

# **Investment decisions, equivalent risk and bounded rationality**

*Carlo Alberto Magni*

Dipartimento di Economia Politica, Università di Modena e Reggio Emilia

Viale Berengario 51, 41100 Modena, Italy

Email: magni@unimo.it

Tel. 0039-059-2056777, fax 0039-059-2056937

## **Abstract.**

The Net Present Value maximizing model, a keystone in economics, is shown to fall prey of fallacies and inconsistencies that may be easily unmasked by performing a cognitive analysis of the decision-making process implied by the maximization problem. The model may be conveniently retrieved if the maximizing version of the criterion is shunt aside and a boundedly rational interpretation is given. The resulting ‘mixed strategy’, currently in use by many real-life decision makers, opens up terrain to a fruitful cooperation between bounded and unbounded rationality. This paper is consistent with a fluid and nondichotomous interpretation of dual-process theories.

PsycINFO code: 2340

JEL code: B40, G11, G31

Keywords. Decision-making, cognition, biases, heuristics, bounded rationality

# Investment decisions, equivalent risk and bounded rationality

## Introduction

This paper shows that the Net Present Value methodology, a widely used tool for investment decisions, is biased. However, interpreting the rule as a satisficing technique one is able to heal the fallacies and retrieve the biased model, thus producing a cognitively and logically impeccable decision-making criterion. As a result, we are left with a criterion generated in the realm of unbounded rationality that takes on a distinctive bounded-rationality flavour. This supports the idea of a fruitful cooperation between bounded and unbounded rationality.

This view is consistent with the idea of two mental systems performing different kinds of cognitive operations: intuitive and reflective (Evans & Over, 1996, Sloman, 1996a; Kahneman & Frederick, 2002; Sloman, 2002; Evans, 2004). The former gives rise to heuristics and to bounded rationality, the latter generates rule-based reasoning and is the rationale for unbounded rationality. This paper just shows a case where the bounded/unbounded dichotomy peters out and turns into a single medal with two sides representing the two concurrent cognitive systems at work. One would expect the *rational* side of the medal to be capable of debiasing the *intuitive* side. Paradoxically, it is quite the contrary: it is just the bounded-rationality interpretation model that heals the fallacies of the unboundedly rational model.

The paper is organized as follows. The first section summarizes the NPV model. Section 2 shows how the NPV criterion is used under uncertainty and introduces the Equivalent Risk tenet. Section 3 unmasks some biases of the NPV maximization process. Section 4 heals the inconsistencies of the maximization model by making recourse to a satisficing interpretation. Section 5 considers two recent papers of the financial literature that show that the satisficing interpretation leads to close-to-optimal results. Section 6 gives some hints of theoretical implications for modelling decision-making and for fluidly interpreting the double-sided system of reasoning. Some remarks conclude the paper.

## 1. Preliminary concepts

The NPV model is used for evaluating investments of any kind, as well as for governance purposes and firms' evaluations (see Bierman & Smidt, 1992; Rao, 1992; Damodaran, 1999; Copeland, Koller & Murrin, 2000; Brealey & Myers, 2000; Fernández,

2002). The concept of Net Present Value (NPV) is the brick of a normative building deep-rooted in the maximizing tradition of economics. The NPV model traces back to Fisher (1930), whose analysis is carried out under assumption of certainty. The idea of net present value maximization is commonplace in economic theory: “The firm attempts to maximize the present value of its net cash flow over an infinite horizon” (Abel, 1990, p. 755); “the net present value rule is also the basis for the neoclassical theory of investment ... Much of the theoretical and empirical literature on the economics of investment deals with issues of this sort” (Dixit & Pindyck, 1994, p. 5). The NPV analysis is equivalent to that of Jorgensen (1963) and to the  $q$  theory of investment (Tobin, 1969): “In all of this, the underlying principle is the basic net present value rule” (Dixit & Pindyck, 1994, p. 5), which formally translates the notion of excess (or economic) profit.

The NPV methodology may be summarized as follows: suppose a decision maker holds a sum  $X$  and wants to invest it for one period. Suppose she has the opportunity of investing  $X$  in a one-period project (let us call it A1) which generates a payoff  $X'$  after one period. Would she invest in such an investment? If she aims at maximizing her wealth she chooses that course of action that maximizes wealth increase (i.e. profit, return) among the available alternatives. Let A2 be the next-best alternative available to the investor and let  $r_2$  be its rate of return. Our decision maker will invest in A1 if

$$X' - X > r_2 X . \quad (1)$$

The left-hand side is the profit generated by project A1, the right-hand side is the profit generated by alternative A2. Eq. (1) may be rewritten as

$$-X + \frac{X'}{(1+r_2)} > 0 . \quad (2)$$

The left-hand side of (2) is called the *Net Present Value* of alternative A1. The NPV rule says that the decision maker should invest in A1 if the Net Present Value of A1 is positive. In fact, if and only if the NPV is positive the investment increases wealth optimally. The discount rate  $r_2$  is the so-called *opportunity cost of capital*: if a decision maker invests in A1 she foregoes the opportunity of earning  $r_2$  on her capital  $X$ . It is the rate of return of the

best available alternative other than the one under consideration. The opportunity cost of capital is then taken as a norm (in the sense of Kahneman & Miller, 1986).

In general, if one faces two or more mutually exclusive courses of action, one should calculate their NPVs by discounting cash flows at the opportunity cost of capital and choose the one with the highest NPV (for the investment with the highest NPV will maximize the increase in wealth). Therefore, letting  $A$  be the set of alternatives available to the decision maker, the NPV procedure implies that the decision maker should take the alternative  $x \in A$  which maximizes the NPV:

$$\max_{x \in A} \text{NPV}(x). \quad (\#)$$

## 2. The Equivalent Risk tenet

Under uncertainty, alternatives cannot be compared in terms of profit unless risk is the same. We cannot say *a priori* that, for example, an *expected* return of 100 is better than an *expected* return of 90 if the former is riskier than the latter. None of the two dominates the other: one is preferable in terms of return, the other one is preferable in terms of risk (risk-aversion is assumed). So, if our alternatives are not equivalent in risk, the NPV rule should not be adopted, since it only creates a *partial* ordering among alternatives. Instead of dismissing the NPV rule, academics and practitioners maintain it with a convenient modification that will be here named the Equivalent Risk (ER) tenet: *the decision maker must compare the alternative at hand with the next-best alternative having equal risk or, in other terms, the decision maker must maximize return with respect to equal-risk alternatives.*

Thus, a two-dimension decision problem (risk/return) is diverted to a one-dimension problem (return) via the ER principle and the NPV is then safeguarded. However, there is now the problem of finding the ‘correct’ opportunity cost of capital a decision maker should use to discount cash flows. Unanimously, the opportunity cost of capital “should equal the return the investor could have earned on other investment opportunities with comparable risk characteristics” (Dixit & Pindyck, 1994, p. 114). Such comparable-risk opportunities are found via the highly appraised *Capital Asset Pricing Model* (Sharpe, 1964; Lintner, 1965; Mossin, 1968), which guarantees, under suitable assumptions, that the (expected) rate of return of comparable opportunities is given by the sum of the risk-free

rate and a risk premium computed as a function of the systematic risk of the project (the so-called beta).

