


Figure 11.5

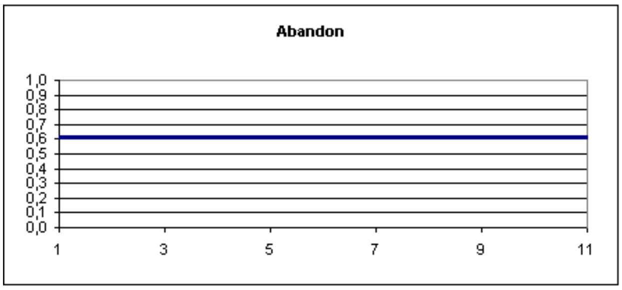


Figure 11.6

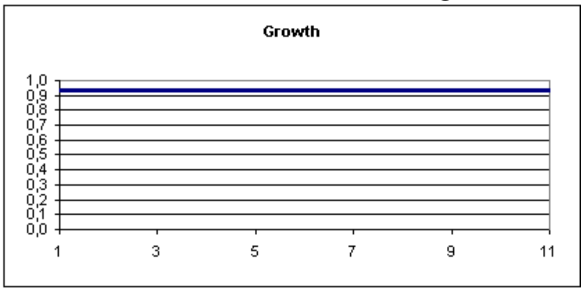


Figure 11.7

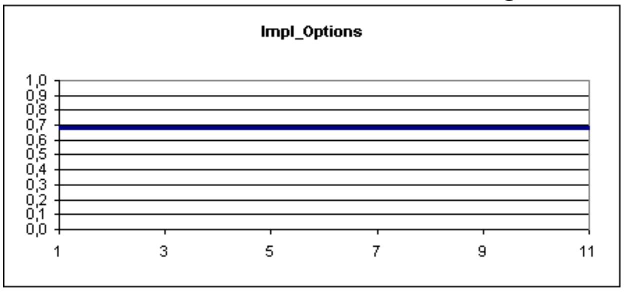


Figure 11.8

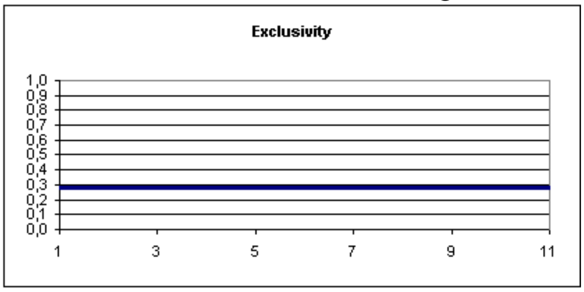


Figure 11.9

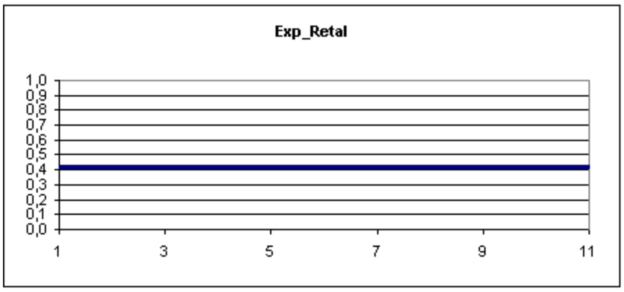


Figure 11.10

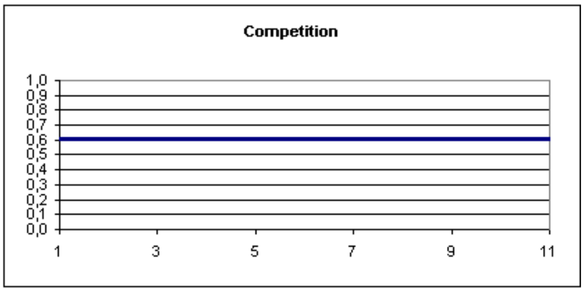


Figure 11.11

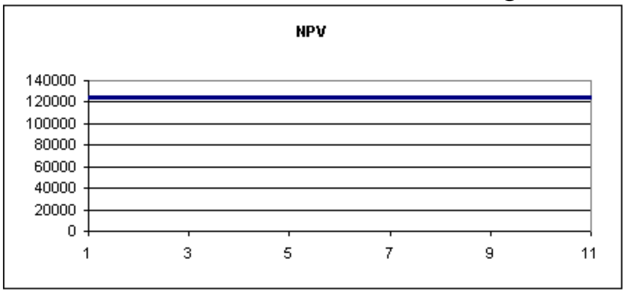


Figure 11.12

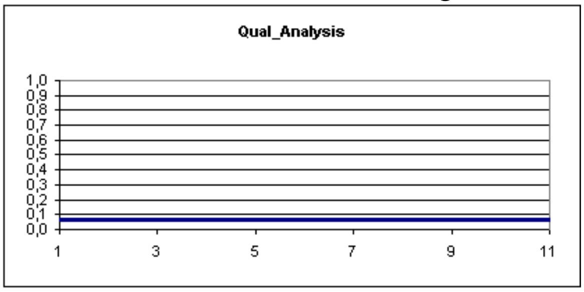


Figure 11.13

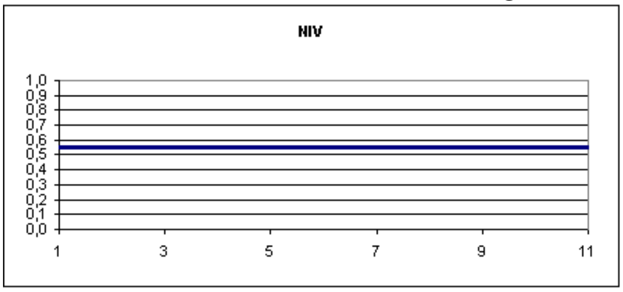


Figure 11.14

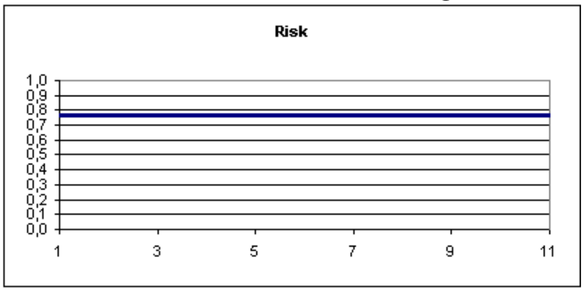


Figure 11.15

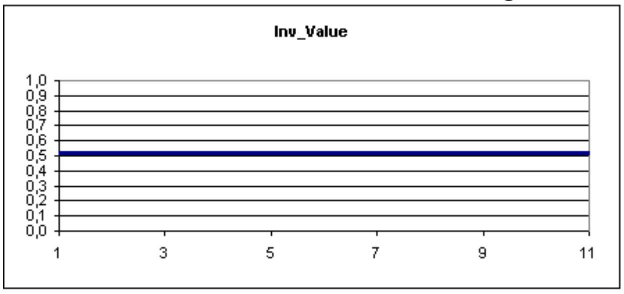


Figure 11.16

6. Conclusive Remarks

This paper presents a model for evaluating a compound strategic option with a fuzzy expert system. This proposal is able to solve some problems arisen in the options-based classical models. The difficulties to practically implement current real option models rest on the complexity of the tools used and on strong and unrealistic implicit assumptions. As Lander and Pinches suggest, a new framework is searched for, apt to handle real situations without, at the same time, increasing the complexity of the model. Our model is a first step in this sense. The fuzzy decision tree we have constructed has several advantages, among which its flexibility and its immediate applicability. The actual fuzzy model we present here can be easily modified in order to satisfy the needs of any decision maker. The decision tree has to be drawn up with the help of the actual decision makers. As we have seen, it easily copes with situations where qualitative parameters play an important role and where the investor has multiple objectives. The model is very sensitive to the subjectivity and personality of the evaluator, which is, in our opinion, an advantage rather than a drawback. In this sense, it is both universal and particular, as it is