Norms of rationality and investment decisions: CAPM, arbitrage and description invariance

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Abstract. The classical Capital Asset Pricing Model (CAPM) represents a well-rooted paradigm of rationality for investment decision-making. Following some anticipations in Magni [European Journal of Operational Research 137 (2002) 206] and Magni [Applied Financial Economics Letters, forthcoming] this paper shows that decision makers abiding by the CAPM prescriptions violate two other standards of rationality: The arbitrage principle and the principle of description invariance. In particular, referring to investments, (i) the notion of value in the CAPM is incompatible with the one employed in arbitrage theory; (ii) CAPM-minded decision makers may fail to exploit arbitrage opportunities; (iii) the principle of additivity is not fulfilled in CAPM-based asset valuation; (iii) the notion of value derived from CAPM is senseless; (iv) CAPM-minded agents fall prey to framing effects. In other terms, this paper fosters the idea that economic agents complying with this consolidated paradigm of valuation and decision-making exhibit biases in investment valuation and choice.

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1 Introduction

Although the Capital Asset Pricing Model (CAPM) has been developed forty years ago (Sharpe, 1964; Lintner, 1965, Mossin, 1966) it is still a bedrock in financial economics and represents the reference model for asset valuation and, in general, for capital budgeting decisions (see Rubinstein, 1973; Copeland and Weston, 1988; Damodaran, 1999; Ross, Westerfield and Jaffe, 1999; Brealey and Myers, 2000; Fernández, 2002). It provides the allegedly correct rate for discounting the cash flows of a project and assessing its value. Alongside its companion NPV (=Net Present Value) it is considered a normative benchmark for decision making, and any empirical deviation of agents' behaviors from its prescriptions is regarded as irrational. This perspective directly stems from the heuristics-and-biases tradition, according to which behaviors are matched against accepted norms of rationality; if behaviors do not conform to rational paradigms they are said to be biased (Kahneman, Slovic and Tversky, 1982; Gilovich, Griffin and Kahneman, 2002; Pohl, 2004). Many contributions have proliferated in the relevant literature dealing with empirical violations of the NPV+CAPM methodology. Some work do find that economic agents use the CAPM for investment decisions. For example, Graham and Harvey (2002), on the basis of responses from 392 companies representing a wide variety of firms and industries, affirm that "most companies follow academic theory and use discounted cash flow (DCF) and net present value (NPV) techniques to evaluate new projects" (p. 9) and their results "indicated that the Capital Asset Pricing Model (CAPM) was by far the most popular method of estimating the cost of equity capital" (p. 12). European CFOs as well as US "determine their cost of capital using the capital asset pricing model" (Brounen, de Jong and Koedijk, 2004, p. 72). Yet, there is some awareness in the literature that the NPV+CAPM methodology is not always used in a correct way and is often replaced by some rules of thumb (McDonald, 2000; Graham and Harvey, 2001, 2002; Jagannathan and Meier, 2002). For example, Brigham (1975) surveyed 33 large, relatively sophisticated firms. Although 94% of them used the DCF methodology to value investments, only 61% of the firms using DCF adopted the cost of capital from the CAPM as the discount rate. Summers (1987) surveyed corporations on investment decision criteria finding that 94% of reporting firms use the NPV rule employing a discount rate independent of risk; McDonald

(2000) writes that "firms making capital budgeting decisions routinely do a number of things that basic finance textbooks say they should not do [among which:] Projects are taken based on whether internal rates of return exceed arbitrarily high discount rates (often called "hurdle rates")" (p. 13). "Finance scholars have always been puzzled by the durability of a host of investment rules that seem to survive and even thrive despite their obvious shortcomings ... including ... the hurdle rate rule" (Ross, 1995, p. 99); in actual facts, "we know that hurdle rates ... are used in practice" (McDonald, 2000, p. 30) and "it appears common for firms to use investment criteria that do not strictly implement the NPV criterion" (*ibidem*, p. 13), so that their "actions do not reflect the application of current financial theory" (Gitman and Mercurio, 1982, p. 29). Graham and Harvey (2002) affirm that "small firms are significantly less likely to use the NPV criterion or the capital asset pricing model and its variants" (p. 22). They find that sometimes the use of hurdle rates is explicitly acknowledged: "Small firms were inclined to use a cost of equity determined by "what investors tell us to require" [and a] majority (in fact, nearly 60%) of the companies said that they would use a single-company wide discount rate to evaluate a new investment project, even though different projects are likely to have different risk characteristics" (*ibidem*, p. 12).