Stating the ER tenet in optimization terms, we may say that decision makers should then maximize NPV with respect to the subset  $A^* \subset A$  of those alternatives which are equivalent in risk:

$$\max_{x \in A^*} \text{NPV}(x) \quad (\#\#)$$

where  $A^*$  is the subset of those alternatives having the same risk (and therefore the same beta).

An analysis of the cognitive content of such a restriction will show that the idea of restricting the function's domain is illusory. Indeed, while apparently flawless, the constrained optimization problem in ( $\#\#$ ) is artificial, cognitively absurd and inapplicable.

### 3. Biases

Suppose a decision maker has  $X$  euros and is willing to invest them somewhere. She comes across investment A1 and uses the NPV+CAPM model to solve the decision problem "to undertake A1 or not". Suppose A2 is the best equivalent-risk alternative other than A1 and suppose the NPV of A1 is negative. According to NPV upholders this means that A1 is to be rejected. A significant feature of this resolution is that the rejection of A1 does not necessarily mean that A2 is undertaken. In other terms, a negative NPV means that A1 is to be rejected, *not that A2 should be undertaken*. Reject does not imply a specific course of action. In an NPV world, rejection suggests a decision maker what she should not do, not what she should do. This leads to several inconsistencies.

(1) If A1 is rejected and A2 is not undertaken, as the NPV rule allows, the optimization problem in ( $\#\#$ ) is not solved in a correct way: alternative A2 belongs to the domain  $A^*$  and the negative NPV indicates that A2 is preferable than A1. Given that A2 is the best alternative in  $A^*$ , then A2 *must* be undertaken: it is just the point of maximum for the function  $\text{NPV}(x)$ . If the decision maker does not select A2, this means that she considers the domain  $A^*$  first, solves the maximization problem but then cancels the solution A2 as it were not feasible. So doing, the maximum in  $A^*$  not reached and, indeed, one does not

know which is the solution. One only knows that A1 is *not* the solution. But why should one consider A2 within the domain of the maximand function  $NPV(x)$  if that very alternative is then possibly dismissed as a solution? The reason is that A2 is a fictitious alternative, artificially introduced to allegedly comply with the equivalent-risk tenet. Actually, A2 just serves the scope of rendering the comparison homogeneous, it is a conventional yardstick that is used to reject A1 if the latter turns out to be worse than A2.

(2) Suppose that, after rejecting A1, the decision maker searches for another investment alternative and comes across alternative B1 (different in risk from A1). She evaluates this course of action against an equivalent-risk alternative, say B2. Suppose the NPV of B1 is positive. Then she invests in B1. But this means that the ER principle is infringed. Indeed, the decision maker owns the amount  $X$  and rejects alternative A1 while accepting alternative B1. So B1 is preferred to A1, which means that B1 and A1 are compared, though indirectly. The decision maker cannot prevent herself to accomplish such a comparison, which is illicit from the very point of view of the ER principle. Indeed, when A1 is discarded (via comparison with A2) the decision process is not over. If a decision maker is willing to invest  $X$  euros the decision process is over *only* once she has selected the investment alternative. So, no matter the way one arrives to the solution of the decision problem, if one deliberately and explicitly discard A1 and choose B1, then one prefers investing in B1 to investing in A1. But if B1 is preferred to A1, then one has compared two alternatives which are different in risk. This is not a problem for all (ordinary) human beings, who in daily life accomplish evaluations of alternatives different in risk, but it is a problem for those decision makers who aver that heterogeneous comparisons must not be made: they contradict themselves, and it cannot be differently, since it is logically impossible to compare only homogeneous alternatives. The line of reasoning just analysed resembles that of Miss Roberta, who is willing to purchase a music CD in a music shop and, at the same time, aims at abiding by an equivalent tenet whereby she should compare only homogeneous alternatives:

*Miss Roberta is in a music shop, willing to purchase a CD. She wonders whether she should buy a CD with music by Wagner (A1) lying on a shelf at the entry of the shop. To decide, she searches for homogeneous CDs, i.e. opera CDs. She finds an opera CD with compositions by Verdi (A2). She prefers Verdi to Wagner, so she does not buy the Wagner CD. Then she goes on browsing through shelves and finds a CD of the rock-band Genesis (B1). To decide whether she should buy it, she looks for a comparable Rock CD. She finds a CD of the famous band Pink Floyd (B2). As she prefers Genesis to Pink Floyd, she buys the Genesis CD.*

This seems to be a grotesque way of solving decision processes. But, regardless of this fact, it is evident that Roberta has discarded Wagner and selected Genesis. So, one is bound to admit that the comparison Wagner/Genesis (i.e. A1/B1) has been actually realized; a comparison that is considered non-homogeneous (and therefore illegitimate) from the very point of view of the decision maker (who regards opera and rock as heterogeneous kinds of music).

