Two-Dimensional Screening: A Case of Monopoly Regulation*

Luigi Brighi[†] and Marcello D'Amato[‡]

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Abstract

This paper deals with the optimal regulatory policy of a monopolist producing two goods and with two-dimensional private information about costs. The case of perfectly and negatively correlated cost's parameters is analyzed and a complete characterization of the optimal mechanism by means of easily interpretable conditions is obtained.

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1 Introduction

Screening problems have been extensively studied under the simplifying assumption that the heterogeneity of preferences in a principal-agent model can be represented by a single parameter. The general properties of the solution are well understood and a number of interesting applications, including problems of nonlinear pricing, monopoly regulation and optimal taxation, are available. There are many cases, however, where the modelling of agent's private information requires more than one parameter. The problem of regulating a monopolist with unknown costs is a case in point. This problem has been analysed in a seminal paper by Baron and Myerson (1982) under the hypothesis that the regulator knows fixed but not marginal costs. As also recognised by Baron and Myerson, a more realistic hypothesis is that both the components of costs are unknown to the regulator, therefore the analysis of the optimal regulatory policy should be carried out by using a model with two parameters of private information.

The analysis of multi-dimensional screening is significantly different and also more difficult than that of the standard one-dimensional model. The main difficulty is due to incentive compatibility constraints. In the one-dimensional setting the ordering of agent's types and the single-crossing assumption allow to identify a well defined pattern of binding constraints at the optimum. On the other hand, with multi-dimensional private information an exogenous ordering will not in general exist and all the possible configurations of binding incentive constraints have to be taken into account. This fact can make the technical analysis of the multi-dimensional case so complex to preclude the possibility of finding a closed-form solution to the screening problem, as it occurs in general models with continuous types.[1]

In a recent paper Armstrong and Rochet (1999) provide a thorough analy-

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[1]For an excellent survey of the literature and an assessment on the state of arts on multi-dimensional screening, see Rochet and Stole (2000).

sis of a screening model with two-dimensional private information and discrete types. Earlier versions of this model also appeared in Spence (1980) and Dana (1993). Armstrong and Rochet offer a complete set of solutions to their model by characterizing the optimal screening mechanisms in terms of correlation and the 'symmetry' of types. As can be expected, however, the economic interpretation of these conditions is not always direct and straightforward.

In the present paper we deal with a special case of Armstrong and Rochet (1999)'s model which admits a straightforward characterization of the optimal mechanism based on easily interpretable conditions.

We study a case of optimal regulation of a multiproduct monopolist with two-dimensional private information about costs. The monopolistic firm produces two goods and has private information on the parameter cost in each product line. The analysis is confined to the case where the cost parameters are perfectly and negatively correlated across goods, i.e. the case with only two types of monopolists which are 'specialized' on different products. The regulator, whose objective is to maximize a measure of social welfare which includes distributional concerns, is entitled to make a take-it-or-leave-it offer of a menu of contracts specifying quantities of each good and transfer payments to the firm.

The paper provides a complete characterization of the optimal regulatory policy. Unlike the 'four-type' model of Armstrong and Rochet we find situations in which the optimal mechanism is ex-post Pareto-efficient, i.e. situations where the regulator is able to implement complete information outcomes. This happens when a simple condition called 'ranking at first-best' is violated. Ranking at first-best holds when there exists a monopolist's type with the lowest costs at every pair of first-best quantities. In such a case the optimal regulatory policy is not ex-post Pareto-efficient. The regulator introduces quite unusual distortions of quantities or prices, such as quantities

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in excess of first-best levels or equivalently prices below marginal costs, and the monopolist may earn a positive informational rent.

A second condition is found which shows when the regulator succeeds in fully extracting the monopolist's informational rent by optimally distorting quantities away from first-best levels. This is another peculiarity of the 'two-type' case, which is not contemplated in single-dimensional screening models.

Within the context of a nonlinear pricing problem by a multiproduct monopolist, Sibley and Srinagesh (1997) and Rochet and Stole (2000) studied a similar 'two-type' model. Sibley and Srinagesh do not offer the same variety of results we obtain, partly because their analysis is restricted to a less general class of mechanisms, i.e. optional two-part tariffs. Rochet and Stole provide a complete analysis of the particular case with linear-quadratic utility and reach very similar results to those found in the present paper. As it will be seen, the economic intuition for our results can be given following the literature on countervailing incentives (see, e.g. Maggi and Rodriguez-Clare (1995) and Lewis and Sappington (1989)).