In this paper I do make use of this very heuristics-and-biases approach but, in contrast, I change perspective and investigate those decision makers that *rigorously adhere to the CAPM prescriptions*. To this end, I recruit two accepted standards of rationality as guiding principles and search for possible deviations of CAPM-minded agents from these standards' requirements.

The first standard I will rely on is the principle of *arbitrage*, which not only is a cornerstone in the finance literature (Modigliani and Miller, 1958; Black and Scholes, 1973; Varian, 1987), but is also central to classical welfare economics, being equivalent to the notion of "Pareto optimality" (Nau, 2004), as well as to competitive equilibrium (Kreps, 1981; Werner, 1987) and to noncooperative game theory (Nau and McCardle, 1990). The no-arbitrage condition is such that a market is rational if and only if there are no arbitrage opportunities in it. Recently, it has been shown that this principle is the fundamental principle of economic rationality, unifying theories of subjective probability, expected utility, and subjective expected utility, as well as competitive equilibrium (Nau

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and McCardle, 1991; Nau, 1999). The implication for capital budgeting decisions is that rational decision makers should undertake a project if and only if there are arbitrage opportunities on it.

The second standard I resort to is the principle of *description invariance*, which characterizes rationality not only in economic domains but in any decision-making setting. Description invariance means that valuation or decision should not change if the problem at hand is differently framed, as long as the descriptions are logically equivalent. Violations of such a principle are called "framing effects" and have received considerable attention in the cognitive and behavioral literature since the birth of the heurisitcs-and-biases tradition (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981; Kahneman and Tversky, 1984). In general, framing effects deal with choice reversals depending on whether the problem is positively framed (in terms of gains) or negatively framed (in terms of losses) and concern several domains of life (see Wang, 1996; Kühberger, 1998; Levin, Schneider and Gaeth, 1998; Kuvaas and Selart, 2004; Gonzalez et al., 2005. See Qualls and Puto, 1989; Roszkowski and Snelbecker, 1990, for framing effects in industrial and financial contexts. See Gold and List, 2004, for a logical treatment). Because framing effects "violate the basic normative principle of 'description invariance', they are widely considered to provide clear-cut evidence of irrationality" (McKenzie, 2005, p. 331). As regards investors, description invariance means that a cash flow should be valued by decision makers univocally, irrespective of the way it is formulated, either as an aggregate or disaggregate quantity. In poor terms, 100 euros are always 100 euros even if one sees it as a portfolio of 60 euros and 40 euros.

Thus, while scholars are usually concerned with the question:

> How well do ordinary agents size up against a well-defined normative benchmark of rationality such as CAPM?

I will instead address the question

> How well do CAPM-minded agents size up against two well-defined normative benchmarks of rationality such as arbitrage and description invariance?

The answer supplied in the paper is that the CAPM paradigm fulfills none of the two.

As a result, the CAPM may not be considered a reliable paradigm for evaluation and decision-making purposes.

Throughout the paper, the analysis is confined to one period and it is supposed that a security market exists where three securities are traded, numbered 1, 2, 3, the latter being a risk-free asset. The market is assumed to be in equilibrium so that all marketed assets lie on the Security Market Line (SML). It is assumed that an economic agent complies with the NPV+CAPM paradigm when dealing with project valuation and selection. In particular, we will deal with projects A, B, C, D, E, not traded in the security market. It is also assumed that one of three states of nature may occur (labelled s_1, s_2, s_3) and that cash flows vary across these states according to prefixed probabilities. The term 'asset' will be employed as a generic term including both projects and securities, and by cost/price of an asset it is meant the outlay required for undertaking/purchasing it.[1] All numbers are rounded off to the second (or sometimes third) decimal. Table 1 collects the notations employed throughout the paper.