(3) Suppose that  $X$  is currently invested in business A3, and the decision maker wonders whether she should withdraw  $X$  from A3 and invest it in A1 or, instead, leave money in A3. To decide, the NPV of A1 is calculated, where the discount rate is obviously given by the expected rate of return of A2, which is comparable in risk to A1. Supposing that A1's NPV is negative, the investor rejects A1, and therefore keeps  $X$  invested in A3. This means that while alternative A2 is *formally* compared to A1 (the opportunity cost is A2's expected rate of return), the former is subsequently forgotten, given that the course of action selected is A3, not A2. The decision maker uses then A2 as a mere fictitious yardstick. In addition to this, she accomplishes a non-homogeneous comparison: A1 is rejected, A3 is selected, and therefore a comparison has been accomplished between A1 and A3, which are, in general, different in risk. The equivalent-risk tenet is violated. This awkward line of reasoning is tantamount to the following one of Miss Roberta:

*Miss Roberta is at home wondering whether she should go out and have a swim with her friends (A1) or not (A3). To decide whether she should go, she retrieves in her mind an alternative that she considers equivalent to 'swimming with friends'. She thinks she may equivalently go out and chat with friends in a park (A2). She prefers chatting (A2) to swimming (A1) so . . . she stays home (A3)!*

This is, again, a grotesque way of dealing with decision processes, but this is what happens if Roberta uses an 'equivalent-some' tenet to back her decisions. However grotesque that may be, Roberta has not correctly solved the optimization problem. Indeed, the real decision process is 'Swimming with friends or staying at home', which are heterogeneous, but Roberta constrains herself to avoid heterogeneous alternatives, so she discards 'chatting with friends' even if she explicitly prefers it to the homogeneous alternative 'swimming with friends'. This implies that the maximization problem is not correctly solved. In addition, she infringes her equivalent-some tenet, given that she selects the

alternative ‘staying at home’, which is not equivalent to ‘chatting with friends’. It is evident that ‘chatting with friends’ is a fictitious alternative, generated only for apparently complying with the equivalent-some tenet.

(4) We have seen the case where A1’s NPV is negative. What if A1’s NPV is positive? In this case, A1 is undertaken. However, suppose that, for whatever reason, the investment is financed by selling some assets the investor owns (say, some shares of firm Y). If the decision maker sells shares of Y and invests the sum obtained in A1, then she prefers invests that amount in A1 to investing it in the firm’s shares. Therefore, alternative A1 has been compared to alternative Y. As the firm’s shares are, in general, different in risk from A1, a comparison has been implicitly accomplished between heterogeneous alternatives.

Whatever decision is taken, the ER principle is not complied with and/or the optimization problem is not correctly solved (see Magni, 2002, 2004, for other biases, and Magni, 2005, for a thorough investigation of NPV fallacies in a behavioural perspective).

#### **4. Bounded rationality and the NPV rule**

The NPV rule belongs to those decision-making criteria that directly descend from the tradition of unbounded rationality. The rational economic agent should maximize the NPV using a discount rate that reflects the risk of the asset (and such a rate is found by applying the Capital Asset Pricing Model). As seen in the previous sections, the methodology does not succeed in dealing with uncertainty.

Is it possible to retrieve the DCF/NPV analysis so that the decision maker does not commit the fallacies above studied? The answer is yes and this section shows that the biases disappear if one gives up maximization and regards the NPV methodology as a heuristic. In particular, one may interpret the NPV criterion as a *satisficing* strategy (Simon, 1955) and let the discount rate be a predetermined cutoff level which is considered satisfactory. Denote with  $r$  the (expected) rate of return of the alternative at hand, and let  $k$  be a hurdle rate subjectively prefixed by the evaluator. A simple heuristic is the following: *undertake the project if  $r > k$ , otherwise search for another alternative until the inequality is satisfied*. It is straightforward to show that such a rule may also be expressed saying that an alternative should be chosen if its NPV is positive, where the hurdle rate  $k$



is used as a discount rate. That is, the decision maker will accept an investment with initial outlay  $X$  and (expected) final return  $X'$  if (and only if)

$$-X + \frac{X'}{(1+k)} > 0 \quad (3)$$

with  $X' = X(1+r)$ . In words, the investment will be undertaken if its NPV at the rate  $k$  is positive, where  $k$  represents the minimum rate of return acceptable by the investor. Eq. (3) is formally analogous to (2) but cognitively different. Replacing  $r_2$  with  $k$  means shifting from the idea of comparing equivalent-risk alternatives to the idea of accepting a satisfactory rate of return. Both  $r_2$  and  $k$  are cutoff rates but the former refers to an objectively available equivalent-risk alternative (computed by making use of the 'rational' CAPM), the latter refers to a subjectively determined aspiration level. In other terms, to passing from (2) to (3) means leaving the realm of unbounded rationality (UR) and reaching the land of bounded rationality (BR). Yet, both equations are expression of an NPV criterion, relying as they are on discounting cash flows at a prefixed discount rate. But while the NPV+ER rule is a cognitively biased UR methodology, the satisficing NPV is a BR procedure and is not biased. The decision maker employing the latter does not commit herself to abide with the inapplicable ER tenet. She selects investment if the value of her payoff function, calculated at the hurdle rate  $k$ , is positive or, which is the same, if the rate of return of the project exceeds the aspiration level  $k$ . By adopting (3) the evaluator does not incur biases because she does not confine herself to homogeneous comparisons. No alternative course of action is called up, therefore no comparison is accomplished, and therefore no maximization is involved.<sup>1</sup> The rate  $k$  is not a rate of return of an existing alternative, and therefore it is not an opportunity cost and does not represent a foregone return. It is a subjective threshold that identifies the minimum required rate of return from the project.

It is true that a decision maker can never avoid (explicit or implicit) heterogeneous comparisons, but while this is a problem for the NPV+ER minded decision maker, it is not for the heuristic NPV-minded reasoner. The former explicitly constrains herself to cope with an unsolvable optimization problem, the latter is uninterested in knowing whether

---

<sup>1</sup> Maximization *implies* comparison: a comparison among mutually exclusive states.

there exists or not an alternative (either homogeneous or heterogeneous) better than the one under examination, she is just interested in reaching that aspiration level, for subjective reasons. In other terms, both kinds of decision makers realize heterogeneous comparisons, but only the NPV+ER-minded reasoner is irrational because she herself avers that she is comparing only homogeneous alternatives. The *satisficer* does not claim that she aims at avoiding heterogeneous comparisons, she just aims at achieving a subjectively predetermined hurdle rate. It is interesting to note that as long as the required rate of return (the discount rate) has the meaning of *opportunity cost* biases pervade the decision process; once the concept of opportunity cost is abandoned (as well as the ER tenet) and the NPV is used as a rule of thumb, then such biases disappear.