applicable to any investor but can be set so as to adequately describe the way of thinking of each single investor (it is *ad personam*). We think it can be appreciated by practitioners for its ease of understanding and applicability. It is particularly suited for complex situations such as strategic investment options, where multiple quantitative and qualitative uncertain variables have to be considered: Our modular approach enables us to subdivide all variables in subsets giving rise to rule blocks for each subsets. The rule blocks present intermediate outputs that are aggregated so as to draw up other intermediate outputs while reducing the number of total rule blocks, until the final tree-trunk is reached.

The sensitivity analysis we have showed tests the robustness of our model and its effective application to practical situations. The result we have reached are consistent with those existing in the literature (see Dixit, Pindyck, *op.cit.*), but they are also more refined for three main reasons: *In primis*, we are able to cope with many variables without any sort of formal complexity, *in secundis* we are not bound to any assumption whatsoever about the path followed by the random variables (we are just bound to the subjectivity of the evaluator), *in tertiis* we can justify step by step the final value by analyzing the behavior of the intermediate outputs as the inputs vary. In this way, different, opposing, reinforcing effects on *ExV* can be directly seen; whenever some rule is considered not fully convincing, the evaluator can modify at any time the model.

Future researches can consist of a development of a more general model, much powerful than the present one as well as examples from real situations. A next paper will be devoted to the actual implementation of this framework to a particular firm.

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