The paper is organized as follows. The basic model is set in Section 2 and the optimal mechanism is fully characterized in Sections 3 and 4.

2   The model

There is a monopolistic firm producing two different goods, respectively, a and b, whose quantities are denoted by $q = (q_a, q_b)$. We assume that each good is produced at constant marginal costs $\theta = (\theta_a, \theta_b)$ and that θ_a and θ_b are firm's private information parameters. For convenience fixed costs are neglected, so that the monopolist's cost function can be simply written as

$$C(q; \theta) = \theta_a q_a + \theta_b q_b \tag{1}$$

For every parameter θ_k, $k = a, b$, there are only two possible realizations, $\bar{\theta}_k$ and $\underline{\theta}_k$, that is, high and low marginal costs, with $\bar{\theta}_k > \underline{\theta}_k > 0$. To simplify notation the difference between high and low cost parameters is normalized to 1, i.e.

$$\bar{\theta}_k - \underline{\theta}_k = 1 \qquad \text{for any} \qquad k = a, b \qquad (2)$$

The model is further specialized by assuming that the marginal costs of the two goods are perfectly and negatively correlated, so that there exist only two types of monopolist. Type 1 is the monopolist characterized by the parameters $\theta_1 = (\underline{\theta}_a, \bar{\theta}_b)$, i.e. low marginal costs in good a and high marginal costs in good b. On the other hand, type 2 monopolist has high marginal costs in good a and low marginal costs in good b, i.e. type 2 is identified by the parameters $\theta_2 = (\bar{\theta}_a, \underline{\theta}_b)$. The probability of the two types is common knowledge and is denoted by $p = Pr(\theta_1)$ and $(1 - p) = Pr(\theta_2)$.

It is important to stress that there is no ordering of types in terms of overall costs in our model and this is what makes the present case different from a standard problem with single-dimensional private information.

Total benefits (or gross consumer surplus) accruing to consumers from the consumption of the two goods are measured in money terms by the additively separable function

$$U(q) = u_a(q_a) + u_b(q_b) \qquad (3)$$

where the sub-utility functions $u_k(\cdot)$ are increasing, strictly concave and differentiable. Total surplus when the monopolist is of type $i = 1, 2$ is given by $S(q; \theta_i) = U(q) - C(q; \theta_i)$. The first-best quantities when the monopolist is of type i, which are denoted by $q_i^* = (q_{ai}^*, q_{bi}^*)$, maximize total surplus $S(q; \theta_i)$. We assume that the first-best quantities are strictly positive and thus are obtained by equating marginal cost and marginal utility for each good.

The regulatory mechanism consists of a menu of contracts specifying quantities or prices of the two goods and a transfer payment to the firm.

In order to unify the treatment of both cases of marketed and non marketed goods (and thus also include the case of procurement) we adopt the accounting convention that the revenues generated by the sale of outputs are collected by the government. Thus, regardless of the nature of the goods produced, the regulated firm's revenues consist only of transfer payments. Accordingly, we assume that the regulator sets quantities rather than prices.

The regulator proposes to the firm a menu of contracts which, for any transfer payment, specify the associated quantities of the two goods to be produced by the firm. The firm is permitted to make a binding choice from this menu of contracts. According to the Revelation Principle, the regulator only needs to consider contracts which are 'direct revelation mechanisms', i.e. contracts conditional to the agent's type. A direct mechanism is denoted by $[t_i, q_i]$, where t_i and $q_i = (q_{ai}, q_{bi})$ are respectively the transfer and the quantities specified by the contract designed for type i monopolist. Truthfully implementable contracts must satisfy individual rationality (IR) and incentive compatibility (IC) constraints. The IR constraint of type i is $t_i - C(q_i; \theta_i) \geq 0$ and is denoted by IR_i. The IC constraint of type i with respect to type j is $t_i - C(q_i; \theta_i) \geq t_j - C(q_j; \theta_i)$ and is denoted by IC_{ij}.

The objective of the regulator is to maximize social welfare which is measured by the sum of (net) consumer surplus and a share α of firm's profits which are weighted less than consumer surplus in order to take account of the regulator's distributional concerns. Social welfare is then given by $W(t, q; \theta) = [U(q) - t] + \alpha[t - C(q, \theta)]$ or equivalently by

$$W(t, q; \theta) = S(q; \theta) - (1 - \alpha)[t - C(q, \theta)] \tag{4}$$

with $0 < \alpha < 1$.