Note also that we have moved from the concept of opportunity cost to the concept of aspiration level leaving the model otherwise untouched. Opportunity cost is formally connected with maximization (and cognitively connected with counterfactuals)<sup>2</sup> and the decision maker solving (2) (which derives from (##)) aims at finding the best counterfactual alternative to use it as a norm (benchmark); the evaluator resting on (3) avoids eliciting equivalent-risk counterfactual alternatives and focuses on an aspiration level totally disengaged from courses of action. The UR-NPV rule shapes the framework as a process of selection of an action from a set of equivalent-risk alternatives. By contrast, to look at the NPV rule with heuristic eyes means to radically mutate the perspective, thus sweeping away any bias. This is just because the satisficing NPV does not merely disregard any issue of equivalent-risk alternatives but neglects the very idea of comparison, which lies at the ground of the concept of maximization.

It is just the substitution of *maximization* with *satisficing* that heals any flaw: we have then an outstanding example of how bounded rationality helps decision makers to avoid fallacies generated by a rigid application of unbounded rationality.<sup>3</sup>

## 5. Does the *satisficing* NPV rule work?

This section gives some hints about the effectiveness of the hurdle-rate (satisficing) NPV as a decision-making tool. There is some awareness in the literature that real-life

---

<sup>2</sup> Opportunity cost is the (expected) rate of return that the investor *would earn if she invested* in the alternative course of action (see also Buchanan, 1969, for a concept of cost as a 'might be').

<sup>3</sup> It is worth noting that two decision makers facing the same project may reach the same solution even if they employ the NPV with the two different perspectives: if they choose the same discount rate (and agree on expected cash flows) their decision will be the same. Yet, the NPV+ER-minded reasoner contradicts herself (for she accomplishes comparisons that are illicit from her very point of view) whereas the heuristic-minded reasoner is in a logically safe position, given that she is uninterested in any comparison.

decision makers do use the NPV rule in the BR version: according to Jagannathan and Meier (2002) (henceforth J&M)<sup>4</sup> “managers use a ... *hurdle rate*” (p. 3) in making decisions. Relying on Poterba and Summers’ (1995) analysis, they find that “hurdle rates are not ... linked to the cost of capital” (p. 22). These statements, in our cognitive terms, boil down to saying that managers do not apply the NPV+ER methodology but the NPV rule as a *heuristic* (a satisficing strategy). Summers (1987) surveyed corporations on investment decision criteria finding that most firms use the NPV rule employing a single rate independently of risk, which is suggestive of the use of hurdle rates (and then of the use of the satisficing NPV rule); McDonald (2000) writes that “we know that hurdle rates and payback rules are used in practice” (p. 30) and that

anecdotal evidence suggests that firms making capital budgeting decisions routinely do a number of things that basic finance textbooks say they should not do:

- Projects are taken based on whether internal rates of return exceed arbitrarily high discount rates (often called “hurdle rates”). (p. 13).

Graham and Harvey (2002) surveyed 392 companies and point out that “small firms ... were more inclined to use a cost of equity determined by “what investors tell us they require”” (p. 12) which seems again suggestive of the use of hurdle rates rather than opportunity costs (and hence of the use of the satisficing NPV rather than the NPV+ER). If future researches will confirm this finding, it means that real-life decision makers are not trapped in biases and their decisions are sound from a cognitive point of view.

The just mentioned authors are not concerned with cognitive issues. They are only concerned with the reliability of the NPV rule as a rule of thumb in decision-making. McDonald (2000) examines “whether the use of seemingly arbitrary investment criteria, such as hurdle rates and profitability indexes, can proxy for the use of more sophisticated real options valuation” (p. 13). Though not explicitly, what McDonald and J&M do in their papers is to analyse the use of a satisficing technique, where the hurdle rate is the aspiration level.<sup>5</sup> In other terms, they study how successful the NPV heuristic is in financial decision-making, wondering whether such “simple rules are relatively robust to changes in project characteristics” and whether “a single hurdle-rate rule [can] yield approximately correct decisions for these projects” (McDonald, 2000, p. 15). McDonald

---

<sup>4</sup> All following quotations from J&M (2002) refer to the internet version.

<sup>5</sup> “A hurdle-rate rule calls for investing when the project’s ... rate of return exceeds the hurdle rate”, (McDonald, 2000, p. 22).

finds that “for a variety of parameters, particular hurdle-rate ... rules can provide close-to-optimal investment decisions. Thus, it may be that firms using seemingly arbitrary “rules of thumb” are approximating optimal decisions” (p. 13). J&M (2002) reach the same conclusion claiming that the use of hurdle rates should not be deemed less reliable than the use of the opportunity cost of capital as a discount rate: “managers ... [take] the right decisions ... because they use a hurdle rate that is higher than the cost of capital to capture the option to wait” (p. 12) and this strategy “is likely to lead near optimal decisions” (p. 21). J&M do not claim that the hurdle rate used by practitioners is always higher than the opportunity cost of capital. They affirm that “companies use low hurdle rates for *strategic projects*” and that, in general, “the cost of capital is not critical in the financial decision-making process” (p. 3). This means that hurdle rates are seen by managers as aspiration levels “that may be gradually adjusted if they become too lax or binding a constraint” (Goodie et al., 1999, p. 351). Although the authors do not recommend the use of aspiration levels and rules of thumb as optimal decisions making criteria<sup>6</sup> it is worthwhile noting that their conclusions implicitly (and unawarely) foster the idea that advantages can be taken by undertaking a simple heuristic:

- (a) “the advantage of using a hurdle rate is that modelling all possible future options is not necessary” (J&M, 2002, p. 4) and decision makers may find it useful to use “a rule that best justifies making intuitively plausible investment decisions” (McDonald, 2000, p. 26)
- (b) “managers *adjust these rules* ... when an investment is strategic and expiring.” (McDonald, 2000, p. 30, italics added)
- (c) “[the] use [of a hurdle-rate rule] in practice might stem from the *success* of apparently arbitrary rules that are *revealed over time to be close to optimal*. Managers likely observe the capital budgeting practices, in their own and other companies, and in many cases probably mimic *what seems to work*” (McDonald, 2000, p. 30, italics added).

It is worth noting that (a), (b) and (c) fit the three premises on which Gigerenzer (2001) bases his notion of adaptive toolbox:

---

<sup>6</sup> “We do not suggest that managers *should* use these rules of thumb” (McDonald, 2000, p. 30).