The paper is structured as follows. In section 2 the notions of cost, value and cost of capital are introduced, on the basis of which all the results of the paper are grounded. Section 3 focuses on the principle of arbitrage and shows that CAPM-minded decision makers' judgments and decisions deviate from those of arbitrage-seeking decision makers (henceforth arbitrageurs). Section 4 focuses on the standard of description invariance; it shows that additivity is not preserved in a CAPM-based valuation, which implies that the value of a project is whatever one wants it to be or, in behavioral terms, that CAPM-minded decision makers fall prey to framing effects. Some remarks conclude the paper.

2 Cost, value, and cost of capital

In capital budgeting, value depends on cost of capital. In the CAPM, cost of capital depends on the beta of the asset:

$$i_l = r_f + \beta_l(r_m - r_f). \tag{1}$$

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[1]In particular, 'cost' is used for projects and 'price' is used for securities.

In turn, beta depends on the rate of return:

$$\beta_l = \frac{\mathrm{cov}(\widetilde{r}_l, \widetilde{r}_m)}{\sigma_m^2}, \tag{2}$$

and the rate of return is univocally determined by cost/price and final cash flow:

$$\widetilde{r}_l = \frac{\widetilde{C}_1^l}{C_0^l} - 1. \tag{3}$$

The value of the asset is therefore

$$V_0^l = \frac{C_1^l}{1 + r_f + \mathrm{cov}\left(\frac{\widetilde{C}_1^l}{C_0^l} - 1, \widetilde{r}_m\right)(r_m - r_f)} \tag{4}$$

and the net present value is

$$\mathrm{NPV}_l = V_0^l - C_0^l. \tag{5}$$

A CAPM-minded decision maker undertakes a project if and only if the NPV is positive.

Let t be a twin security (or replicating portfolio) of l, such that $\widetilde{C}_1^l = \theta\widetilde{C}_1^t$ for some nonzero θ. Standard results of arbitrage pricing tell us that the value of the asset is

$$v_0^l = \theta v_0^t.$$

If absence of arbitrage is assumed, then $v_0^t = C_0^t$; as $C_0^t = \dfrac{C_1^t}{1 + r_t}$, we have

$$v_0^l = \frac{\theta C_1^t}{1 + r_t} = \frac{C_1^l}{1 + r_t} \tag{6}$$

and the net present value is

$$\mathrm{npv}_l = \frac{C_1^l}{1 + r_t} - C_0^l. \tag{7}$$

The project is worth undertaking if there exist arbitrage opportunities, i.e. if value is greater than cost, or, equivalently, if the net present value in (7) is positive. The rate r_t is the expected rate of return of the twin security and acts as the cost of capital, so that[2] $j_l = r_t$. If one assumes that t lies on the SML (as it is done in this paper), we also have $r_t = r_f + \beta_t(r_m - r_f)$.

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[2] In arbitrage pricing it is standard to use the risk-free rate as a discount rate and adjust the distribution of cash flow (risk-neutral valuation). In this context it is preferred to adjust the cost of capital (as known, the two ways are formally equivalent in valuing assets).

Let us summarize the above relations. The CAPM-based value of an asset is

$$V_0^l = \frac{C_1^l}{1 + i_l};$$

(8)

the arbitrage-based value of an asset is

$$v_0^l = \frac{C_1^l}{1 + j_l}$$

(9)

with $i_l = r_f + \beta_l(r_m - r_f)$ and $j_l = r_f + \beta_t(r_m - r_f)$. In the following section it is shown that β_l and β_t differ, and therefore the same is true for the costs of capital and, hence, for the values.

3   CAPM versus arbitrage

Table 2 describes a market in which three securities are traded, conventionally labelled 1, 2, 3. Note that the market is complete, i.e. the asset span equals the whole space $\mathbb{R}^3$. The input variables are: The payoffs of the securities, the probabilities of the states, the price of the securities and the number of shares outstanding in the market. Rates of return, expected rate of return, beta, costs of capital, values and net present values are obtained via application of eqs. (3) to (9). Note that the three securities have the following features:

- expected rates of return and costs of capital coincide: $r_l = i_l = j_l$ for all l=1, 2, 3.