3   The optimal mechanism

To simplify subsequent analysis let us introduce some more notation and definitions. By $R_i = t_i - C(q_i; \theta_i)$ we denote the rent of type i under the direct mechanism $[t_i, q_i]$. The *incremental cost* at q, $\delta(q)$, is the difference between the cost of type 2 and the cost of type 1 for producing the output vector q, i.e.

$$\delta(q) = C(q; \theta_2) - C(q; \theta_1) = q_a - q_b$$

where the last equality follows by (2). The IR and IC constraints are rewritten in terms of rents and incremental costs as follows:

$$R_1 \geq 0 \tag{5}$$
$$R_2 \geq 0 \tag{6}$$
$$R_1 - R_2 \geq \delta(q_2) \tag{7}$$
$$R_2 - R_1 \geq -\delta(q_1) \tag{8}$$

Accordingly, we will substitute rents for transfers in the direct mechanism and consider the contracts $[R_i, q_i]$.

The regulator's problem is to find a direct revelation mechanism that gives the highest expected social welfare. Thus, the menu of optimal contracts is determined by solving the following optimization program

$$\max_{R_1, q_1, R_2, q_2} \quad p[S(q_1, \theta_1) - (1-\alpha)R_1] + (1-p)[S(q_2, \theta_2) - (1-\alpha)R_2]$$

subject to the IR and IC constraints $(5) - (8)$.

In order to find a solution to the regulator's program we have to identify the set of binding constraints at the optimum. This problem is readily solved in standard one-dimensional screening models where a natural ordering of

types exists. Indeed, in this case, only adjacent constraints are binding and they must bind in the same direction, i.e. either downward or upward. On the other hand, when no exogenous ordering of types exists, any possible configuration of binding constraints has to be taken into account. In such a case, the optimal allocations and the pattern of binding constraints at the optimum are jointly determined, as it occurs in general multi-dimensional models or in models with type-dependent participation constraints [see, e.g., Maggi and Rodriguez-Clare (1995) or Jullien (2000)]. In our model, however, this problem is easily solved. Indeed, it turns out that the pattern of binding constraints at the optimum is determined by the sign of incremental costs at first-best allocations.

First notice that, by a standard revealed preference argument, we have

$$C(q_1^*; \theta_1) - C(q_2^*; \theta_1) \leq U(q_1^*) - U(q_2^*) \leq C(q_1^*; \theta_2) - C(q_2^*; \theta_2)$$

which, in turn, yields the following inequality between the incremental costs at first-best:

$$\delta(q_2^*) \leq \delta(q_1^*) \tag{9}$$

There are three exhaustive and mutually exclusive cases to be considered:

$$\delta(q_2^*) \quad > \quad 0 \tag{10}$$

$$\delta(q_1^*) \quad < \quad 0 \tag{11}$$

$$\delta(q_2^*) \quad \leq \quad 0 \leq \delta(q_1^*) \tag{12}$$

If (10) holds type 2 has higher costs than type 1 at any first-best allocation, since by (9) we also have $\delta(q_1^*) > 0$. The monopolist's types are said to be *'ranked at first-best'*, type 1 is called 'superior' and type 2 is called 'inferior'. Condition (11) identifies the symmetric case of 'ranking at first-best' where the ordering of types is reversed, i.e. type 2 is 'superior' and type 1 is 'inferior'. Finally, when condition (12) holds none of the monopolists types

is able to produce the first-best quantities associated with the other types at lower costs, therefore we say that types are 'not ranked at first-best'.