- (A) psychological plausibility: models of decision-making should have adequate regard “for the constraints in time, knowledge, and computational capacities that humans face” (Gigerenzer, 2001, p. 38)
- (B) domain specificity: “each heuristic is specialized for certain classes of problems, which means that most of them are not applicable in a given situation” (Gigerenzer & Todd, 1999, p. 32) and “what works to make quick and accurate inferences in one domain may well not work in another. Thus, different environments can have different ... heuristics” (Gigerenzer, Todd & the ABC Research Group, 1999, p. 18)
- (C) ecological rationality: this consists in “the match between the structure of a heuristic and the structure of an environment” (Gigerenzer & Selten, 2001a, p. 9). Heuristics are ecologically rational in that “they are adapted to particular environments ... [and] can be fast, frugal and accurate by exploiting the structure of information in natural environments” (*ibidem*).

Sentences in (a) above are evidently redolent of the psychological plausibility (A) of the NPV heuristic (decision makers are not able to take into account all possible future opportunities and outcomes).

Sentence (b) reveals that these rules are domain-specific (B) in the sense that when the context changes (strategic investments versus nonstrategic investments, expiring versus nonexpiring projects) the rule is changed or is combined with other rules of thumb<sup>7</sup> (and that the aspiration levels are adjusted too, as suggested by J&M. See above). For example, while the NPV heuristic seems to work well for a variety of industrial projects, the recognition heuristic (Borges et al., 1999; Goldstein & Gigerenzer, 1999, 2002; Borges et al., forthcoming) is inapplicable in such frameworks.<sup>8</sup> A combination of rules may also be adopted in specific domains to form a different heuristic: “firms in practice might find it useful to use multiple rules at once” (McDonald, 2000, p. 26).

Sentence (c) reveals that the use of the NPV heuristic is ecologically rational (C) due to success over time. McDonald’s words also implicitly suggest that the rules of thumb analysed may be the result of imitation and social learning. In other terms, his words seem

---

<sup>7</sup> McDonald is actually concerned with three rules of thumb: hurdle-rate (our NPV heuristic), profitability indexes and payback period.

<sup>8</sup> Possibly, the reverse is true for investment in capital markets.

to encourage the view that the satisficing NPV rule is a “do-what-others-do” (or “do-what-successful-people-do”) strategy (see Laland, 2001).

## **6. The two rationalities and the two systems of reasoning**

The heuristics-and-biases program draws attention on logical fallacies of human reasoning which cloud human minds (Tversky & Kahneman, 1974; Kahneman, Slovic & Tversky, 1982; Kahneman & Tversky, 1996) and claims superiority of unboundedly rational models for decision-making; conversely, advocates of bounded rationality are concerned with showing that heuristics may often prove useful in decision-making (Gigerenzer, Todd & the ABC Research Group, 1999). Also, they aver that cognitive illusions disappear once heuristics are seen as adaptive tools rather than logical devices for solving decision-making processes (Gigerenzer, 1996; Gigerenzer, 2000). The literature on bounded rationality has so far focussed on the two blades of Simon’s scissors: cognitive limitations of human beings and environmental bindings. The relation between the two scissors is exemplified by the notion of ecological rationality. Gigerenzer and the ABC group endorse the idea that human beings’ rationality is ecological rather than logical and their fast-and-frugal-heuristics program aims at presenting a number of heuristics that are successfully applied in real-life in specific environments.

The main concern of this paper is to take another route to enter the debate between bounded and unbounded rationality. The very heuristic-and-biases principles are here applied to show that biases do arise in an optimization model. This work therefore inductively suggests the idea that there may be models, well-rooted in the maximizing tradition of unbounded rationality, which are biased. My stance is then compatible with that of Gigerenzer and the other advocates of bounded rationality, but it is different as for the arguments used: Kahneman and Tversky’s program suggests that rationality should be evaluated according to adherence to principles of internal consistency, and this paper shows that the NPV maximization rule breaks down just because of its inconsistency. The fallacies encountered do not merely imply a distortion of the decision process and a solution based on a bizarre (to say the least) representation of it: they also reveal that the decision maker rests on heterogeneous comparisons which the ER tenet is just supposed to rule out. “The classical theory does not tolerate the incomparability of oranges and apples ... it requires a complete ordering of pay-offs” (Simon, 1955, p. 108). And so, when

uncertainty is introduced, the maximization of the NPV (being the latter a maximizing model), “breaks down for lack of a complete ordering of the pay-offs” (Simon, 1955, p. 109). This is the reason why financial economists have devised the ER tenet as an indispensable companion of the NPV rule: they aimed at rendering NPV maximization a reliable unboundedly rational model. By analysing the procedure from a cognitive point of view one is able to discover that the complete ordering induced by such a methodology is illusory. Resting on the concept of opportunity cost (intended as the best alternative equivalent-risk rate of return) and on the NPV maximization (that is, on a comparison between equivalent-risk alternatives) amounts to forgetting that human beings cannot avoid implicit or explicit comparisons with different-risk alternatives. As a result, the decision maker pretends to compare alternative course of actions which are equivalent in risk, whereas she is bound to compare alternative courses of action which are not comparable in terms of risk.

Paradoxically, we have seen that not only a maximizing model leads to biases, but if we ever want to salvage that model, then a possible way to do it is to follow a boundedly rational perspective. As seen, if the NPV rule is employed as a satisficing strategy, the biases disappear. Or, in other terms, the use of the NPV rule as a simple ecological heuristic, where no maximization is involved and no equal-risk alternatives are recruited, heals the inconsistencies of the criterion and legitimates the model from the very point of view of an unbounded-rationality upholder. In conditional form, the satisficing-NPV decision maker applies the following rule:

*if the alternative at hand has a rate of return greater than the (satisfying) prefixed rate of return, then the alternative will be undertaken*

which implies no intrinsic fallacies.

A bounded-rationality approach seems then to be able to debias a biased unboundedly rational model. One may be surprised in learning that a heuristic leads to results that are close to optimal, as the recent literature seems to corroborate.<sup>9</sup>

If efforts will be spent by scholars in analysing maximizing models from a cognitive point of view, we will perhaps find that biases arise in those very models, and,

---

<sup>9</sup> Some authors, concerned with biases in subjective real option evaluation, are surprised in noticing that although decision makers do not follow normative models, “managers’ investment decisions may be ‘directionally correct’” and consider this fact “an intriguing paradox” (Miller & Shapira, 2004, p. 281).

possibly, that a rule-of-thumb perspective will either heal such biases and be successful for decision-making.