- values and prices coincide: $C_0^l = V_0^l = v_0^l$ for all l=1, 2, 3.

- net present values coincide and are equal to zero: NPV$_l$=npv$_l$=0 for all l=1, 2, 3.

Each of these points boils down to saying that the three securities lie on the SML and arbitrage is not possible (see also footnote 5).

Given the presence of such a market, suppose a decision maker faces project A, described in Table 3. Project A has the same beta as security 1 ($\beta_A=\beta_1=1.094$). This reflects in a CAPM-based cost of capital equal to

$$i_A = 0.0433 + 1.094(0.1547 - 0.0433) = 0.1652,$$

which implies a value equal to

$$V_0^A = \frac{0.5(1200) + 0.1(1000) + 0.4(800)}{1 + 0.1652} = 875.33$$

and a Net Present Value equal to $\text{NPV}_A = 875.33 - 738.48 = 136.85$. But note that project A's payoffs may be replicated by purchasing two shares of security 2 ($\widetilde{C}_1^A = 2\widetilde{C}_1^2$). The arbitrage-based cost of capital is then $j_A = r_2 = 0.1443$ so that project A's value is

$$v_0^A = \frac{0.5(1200) + 0.1(1000) + 0.4(800)}{1 + 0.1443} = 891.32$$

and its NPV is $\text{NPV}_A = 891.32 - 738.48 = 152.84$. This fact is striking, since we have two different costs of capital ($0.1652 \neq 0.1443$) and therefore two different values (and NPVs) for the same project. This simple counterexample allows us to state that CAPM-based valuation is not consistent with arbitrage-based valuation.

Formally, we may easily prove that CAPM-based evaluations are different from arbitrage-based evaluations;[3] we just have to prove that the costs of capital i_l and j_l in the two paradigms are different, and thus the same holds for the values V_0^l and v_0^l. To prove the result we need the following

Lemma 1. *Let A be a project and let t be a security (or portfolio) such that t lies on the SML and replicates A's cash flows in every state of nature. If $\beta_A = \beta_t$, then $V_0^A = \theta V_0^t$.*

Proof. Using (4) we have $V_0^A = \dfrac{C_1^A}{1 + r_f + \beta_A(r_m - r_f)}$. As the betas are equal by hypothesis, we also have

$$V_0^A = \frac{C_1^A}{1 + r_f + \beta_t(r_m - r_f)}.$$

This implies $V_0^A = \dfrac{C_1^A}{1 + r_t}$, because t lies on the SML. Hence,

$$V_0^A = \frac{\theta C_1^t}{1 + r_t} = \theta C_0^t \tag{10}$$

for some nonzero θ. As $C_0^t = V_0^t$ (t lies on the SML) we finally have $V_0^A = \theta V_0^t$. $\qquad \square$

We have then the following

Proposition 1. *Let A be a project and let t be a security (or portfolio) lying on the SML replicating A's cash flows in every state of nature. Then $\beta_A \neq \beta_t$, as long as $C_0^A \neq V_0^A$ (i.e. as long as A does not lie on the SML).*

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[3]Strictly speaking, a single counterexample (as that of Table 3) is sufficient to prove such a difference, but formalization favours insight in the issue, so I will also prove the result with no reference to counterexamples.

Proof. Suppose the thesis is not true. Then, the equality $\beta_A = \beta_t$ implies

$$\text{cov}\Big(\frac{\widetilde{C}_1^A}{C_0^A}, \widetilde{r}_m\Big) = \text{cov}\Big(\frac{\widetilde{C}_1^t}{C_0^t}, \widetilde{r}_m\Big)$$

which implies

$$\text{cov}\Big(\frac{\theta\widetilde{C}_1^t}{C_0^A}, \widetilde{r}_m\Big) = \text{cov}\Big(\frac{\widetilde{C}_1^t}{C_0^t}, \widetilde{r}_m\Big)$$

which in turn entails

$$\frac{\theta}{C_0^A}\,\text{cov}(\widetilde{C}_1^t, \widetilde{r}_m) = \frac{1}{C_0^t}\,\text{cov}(\widetilde{C}_1^t, \widetilde{r}_m)$$

whence $C_0^A = \theta C_0^t = \theta V_0^t$. From Lemma 1 we also have $V_0^A = \theta V_0^t$, so that $C_0^A = V_0^A$. But this contradicts the assumption $C_0^A \neq V_0^A$. $\qquad\square$