$IC_{12}: R_2 = R_1 - \delta(q_2^*)$

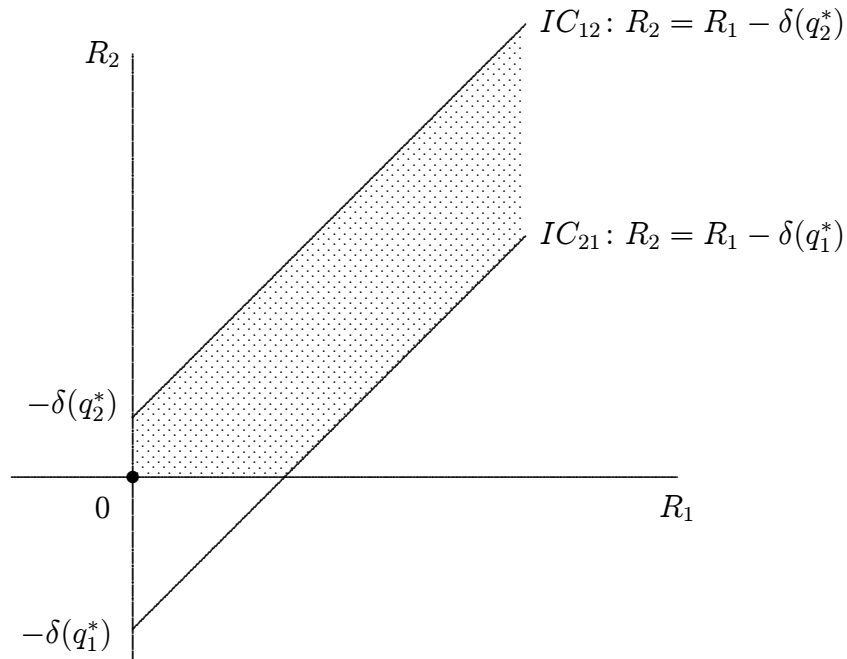$IC_{21}: R_2 = R_1 - \delta(q_1^*)$

Figure 1. No ranking at first-best

With the help of some diagrams we show how the above conditions determine the pattern of binding constraints at the optimum. The dotted area in Figure 1 represents the set of rents implementing the first-best output vectors of the two types when condition (12) holds, i.e. when there is 'no ranking at first-best'. Points on the axes represent combinations of rents at which one of the IR constraints binds, while points along the upward sloping straight lines correspond to rents associated with a binding IC constraint. As it is easily seen, irrespectively of the probability of types, the origin of the axes is the point where the first-best allocations can be implemented at the

minimum expected rents. Thus when 'no ranking at first-best' holds both IR constraints must be binding at the optimum while IC constraints are slack.
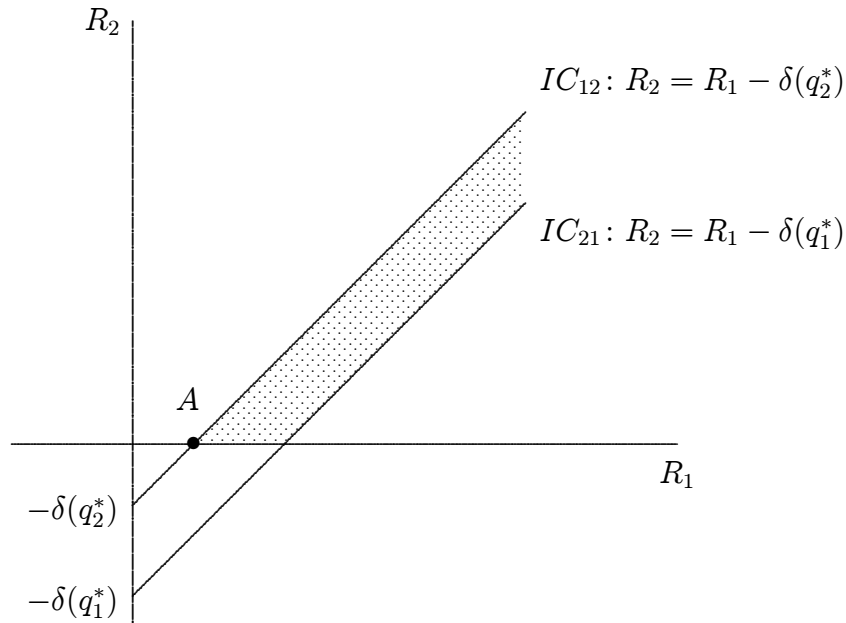

Figure 2. Ranking at first-best, condition (10)

The picture changes when 'ranking at first-best' holds. Specifically, Figure 2 shows the set of rents implementing the first-best allocations when (10) is satisfied. Point A corresponds to the rents implementing q_1^* and q_2^* at the minimum expected value and shows that IR_2 and IC_{12} are binding. In this case, however, the implementation of first-best output vectors may not be a solution, since the regulator may be willing to trade off informational rents to type 1 for efficiency. It will be seen below that the IR_2 and IC_{12} constraints not only bind at first-best but also at the optimum.

When 'ranking at first-best' holds under condition (11) we have still another pattern of binding constraints. By using the same sort of graphical

analysis it is easily seen that the candidate binding constraints at the optimum are IR_1 and IC_{21}.