But the results we have reached are by no means a dismissal of unbounded rationality. The entire paper may be fruitfully interpreted by taking a nonnarrow “two-process theories” perspective. The classical idea that cognitive process may be divided into two main classes is of some concern in cognitive psychology and, in particular, in the heuristics-and-biases program (see Gilovich, Griffin & Kahneman, 2002). According to this view, mental processes make use of a heuristic-based system of reasoning and of a rule-based system of reasoning, the former being associative and intuitive, the latter being analytic and reflective. The former employs automatic, effortless, rapid strategies for decision-making, the latter rests on controlled, effortful, deductive processing (Evans, 2004; Sloman, 1996a, 1996b; Sloman, 2002; Kahneman & Frederick, 2002). Each of the two systems has its specific domain, but it is plausible that the two systems are interactive and “function as two experts who are working cooperatively to compute sensible answers.” (Sloman, 2002, p. 383). As a matter of fact, the unbiased NPV heuristic is the result of two independent but concurring sources: Fisher’s rigorous and analytic model and Simon’s satisficing strategy. The former lies in the rigorous realm of unbounded rationality and recruits the rule-based system for providing an answer to the decision problem; the latter represents a simple heuristic, consistent with the bounded rationality of the ecological decision maker, generated by the associative system. Taking Fisher’s NPV procedure and attributing it a heuristic perspective means that the two systems work cooperatively. But I would like to stress two important differences in which the results here shown are consistent with two-process theories. Firstly, the cooperation is of a different kind from the one suggested in the literature: according to Sloman (2002) “associative paths ... can be a source of creativity, whereas more careful and deliberative analyses can provide a logical filter guiding through to productive ends” (p. 395). The combination of logic and creativity in the NPV is instead reversed: creativity is in Fisher’s NPV and is the logical result of applying rule-based arguments to decision-making, while the associative heuristic-based reasoning constitutes the filter to debias the model. In other words, the model, in its elegance, simplicity and creative strength derives from Fisher’s using the rule-based, normative reasoning, but its legitimation as an analytically faultless methodology derives from the application of a way of reasoning pertaining to the associative system. Secondly, this work’s results endorse the idea that decision models



may be produced by means of a *mixed* strategy involving the use of both systems:<sup>10</sup> “Mathematics, law, and (probably) all disciplines demand this combination of creativity and rigorous rule application” (Sloman, 2002, p. 395). As for the NPV, such a combination occurs in a nondichotomous and nontrivial way: nondichotomous since both types of reasoning converge and mix up to produce a reliable decision model; nontrivial in the sense that it is the rule-based system that came up with the NPV model, but it is the heuristic-based system that plays a role in validating it.

If, in addition, future researches will definitely show that managers and individuals actually employ the heuristic-based NPV, this means that they have grasped the intrinsic usefulness of the rule-based NPV model and they themselves have transplanted it to the ecological environment where they live. That is, they have spontaneously restored a biased methodology by taking account of their own bounded/ecological rationality and, at the same time, by realizing the powerfulness of the rule-based model. The heuristic-based NPV methodology is then based on the fruitful cooperation of both systems. Hence, both heuristics’ lovers and rule-based-reasoning’s supporters will have a common road to go along. “There is no deep-rooted conflict between an evolutionary perspective on human cognition and the heuristics and biases approach ... Both acknowledge that many cognitive problems essential to survival are typically solved with efficiency and precision. And both can accept the existence of pockets of (particularly informative) bias and error in human judgment” (Gilovich & Griffin, 2002, p. 10). Yes, but such biases do not necessarily lie on the heuristic side of the street.

## **Conclusions**

This work draws attention on a model which is a keystone in economics for decision-making purposes. This model presupposes maximization of an index in order to achieve maximization of the investor’s wealth. Such an index is called Net Present Value and measures the return (the profit, the increase in wealth) generated by a particular course of action. Under uncertainty, it is associated with another keystone in the literature, the Capital Asset Pricing Model, which provides the ‘correct’ discount rate a rational decision maker should use for solving the decision process. The main results of this work are:

---

<sup>10</sup> See Magni, Mastroleo and Facchinetti (2002); Magni et al. (2004); Magni, Malagoli and Mastroleo (forthcoming); Malagoli, Magni and Mastroleo (in press) for a decision-making approach where both systems of reasoning are implicitly applied.

- (1) The NPV maximizing procedure for risky investment is biased. The ER is always violated and the optimization problem is not correctly solved
- (2) A bounded rationality perspective is able to heal inconsistencies and originate a reliable heuristic-based NPV.

In such a way, the paper enters the debate as to whether unbounded or bounded rationality should be judged more apt to help decision makers. The latter rests primarily on the idea of heuristics as fruitful tools to be used in specific environments, the former focuses on the equation heuristics=biases, in the sense that heuristics are often considered internally inconsistent and for this reason inappropriate for decision-making. While this paper is perfectly consistent with a bounded/ecological rationality approach (and, indeed, it recommends bounded and ecological rationality as a fruitful and indispensable vision of rationality) it does not concentrate on its successfulness in some environments (even though some evidence in this sense may be gleaned from the papers cited in section 4). Above all, it aims at showing that biases are not specific of bounded rationality but may be found in optimization models too, and that a heuristic-based interpretation may heal inconsistencies of an optimization model. In this sense, the approach is quite new and gives advocates of bounded rationality the opportunity of counterobjecting upholders of unbounded rationality by using their same arguments: optimization models may lead to biases. As for bounded rationality, it is true that “the heuristics themselves are sensible estimation procedures that are by no measure “irrational”.” (Gilovich and Griffin, 2002, p. 3). But there is something more: a heuristic-based approach can debias a model.