Proposition 2. *Let A be a project. The CAPM-based cost of capital i_A differs from the arbitrage-based cost of capital j_A.*

Proof. Let t be again the twin security lying on the SML. We have

$$j_A = r_t = r_f + \beta_t(r_m - r_f).$$

From Proposition (1) we have $\beta_A \neq \beta_t$ so that

$$j_A = r_f + \beta_t(r_m - r_f) \neq r_f + \beta_A(r_m - r_f) = i_A.$$

$\qquad\square$

Corollary 1. *CAPM-based valuation and arbitrage-based valuation are nonequivalent.*

Proof. Use eqs. (8), (9) and Proposition 2. $\qquad\square$

Remark 1. Proposition 1 shows that if a project's payoffs are proportional to a security traded in the capital market, then the beta of the two assets are different or, equivalently, two equal-beta assets do not have proportional payoffs (as long as the project does not lie on the SML, in which case the NPV is zero and the decision process reduces to an idle issue). This difference reverberates on the costs of capital and therefore on the values, as Corollary 1 shows.

Remark 2. It is worth noting that value in the CAPM is a function of the cost of capital i_A, which depends on the beta β_A, which in turn depends on cost C_0^A. Therefore value in the CAPM depends on cost C_0^A (see eq. (4)). By contrast, in arbitrage-based valuation value does not depend on the asset's cost, because the cost of capital j_A does not depend on the asset's beta β_A (however, it does depend on the beta of the twin security, and hence on the cost of the twin security).

True as it is, in the example of Table 3 valuation is twofold, but in decisional terms CAPM and arbitrage theory lead to the same behavior: The project is accepted. Is it the case that CAPM-based valuations lead to decisions compatible to those obtained with arbitrage pricing? The answer is no. In fact, suppose a project l is such that $V_0^l < C_0^l < v_0^l$, so that $\text{NPV}_l < 0 < \text{npv}_l$. This means:

- a rational decision maker undertakes l, because arbitrage profits are possible ($\text{npv}_l > 0$)

- a CAPM-minded decision maker does not undertake l ($\text{NPV}_l < 0$) and therefore fails to exploit an arbitrage opportunity.

That such a case is actually possible is attested by the following example. Assume a decision maker is offered an investment composed of projects B and C (the investment must be *fully* accepted or *fully* rejected). Looking at Table 4, the net present value of this alternative for a CAPM-minded evaluator is

$$\left(-926 + \frac{935}{1 + 0.0433}\right) + \left(-64 + \frac{0.5(466) + 0.1(338.6) + 0.4(-73)}{1 + 0.0433 + 17.21(0.1547 - 0.0433)}\right) = -13.46. \quad (11)$$

The CAPM-minded evaluator rejects this business, because the net present value is negative. But this decision conflicts with the decision taken by a rational investor. The latter accepts to invest in the investment because it gives arbitrage opportunities. Indeed, an arbitrageur would sell short 0.77 shares of security 1 receiving 1006.65=0.77(1307.34) and use the sum to invest in the business at 990=926+64, so gaining a 16.65 profit.[4] At time 1, the arbitrageur will use the payoffs from the investment to close off the position on security 1, so that final net cash flow is zero. Consequently, not only the two paradigms

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[4]Putting it differently, 1006.65 is the value of the project, 990 is the cost, and 16.65 is the (arbitrage-based) net present value of the project.

offer different evaluations, but they may also give rise to different choice behaviors. In particular, CAPM-minded decision makers do not systematically profit from arbitrage opportunities.

In this section we have seen that:

- Valuation via CAPM conflicts with valuation via arbitrage pricing

- CAPM-minded decision makers happen to miss arbitrage opportunities.[5]

In the next section we will see that:

- Value as derived from CAPM is non additive, which means that CAPM-minded decision makers are subject to framing effects.