Proposition 1. *(i) If types are 'not ranked at first-best', i.e. (12) holds, the optimal contract implements the first-best allocations and the monopolist does not earn any rents, i.e.*

$$q_i = q_i^* \qquad and \qquad R_i = t_i - C(q_i^*; \theta_i) = 0, \qquad i = 1, 2.$$

(ii) If types are 'ranked at first best', i.e. either (10) or (11) hold, the optimal contract implements the efficient quantities of the 'superior' type and inefficient quantities of the 'inferior' type. Specifically, if (10) is satisfied, the optimal allocation of output vectors is given by

$$q_1 = q_1^*, \qquad q_{a2} \le q_{a2}^*, \qquad q_{b2} > q_{b2}^* \quad with \quad q_{a2} \ge q_{b2}$$

and the optimal transfers are obtained by the binding constraints IC_{12} and IR_2, i.e.

$$R_1 = t_1 - C(q_1^*; \theta_1) = \delta(q_2), \qquad R_2 = t_2 - C(q_2; \theta_2) = 0.$$

For the proof see the Appendix.

According to Proposition 1, there are two optimal regulatory policies. If monopolist's types cannot be 'ranked at first-best', all the quantities set by the contract are Pareto-efficient and the monopolist does not earn any rents from his private information, since transfer payments exactly cover total costs. On the other hand, if one of the monopolist's types is 'superior' the contract sets first-best quantities for the 'superior' type, but inefficient quantities for the 'inferior' type, so that the optimal policy is not ex-post Pareto-efficient. Specifically, if (10) holds, the output of the good produced by the 'inferior' type at a low marginal cost (good b) is set in excess of the

efficient level and the output of the good produced at a high marginal cost (good a) is set below the efficient level.[2] In this case the 'superior' type may earn a positive informational rent which is given by

$$\delta(q_2) = q_{a2} - q_{b2} \geq 0. \tag{13}$$

The economic intuition behind Proposition 1 is quite simple. Owing to the presence of perfect negative correlation of cost parameters across goods, the monopolist is faced with 'countervailing incentives' [see, e.g., Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995)]. From the point of view of type 1, there is an incentive to *overstate* the marginal cost in good a, since this would bring him an extra profit of $(\bar{\theta}_a - \underline{\theta}_a)$ for each unit of good a produced. By so doing, however, type 1 is implicitly *understating* the marginal cost in good b, so that he will incur in a loss of $(\bar{\theta}_b - \underline{\theta}_b)$ for each unit of good b produced. Thus, type 1 faces a countervailing effect which mitigates the incentive to deviate from truthtelling.

The 'strength' of the countervailing incentives for type 1 depends on the quantities selected for type 2 and is measured by the incremental cost $\delta(q_2)$. A negative value of $\delta(q_2)$ indicates a strong countervailing effect which overcomes the benefits from misreporting, while a positive value of the incremental cost at q_2 reflects weak countervailing incentives for type 1. Clearly, a symmetric argument applies to the strength of the countervailing incentives for type 2, which is measured by $-\delta(q_1)$.

'Ranking' and 'no ranking' conditions (10) – (12) are now easily interpreted in terms of countervailing incentives at first best allocations. When $\delta(q_2^*) \leq 0$ countervailing incentives for type 1 are so strong that it is unprofitable for him to deviate, and similarly when $\delta(q_1^*) \geq 0$ type 2 has no benefits from misreporting his type. Thus 'no ranking at first-best', i.e. (12),

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[2]For marketed goods the optimal contract may be cast in terms of prices rather than quantities. The regulator adopts marginal cost pricing for type 1. As for type 2, the prices of goods a and b are set respectively above and below their marginal costs.

identifies the situation where both the monopolist's types face strong countervailing incentives at first-best allocations. On the other hand, ranking at first-best, i.e. (10) or (11), refers to situations where the countervailing incentives are strong for one type (the inferior monopolist) and weak for the other type (the superior monopolist). In this case the presence of countervailing incentives also explains the unusual pattern of quantity distortions at the optimum.

Suppose, for example, that (10) holds so that to implement first-best allocations the regulator should pay rents to type 1 given by $\delta(q_2^*) = q_{a2}^* - q_{b2}^*$. As in the standard model, the regulator is willing to trade off a distortion in the quantities produced by the 'inferior' type for a reduction of the informational rents to the 'superior' type. However, here a unit downward distorsion in the quantities of both goods produced by type 2 would not achieve any saving in the rents paid to type 1. To save on rents by keeping distortions at the minimum, the regulator decreases q_{a2} and increases q_{b2}. Indeed, under-production of good a (with respect to the first-best level of type 2) reduces the extra benefits for type 1 from misreporting (standard downward distortion) and over-production of good b will increase the losses for type 1 from misreporting (upward distortion due to the presence of the countervailing effect).