True as it is, this paper concerns only one model and the two points above should not be taken as the idea that all (or even a large number of) models in unbounded rationality are intrinsically biased nor that all biased models can be retrieved by bounded rationality. However, this paper may suggest a direction for research and act as a stimulus for further inquiries. The scientific niche that might be disclosed could offer unexpected views about bounded and unbounded rationality and their interrelations. A possible payoff of such a view is not solely that internal consistency is a double-edged weapon (and therefore one should not entirely base decision-making on it) but that bounded rationality and unbounded rationality are not necessarily rivals. This view is then consistent with a nondichotomous dual-process theory of reasoning, according to which individuals are furnished with two cognitive systems: a heuristic-based (intuitive) system and rule-based

(analytic) systems that cooperate in producing answers to decision problems. Bounded and unbounded rationality may be viewed as two sides of the same medal. This seems to be the new direction of the heuristics-and-biases program. But the latter seems also interested to stress that System 1 (rule-based) and System 2 (heuristic-based) do cooperate even though “the accuracy of (System 2) decision rules rests on the validity of a System 1 computation” (Gilovich & Griffin, 2002, p. 17). That System 1 is always superior to System 2 is here disconfirmed, since the rule-based model is biased and the heuristic-based NPV is capable of retrieving it: the validity of System 1’s decision rules depends here on System 2. Also, to rigidly split our reasoning processes in two Systems may prove a useful metaphor for scientific research but often results in a simplistic explanation. One never knows where either System begins and where it ends, and the distinction between the two is rather fuzzy: it is not simple to ascertain what an association is and what a rule is, and to find clear-cut differences between them (Gigerenzer & Regier, 1996). Besides this, it is not even exact to say that bounded rationality pertains to the associative system whereas unbounded rationality relates to the rule-based system. The Fisherian NPV is the result of deduction but creativity (and thus System 2) has played a role in suggesting Fisher which assumptions to take at the outset and which deductive chains to perform. Likewise, Simon’s satisficing strategy is a heuristic but the way he presents it and shows its reliability is entirely logical and based on the reflective System 1.

Unbounded rationality itself, as derived from logic and mathematics, may not abstain to consider itself as a derivation of ecological rationality: logic and mathematics should be intended as the most advanced step of human *simulation*, and the impressive degree we have achieved in such an ability may be seen as the result of an evolutionary process, in which its surviving value has been adaptively tested. Simulation is a tool for anticipating and as such is indispensable for discovering and creating. Therefore logic and mathematics, as symbolic tools, assemble the experience of our ancestors (Monod, 1970, pp. 171-172), or, to put it in a nutshell, even logic is ecological.

If a narrow interpretation of the two-process theory means that the two system always give contradictory answers, then this paper aims at showing the opposite; if a less rigid interpretation of it is followed, then a dual-process theory may actually be of some help to understand (decision-making and) decision-making models. Evidently, such a theory cannot substitute a meticulous analysis of cognitive processes where focus is not placed on the opposition of the two systems but rather on the interaction between the two

and on the mechanisms that combine them, as well as on the theoretical question concerning how much of each reposes in the other. Certainly, “the two-systems view ... helps to clarify the differences and similarities between the ... “heuristics and biases” program and ... the “fast and frugal heuristic” program” (Gilovich & Griffin, 2002, p. 16) and its “potential usefulness of analyses of this sort” (Gigerenzer & Regier, 1996) is undeniable. But

collecting more and more binary oppositions—and labeling these “two-process theories”—does not necessarily enhance clarity. Dichotomies can be an important step, but they cannot substitute for theories of cognitive processes. (Gigerenzer, 2000, p. 293).

More “fluid theories” and more mixed strategies will perhaps help us understand decision makers and help decision makers cope with complex decision problems (with the pleasant by-product of conciliating the two rival parties).

## References

- Abel, A. B. (1990). Consumption and Investment. In B. M. Friedman & F. H. Hahn (Eds.), *Handbook of Monetary Economics*. Amsterdam: North-Holland.
- Bierman, H. Jr. & Smidt, S. (1992). *The Capital Budgeting Decision*. MacMillan Company, eighth edition.
- Borges, B., Goldstein, D. G., Ortmann, A. & Gigerenzer, G. (1999). Can Ignorance Beat the Stock Market?. In *Simple Heuristics That Make Us Smart*. See Gigerenzer, Todd & the ABC Research Group (1999).
- Borges, B., Goldstein, D. G., Ortmann, A. & Gigerenzer, G. (forthcoming). The Recognition Heuristic: A Fast and Frugal Way to Investment Choice? In Plott, C. R. & Smith, V. L. (Eds.), *Handbook of experimental economics results*. Amsterdam, Netherlands: Elsevier Press/North Holland.
- Brealey, R. & Myers, S. C. (2000). *Principles of Corporate Finance*, McGraw-Hill.
- Buchanan, J. (1969). *Cost and Choice. An Inquiry in Economic Theory*. Chicago: Markham. Republished as Midway reprint, Chicago: University of Chicago Press, 1977.
- Copeland, T. E., Koller, T., & Murrin, J. (2000). *Valuation: Managing the Value of Companies*, New York: John Wiley & Sons, third edition.
- Damodaran, A. (1999). *Applied Corporate Finance: A User's Manual*. New York: Wiley.

- Dixit, A. & Pindyck, R. (1994). *Investment under Uncertainty*, Princeton NJ: Princeton University Press.
- Evans, J. St. B. T. & Over, D. E. (1996). *Rationality and Reasoning*. Hove, England: Psychology Press.
- Evans, J. St. B. T. (2004). History of the dual process theory of reasoning. In K. Manktelow & M. C. Chung, (Eds.), *Psychology of reasoning*. New York, NY: Psychology Press.
- Fernández, P. (2002), *Valuation Methods and Shareholders Value Creation*, San Diego: Academic Press.
- Fisher, I. (1930). *The Theory of Interest*. New York: The Macmillan Company; (reprinted) Clifton, NJ: Augustum M. Kelley Publishers, 1974.
- Gigerenzer, G. (1996) On narrow norms and vague heuristics: A reply to Kahneman and Tversky. *Psychological Review*, 103, 650–669.
- Gigerenzer, G. (2000), *Adaptive Thinking. Rationality in the Real World*. Oxford University Press: New York, USA.
- Gigerenzer, G. (2001). The Adaptive Toolbox. In *Bounded Rationality*. See Gigerenzer & Selten (2001b).
- Gigerenzer, G. & Regier, T. (1996), How do we tell an association from a rule?. *Psychological Bulletin*, 119 (1), 23–26.
- Gigerenzer, G. & Selten, R. (2001a). Rethinking Rationality. In *Bounded Rationality*. See Gigerenzer & Selten (2001b).
- Gigerenzer, G. & Selten, R. (Eds.) (2001b), *Bounded Rationality. The Adaptive Toolbox*. The MIT Press: Cambridge, Massachusetts, USA
- Gigerenzer, G. & Todd, P. M. (1999). Fast and Frugal Heuristics: The Adaptive Toolbox. In *Simple Heuristics That Make Us Smart*. See Gigerenzer, Todd, the ABC Research Group (1999).
- Gigerenzer, G., Todd, P. M. & the ABC Research Group (1999). *Simple Heuristics That Make Us Smart*. New York: Oxford University Press
- Gilovich, T. & Griffin, D. (2002). Introduction – Heuristics and Biases: Then and Now. In *Heuristics and Biases*. See Gilovic, Griffin & Kahneman (2002).
- Gilovich, T., Griffin, D. & Kahneman, D. (Eds.) (2002). *Heuristic and Biases. The Psychology of Intuitive Judgment*. Cambridge, UK: Cambridge University Press.