4 CAPM versus description invariance

Consider the following tasks administered to CAPM-minded decision makers:

Task 1

> You are offered the opportunity of investing 990 euros today and receiving, in one period, 1200, 1100, 970 if state s_1, s_2, s_3 respectively occurs. The security market is described in Table 2. Do you undertake the investment?

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[5]Dybvig and Ingersoll (1982) show that if (i) the CAPM pricing relation holds for all securities in the market, (ii) the market is complete, (iii) the probability that $\widetilde{r}_m > r_m + \frac{\sigma_m^2}{r_m - r_f}$ is positive, then arbitrage opportunities arise. In the security market of Table 2 condition (iii) is not satisfied, so arbitrage opportunities do not arise *within the security market*. However, the example above presented indicates that arbitrage opportunities do arise, though they involve assets that exist *outside* the security market. The relevant fact here is that a CAPM-based valuation does not signal such opportunities (Dybvig and Ingersoll's results do not refer to project selection, but to pricing of financial assets; in other terms they refer to arbitrage opportunities arising *within a security market* where CAPM holds).

Task 2

> You are offered the opportunity of investing in a business composed of two projects. The first project requires an outlay of 930 today whereby you will get, in one period, the certain amount 990. The second one requires an outlay of 60 euros today whereby you will receive 250, 150, 20 euros if state s_1, s_2, s_3 respectively occurs. The security market is described in Table 2. Do you undertake the business?

CAPM-minded decision makers will easily solve these decision problems. In Task 1 the beta of the asset is equal to 0.47 so that its NPV is

$$-990 + \frac{0.5(1200) + 0.1(1100) + 0.4(970)}{1 + 0.0433 + 0.47(0.1547 - 0.0433)} = 12.01 \tag{12}$$

(see project F in Table 5). As a result, subjects will accept the investment. The investment in the second Task is just portfolio D+E (see Table 5), whose NPV is negative ($-19.47+17.54<0$). As a result, subjects will reject the investment.[6]

To sum up, CAPM-minded agents undertake investment in Task 1 and reject investment in Task 2. Yet, the two tasks are extensionally equivalent since the two courses of action share the same cash flows (1200 in state s_1, 1100 euros in state s_2, 970 euros in state s_3) and the same cost (990 euros). A choice reversal is then occurred, even if the course of action is the same.

This is a bizarre result indeed: Using the CAPM approach the decision is not invariant under changes in the framing of the problem. Or, to say it in different terms, the same alternative is considered worth undertaking and not worth undertaking depending on the way it is described to (or by) the decision maker.[7] This striking result implies that the allegedly rational CAPM-minded evaluator undergoes framing effects.

This bias is systematic and predictable. Although I have considered a thought experiment, a real experiment can be conducted among managers, students, scholars, profes-

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[6]Obviously, it is assumed that decision is taken on the whole business, i.e. the course of action is to be fully rejected or fully accepted.

[7]The agent may come across an exogenously framed investment or, rather, may frame it herself as she subjectively perceives it: "The choice [of a particular framing] depends on the economic conditions giving rise to that particular net cash flow and on the psychological factors that influence the cognitive perception of the decision maker" (Magni, 2002, p. 211).

sionals that claim to make use of the CAPM. Evidently, those subjects who turn out not to conform to the solution given above are false CAPM-minded agents, so their answers will be discarded (we are aiming at testing only CAPM-minded reasoners). All the other ones inevitably incur the framing effect.

It is possible to prove formally that a framing bias is intrinsic in the CAPM approach. To this end, we just have to show that the fundamental principle of additivity does not hold in the CAPM approach.[8]