4 Full extraction of informational rent

If the monopolist's types can be ranked at first best it may well happen that the 'superior' monopolist does not earn any informational rent even though the optimal quantities of the 'inferior' type are distorted. This fact is illustrated by the following example.

Example. Let us specialize our model by assuming a quadratic utility, i.e.

$$U(q_a, q_b) = (Aq_a - \frac{1}{2}q_a^2) + (Bq_b - \frac{1}{2}q_b^2)$$

where $A > \bar{\theta}_a$ and $B > \bar{\theta}_b$ so that the first-best quantities are strictly positive and are given by $q_{ai}^* = A - \theta_{ai}$ and $q_{bi}^* = B - \theta_{bi}$ with $i = 1, 2$.

Let $\underline{\theta}_a = \underline{\theta}_b = 1$ so that $\bar{\theta}_a = \bar{\theta}_b = 2$ and set $A = 5$, $B = 3$. The first-best outputs of the two types are respectively $q_1^* = (4, 1)$ and $q_2^* = (3, 2)$. Since $q_{a2}^* > q_{b2}^*$ condition (10) holds and the monopolist's types are ranked at first-best. If $\alpha = 1/4$ and $p = 1/2$ it can be seen that the unique solution to the regulator's problem sets $q_{a2} = q_{b2} = 5/2$, so that type 2's quantities are not Pareto-efficient and, by (13), the informational rent of type 1 is equal to zero.

Under what conditions does the full extraction of monopolist's rent occur? For convenience, we answer this question by focusing on the case of 'ranking at first-best' under condition (10). The analysis of the symmetric case where (11) holds is similar and will be omitted.

By Proposition 1.(ii), the informational rent of type 1 is zero when the optimal quantities of type 2 can be produced by both types at exactly the same costs and, specifically, when $q_{a2} = q_{b2}$. To see when such a case occurs let us consider the problem of maximizing total surplus of type 2, i.e. $S(q_a, q_b; \theta_2)$, under the 'no rent' constraint $q_a - q_b \leq 0$. Since (10) holds, the maximum is obtained at $q_a = q_b = \hat{x}$, where $\hat{x}$ is defined by[3]

$$u_a'(\hat{x}) - \bar{\theta}_a = \underline{\theta}_b - u_b'(\hat{x}) \tag{14}$$

and

$$\mu = u_a'(\hat{x}) - \bar{\theta}_a \tag{15}$$

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[3]It is easy to see that $\hat{x}$ is well defined, i.e. exists, is unique and $q_{b2}^* < \hat{x} < q_{a2}^*$.

is the Lagrange multiplier. μ measures the increase in type 2 total surplus when the 'no rent' constraint is relaxed, that is when the regulator allows type 1 to get some rent.

From the regulator's point of view, $(\hat{x}, \hat{x})$ is the best bundle of goods to be implemented by type 2 without paying any informational rent to type 1. However, in order to guarantee that $(\hat{x}, \hat{x})$ is also a solution to the regulator's problem, we have to check that social welfare does not increase when the 'no rent' constraint is relaxed. As it is easily seen, by increasing the informational rent of type 1 by one unit, the approximated expected gain in social surplus from type 2 is $(1-p)\mu$ and the expected loss from type 1 is $p(1-\alpha)$. Therefore, if gains are not greater than losses the regulator will implement the bundle $(\hat{x}, \hat{x})$ and the optimal contract will not allow for a positive informational rent to type 1. This is the basic argument behind the following result.

Proposition 2. *Let (10) hold, i.e. the monopolist's types are ranked at first-best. The informational rent of the 'superior type' under the optimal contract is zero, i.e. $R_1 = 0$, if and only if*

$$\mu \leq \frac{p}{1-p}(1-\alpha) \tag{16}$$

where μ is as defined by (14) and (15).

The proof is in the Appendix.

Condition (16) (together with (10)) characterizes the case where three constraints are binding at the optimum, and precisely IR_1, IR_2 and IC_{12}. Conversely, a violation of (16) implies that IR_1 is slack and that the 'superior' monopolist will earn a strictly positive informational rent. In both cases, however, Proposition 1.(ii) shows that the optimal allocation is not Pareto-efficient since quantities for the 'inferior' monopolist will be distorted.