- Goldstein, D. G. & Gigerenzer, G. (1999). The Recognition Heuristic: How Ignorance Make Us Smart. In *Fast and Frugal Heuristics That Make Us Smart*. See Gigerenzer, Todd & the ABC Research Group (1999).
- Goldstein, D. G. & Gigerenzer, G. (2002). Models of Ecological Rationality: The Recognition Heuristic. *Psychological Review*, 109 (1), 75–90.
- Goodie A. S., Ortmann, A., Davis, J. N., Bullock, S. & Werner, G. M. (1999). Demons Versus Heuristics in Artificial Intelligence, Behavioral Ecology, and Economics. In *Simple Heuristics That Make Us Smart*. See Gigerenzer, Todd & the ABC Research Group (1999).
- Graham, J. & Harvey, C. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics*, 60, 187–243.
- Graham, J. & Harvey, C. (2002). How do CFOs make capital budgeting and capital structure decisions?. *Journal of Applied Corporate Finance*, 15 (1), 8–22.
- Jagannathan, R. & Meier, I. (2002). Do we need CAPM for Capital Budgeting?, *Financial Management*, 31 (4), 55–77. Internet version available on <http://www.nber.org/papers/w8719>.
- Jorgensen, D. (1963). Capital Theory and Investment Behavior. *American Economic Review*. 53, 247–259, May.
- Kahneman, D. & Frederick, S. (2002). Representativeness Revisited: Attribute Substitution in Intuitive Judgment. In *Heuristics and Biases*. See Gilovic, Griffin & Kahneman (2002).
- Kahneman, D. & Miller, D. T. (1986), Norm Theory: Comparing Reality to Its Alternatives, *Psychological Review*, 93 (2), 136–153.
- Kahneman. D., Slovic, P. & Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases*. Cambridge, UK: Cambridge University Press.
- Kahneman. D. & Tversky, A. (1996). On the reality of cognitive illusions: A reply to Gigerenzer’s critique. *Psychological Review*, 103, 582–591.
- Laland, K. N. (2001). Imitation, Social Learning, and Preparedness as Mechanisms of Bounded Rationality. In *Bounded Rationality*. See Gigerenzer & Selten (2001b).
- Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economic and Statistics*, 47, 13–37.

- Magni, C. A. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operational Research*, 137 (1), 206–217.
- Magni, C. A. (2004). Antinomies e illusioni cognitive nel criterio del valore attuale netto, *Analisi Finanziaria*, 54, 2° trim. 26–42.
- Magni, C. A. (2005). Reasoning the ‘net-present-value’ way: Biases and how to use psychology for falsifying decision models. *Materiali di discussione*, Dipartimento di Economia Politica, Università di Modena e Reggio Emilia.
- Magni, C. A., Malagoli, S. & Mastroleo, G. (forthcoming). An alternative approach to firms’ evaluation: expert systems and fuzzy logic. *International Journal of Information Technology and Decision Making*.
- Magni, C. A., Mastroleo, G., & Facchinetti, G. (2002). A Fuzzy Expert System for Solving Real Option Decision Processes. *Fuzzy Economic Review*, VI (2), 51–73.
- Magni, C. A., Mastroleo, G., Vignola, M. & Facchinetti, G. (2004), Strategic options and expert systems: a fruitful marriage. *Soft Computing*, 8 (3), 179–192, January.
- Malagoli, S., Magni, C. A. & Mastroleo, G. (in press). The use of fuzzy logic and expert systems for rating and pricing firms: a new perspective on valuation. *Managerial Finance*.
- McDonald, R. L. (2000). Real Options and Rules of Thumb in Capital Budgeting, In M. J. Brennan & L. Trigeorgis (Eds.), *Project flexibility, agency and competition: new developments in the theory and application of real options*. New York: Oxford University Press.
- Miller, K.D. & Shapira Z. (2004). An empirical test of heuristics and biases affecting real option valuation. *Strategic Management Journal*, 25, 269–284.
- Monod, J. (1970). *Le hasard et la nécessité. Essay sur la philosophie naturelle de la biologie moderne*. Paris: Éditions du Seuil.
- Mossin, J. (1968). Equilibrium in a Capital Asset Market, *Econometrica*, 34 (4) 768–783.
- Poterba, J. M. & Summers, L. H. (1995), A CEO survey of US companies’ time horizons and hurdles rates, *Sloan Management Review*, 37 (1), 43–53.
- Rao, R. (1992). *Financial Management*, MacMillan Publishing Company.
- Sharpe, W. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance*, 19, 425–442.
- Simon, H. (1955) A behavioral model of rational choice. *Quarterly Journal of Economics*, 69, 99–118.

- Simon, H. (1956) Rational choice and the structure of the environment. *Psychological Review*, 63, 129–138.
- Sloman, S. A. (1996a). The empirical case for two systems of reasoning. *Psychological Bulletin*, 119 (1), 3–22.
- Sloman, S. A. (1996b). The probative value of simultaneous contradictory belief: Reply to Gigerenzer & Regier (1996). *Psychological Bulletin*, 119 (1), 27–30.
- Sloman, S. A. (2002). Two Systems of Reasoning. In *Heuristics and Biases*. See Gilovich, Griffin & Kahneman (2002).
- Summers, L. H. (1987), Investment Incentives and the Discounting of Depreciation Allowances. In M. Feldstein (Ed.), *The Effects of Taxation on Capital Accumulation*. Chicago: University of Chicago Press.
- Tobin, J. (1969). A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, 1, 15–29, February.
- Tversky, A. & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, 185, 1124–1131.