Proposition 3. *Value additivity is not preserved in a CAPM-based valuation*

Proof. Value additivity holds in a CAPM-based valuation if and only if, for *any* asset y and k, we have

$$V_y + V_k = V_{y+k}.$$

Let us consider risky assets x and y and a nonrisky asset k such that $\widetilde{C}_1^x = \widetilde{C}_1^y + C_1^k$ and $C_0^x = C_0^y + C_0^k$. In other terms, $x = y + k$. Applying (4) we have

$$V_x = \frac{C_1^x}{1+i_x} = \frac{C_1^x}{1+r_f+\beta_x(r_m-r_f)}$$

$$V_y = \frac{C_1^y}{1+i_y} = \frac{C_1^y}{1+r_f+\beta_y(r_m-r_f)}$$

$$V_k = \frac{C_1^k}{1+r_f}.$$

Assume $C_0^k = 0$, so that $C_0^y = C_0^x$. Then

$$\beta_y = \frac{\text{cov}(\widetilde{r}_y, \widetilde{r}_m)}{\sigma_m^2}$$

$$= \frac{1}{C_0^y} \frac{\text{cov}(\widetilde{C}_1^y, \widetilde{r}_m)}{\sigma_m^2}$$

$$= \frac{1}{C_0^x} \frac{\text{cov}(\widetilde{C}_1^x - C_1^k, \widetilde{r}_m)}{\sigma_m^2}$$

$$= \frac{1}{C_0^x} \frac{\text{cov}(\widetilde{C}_1^x, \widetilde{r}_m)}{\sigma_m^2}$$

$$= \frac{\text{cov}(\widetilde{r}_x, \widetilde{r}_m)}{\sigma_m^2} = \beta_x$$

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[8]Again, the counterexample just presented could be sufficient to prove that CAPM-based valuation are not additive.

which implies $i_x=i_y$ so that $V_y = \dfrac{C_1^y}{1+i_x}$ whence

$$V_y + V_k = \frac{C_1^y}{1+i_x} + \frac{C_1^k}{1+r_f} = \frac{C_1^x - C_1^k}{1+i_x} + \frac{C_1^k}{1+r_f}. \tag{13}$$

But we have $y + k=x$ so that $V_{y+k} = V_x = \dfrac{C_1^x}{1+i_x}$ whence $V_y + V_k \neq V_{y+k}$ (as long as $i_x \neq r_f$). $\qquad\square$

Remark 3. The condition $C_0^k=0$ used in the proof of Proposition (3) is sufficient to prove the result but not necessary, as testified by Task 2, where project D's cost is nonzero.

Corollary 2. *The value of an asset in the CAPM approach is any real number.*

Proof. Any asset x may be seen as a portfolio of a nonrisky asset k and a risky asset $y=x-k$, where k is arbitrary. Let V^* be an arbitrary real number and choose k so that $C_0^k=0$ and

$$C_1^k = \frac{V^*(1+i_x) - C_1^x}{i_x - r_f}(1+r_f).$$

Substituting in eq. (13) we find that V^* is the value of $x=y+k$. $\qquad\square$

It is also obvious from the above Corollary that

Corollary 3. *The CAPM-based net present value of an asset is any real number.*

Remark 4. It is standard in the behavioral literature to encounter *valence-based* framing effects, where the different frames convey the same information in either a positive or a negative light (e.g. an amount of money may be seen as a gain or as a loss). In contrast, we see here that framing effects arise even in situations where the positive/negative dichotomy plays no role. In our case, the framing effect depends on how outcomes are *partitioned* (an amount of money may be seen as an aggregate or disaggregate quantity and, in the latter case, there are many infinite ways to part it). In the example above, investment in Task 2 is obtained from investment in Task 1 by partitioning it into a portfolio of two assets, one risky and the other one certain. The bias we are studying is then, so to say, a *partition-based* framing effect.

Remark 5. Contrary to CAPM-minded agents, arbitrage-seeking decision makers do not incur framing effect. This is also exemplified by Modigliani and Miller's (1958) Proposition 1 which proves that the debt-equity mix is irrelevant in a firm's value. A firm may be seen as a two-asset (equity-debt) portfolio or as a unique asset. In our perspective, we may imagine asset x as a firm consisting of equity y and (riskless) debt k such that