In Section 3 we have seen that the nature of the optimal regulatory policy, which is determined by the 'ranking at first-best' conditions, does not

depend on the probability distribution of types or the distributional concerns of the regulator. These factors only affect the magnitude of informational rents and the extent of inefficiency in the allocation of goods. In particular, Proposition 2 shows the role played by the parameters p and α in the determination of rents. Clearly, (16) is more likely to be violated the lower is p and the higher is α. Accordingly, a strictly positive informational rent can be expected when the 'superior' type of monopolist is less likely and the regulator's distributional concerns are less important.

As a final remark we compare Proposition 2 to a similar result found by Rochet and Stole (2000) in the analysis of a 'two-type' model of nonlinear pricing by a multiproduct monopolist, where consumers' utility function is linear in private information parameters and quadratic in quantities.

If the functional form of utility is quadratic, as in the Example, condition (16) can be written more directly in terms of incremental costs at first-best. Indeed, it is easily seen that $\hat{x} = (q_{a2}^* + q_{b2}^*)/2$ and $\mu = (q_{a2}^* - q_{b2}^*)/2 = \delta(q_2^*)/2$. Therefore, (16) becomes

$$\delta(q_2^*) \leq 2\frac{p}{1-p}(1-\alpha)$$

Moreover, for $\alpha = 0$, (10) and (16) can be simply written as follows[4]

$$0 < \delta(q_2^*) \leq p\delta(q_1^*)$$

An analogous condition is also found by Rochet and Stole (2000) [see p. 24].

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[4]Recall that, by (2), we have $(\bar{\theta}_a - \underline{\theta}_a) + (\bar{\theta}_b - \underline{\theta}_b) = 2$ or, equivalently, $2 + (\underline{\theta}_b - \bar{\theta}_a) = \bar{\theta}_b - \underline{\theta}_a$. Finally, noticing that $\delta(q_i^*) = (A - B) + \theta_{ib} - \theta_{ia}$, we obtain $[2 + \delta(q_2^*)] = \delta(q_1^*)$.

Appendix

Proof of Proposition 1.

Since the objective function is concave and the constraints are linear the Kuhn-Tucker conditions are both necessary and sufficient for a global maximum. Rearranging the first-order conditions yields

$$u'_a(q_{a1}) = \underline{\theta}_a - \gamma_{21}/p \tag{17}$$

$$u'_b(q_{b1}) = \bar{\theta}_b + \gamma_{21}/p \tag{18}$$

$$u'_a(q_{a2}) = \bar{\theta}_a + \gamma_{12}/(1-p) \tag{19}$$

$$u'_b(q_{b2}) = \underline{\theta}_b - \gamma_{12}/(1-p) \tag{20}$$

$$\lambda_1 + \lambda_2 = 1 - \alpha \tag{21}$$

$$\lambda_2 = (1-p)(1-\alpha) + \gamma_{12} - \gamma_{21} \tag{22}$$

where, λ_1 and λ_2 are the Lagrange multipliers of IR constraints and γ_{12} and γ_{21} are respectively the multipliers of the IC constraints (7) and (8). All the multipliers must be non negative and complementary slackness conditions must be satisfied. In order to derive the solutions we have to identify the binding constraints or, equivalently, the values of Lagrange multipliers. We proceed in three steps.

Step 1. $\gamma_{ij} > 0$ implies $\gamma_{ji} = 0$ and $\lambda_j > 0$, for $i \neq j$ and $i, j = 1, 2$. To prove the first implication let us suppose that both IC constraints are binding. Adding and rearranging equalities (7) and (8), using (2), yields

$$q_{a1} - q_{a2} = q_{b1} - q_{b2}.$$

By (17) and (19) the term on the LHS is strictly positive and by (18) and (20) the RHS is strictly negative and a contradiction follows. Hence $\gamma_{ij} > 0$ implies $\gamma_{ji} = 0$. Finally, $\lambda_j > 0$ is easily shown by using (21) and (22).

Step 2. (a) $\delta(q_1^*) > 0$ *implies* $\gamma_{21} = 0$. *(b)* $\delta(q_2^*) < 0$ *implies* $\gamma_{12} = 0$.

(a) Let us suppose that $\delta(q_1^*) = q_{a1}^* - q_{b1}^* > 0$ and $\gamma_{21} > 0$. From the FOC (17) and (18) and strict concavity of u_k we have $q_{a1}^* < q_{a1}$ and $q_{b1} < q_{b1}^*$. Subtracting these inequalities term by term yields $\delta(q_1) = q_{a1} - q_{b1} > q_{a1}^* - q_{b1}^*$, thus $\delta(q_1) > 0$. Moreover, by Step 1, $\gamma_{21} > 0$ implies $\gamma_{12} = 0$ and $\lambda_1 > 0$, so that from (6) and the binding constraints (5) and (8) it follows $0 \leq R_2 = -\delta(q_1)$, i.e. $\delta(q_1) \leq 0$ and we obtain a contradiction. Hence $\gamma_{21} = 0$.

The proof of Step 2 (b) is similar and will be omitted.

Step 3. (a) (10) holds if and only if $\gamma_{12} > 0$. *(b) (11) holds if and only if* $\gamma_{21} > 0$.

(a) Let us suppose that (10) holds and $\gamma_{12} = 0$. From (9) and (10) it follows $\delta(q_1^*) > 0$ so that, by Step 2(a) we also have $\gamma_{21} = 0$. Thus, from FOC (17) – (22) we notice that quantities are set at their first-best level and $R_1 = R_2 = 0$. Finally, from (7) we have $\delta(q_2^*) \leq 0$ which violates (10). Hence $\gamma_{12} > 0$.

To show the converse let $\gamma_{12} > 0$, then from FOC (19) and (20) and strict concavity of utility we have $q_{a2} < q_{a2}^*$ and $q_{b2} > q_{b2}^*$. Subtracting term by term yields $q_{a2} - q_{b2} < q_{a2}^* - q_{b2}^*$, i.e. $\delta(q_2) < \delta(q_2^*)$. Moreover, by Step 1, $\lambda_2 > 0$ and thus from (5) and equalities (6) and (7) we obtain $0 \leq R_1 = \delta(q_2)$. Combining this with the above inequality yields $\delta(q_2^*) > 0$, and (10) holds.

The proof of Step 3 (b) is similar and will be omitted.

Proposition 1.(i) is proved as follows. Steps 3 gives $\gamma_{12} = \gamma_{21} = 0$ and then, by (21) and (22), we have $\lambda_1 > 0$ and $\lambda_2 > 0$. Proposition 1.(ii) is proved by combining Step 1 and 3. For example, if (10) holds Step 3(a) gives $\gamma_{12} > 0$ and Step 1 $\gamma_{21} = 0$, and $\lambda_2 > 0$. The optimal quantities and transfers are then determined from the first-order conditions and the binding constraints. Finally, $q_{a2} \geq q_{b2}$, i.e. $\delta(q_2) \geq 0$, was established in the proof of Step 3(a). $\qquad \square$

Proof of Proposition 2.

Let the informational rent of type 1 be zero so that, by Proposition 1 (ii) and (13), we have $q_{a2} = q_{b2}$. Equations (19), (20), (14) and (15) yield $\hat{x} = q_{a2} = q_{b2}$ and $\gamma_{12} = \mu(1-p)$. From equations (21) and (22) we have $\lambda_1 = p(1-\alpha) - \gamma_{12}$ and finally, substituting γ_{12} and using $\lambda_1 \geq 0$, we obtain

$$\mu \leq \frac{p}{1-p}(1-\alpha)$$

i.e. $R_1 = 0$ implies (16).

To show the converse let (16) hold and $R_1 > 0$, i.e. $q_{a2} > q_{b2}$. Then the IR constraint (5) cannot be binding and $\lambda_1 = 0$. Therefore, by (21) and (22) we have $\gamma_{12} = p(1-\alpha)$. Substituting γ_{12} into (19) yields

$$u'_a(q_{a2}) - \bar{\theta}_a = \frac{p}{1-p}(1-\alpha)$$

It is easily seen that $\hat{x}$, defined by (14), must lay in between q_{b2} and q_{a2}, i.e. $q_{b2} < \hat{x} < q_{a2}$, thus, by strict concavity of u_a we have

$$\mu \equiv u'_a(\hat{x}) - \bar{\theta}_a > u'_a(q_{a2}) - \bar{\theta}_a = \frac{p}{1-p}(1-\alpha)$$

and (16) is violated. Hence (16) implies $R_1 = 0$ and this completes the proof.

$\square$

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