$$v_y = \frac{C_1^y}{1 + j_y}$$

$$v_k = \frac{C_1^k}{1 + r_f}$$

$$v_{y+k} = \frac{C_1^y + C_1^k}{1 + j_{y+k}}$$

where C_1^y is the equity cash flow, C_1^k is the cash flow to debt, $C_1^y + C_1^k$ $(=C_1^x)$ is the capital cash flow,[9] v_y is the equity value, v_k is the debt value, v_{y+k} is the firm value, and j_{y+k} $(=j_x)$ is the Weighted Average Cost of Capital, that is $j_{y+k} = \frac{j_y v_y + r_f v_k}{v_y + v_k}$. Applying Modigliani and Miller's Proposition 1 to our case, we obtain

$$v_y + v_k = v_{y+k}.$$

Therefore, arbitrage-based valuations (and choice behaviors) are invariant under changes in framing: The value of the asset at hand remains unvaried no matter whether we see the asset as unique or as a two-asset (e.g. equity-and-debt) portfolio.

5 Conclusions

Economic agents' behaviors are sometimes found to infringe normative standards of rational judgment and decision-making. This paper does focus on a possible mismatch between agents' behavior and accepted standards of rational judgment. But, in contrast to the usual position, it just draws attention to CAPM-minded agents and sizes them up against two fundamental norms of rationality: The principle of arbitrage and the principle of description invariance. The results refer to investment valuation and selection, and may be summarized as follows:

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[9]Capital cash flow is the sum of equity cash flow and cash flow to debt. If there are no taxes (as in Modigliani and Miller's Proposition 1) capital cash flow coincides with free cash flow (see Ruback, 2002; Fernández, 2002; Tham and Vélez-Pareja, 2004).

- The CAPM-based notion of value is inconsistent with that of arbitrage theory (see Table 3, Proposition 2 and Corollary 1); the same is consequently true for the notion of net present value

- A CAPM-minded agent may fail to exploit arbitrage opportunities (see the example of Table 4)

- Asset valuation based on the CAPM is ambiguous, because additivity is not preserved (see Proposition 3)

- The notions of value and net present value in a CAPM-based approach are meaningless: They are whatever one wants them to be (see Corollaries 2 and 3)

- Due to nonadditivity, CAPM-minded decision makers are subject to framing effects, i.e. their evaluations (and choice behaviors) are not invariant under changes in framing (see Task 1 and Task 2), whereas arbitrageurs' behaviors are frame-independent (see Remark 5).

CAPM-minded decision makers are then irrational from two points of view: They are irrational because they do not systematically exploit arbitrage opportunities and because they are subject to framing effects. Therefore, we should start rethink the CAPM as a tool for making capital budgeting decisions.

Finally, as an interesting byproduct, deviations of decision makers' behaviors from the CAPM, massively recorded in the current literature, should be seen under a new light: They are just violations of a biased benchmark. In other terms, the empirical finding that economic agents use a rule of thumb instead of the CAPM should not be seen as a greater sign of irrationality than the finding that some decision makers (the CAPM-minded ones) do use such a procedure.

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Table 1. Notations

C_0^l	Cost/price of project/security l (i.e. outlay for undertaking/buying l)
$\widetilde{C}_1^l$	Payoff released by asset l at time 1
C_1^l	Expected payoff released by asset l at time 1
$\widetilde{r}_l$	Rate of return of asset l
r_l	Expected rate of return of asset l
$\widetilde{r}_m$	Market rate of return
r_m	Expected market rate of return
σ_m^2	Variance of market rate of return
r_f	Risk-free rate in the capital market
β_l	Beta of asset l
V_0^l	CAPM-based value of asset l
v_0^l	Arbitrage-based value of asset l
NPV_l	CAPM-based net present value of asset l
npv_l	Arbitrage-based net present value of asset l
i_l	CAPM-based cost of capital of asset l
j_l	Arbitrage-based cost of capital of asset l
cov	covariance

$$l = 1, 2, 3, A, B, C, D, E, x, y, k$$

Table 2. The security market

	Security			Market		
	1	2	3			
Outstanding shares	10,000	21,000	3,000	34,000		
Price	1307.34	445.66	119.80			
					state	probability
Cash Flow	1820	600	125	31,175,000	s_1	0.5
	1654	500	125	27,415,000	s_2	0.1
	1120	400	125	19,975,000	s_3	0.4
Rate of return (%)	39.21	34.63	4.33	36.78	s_1	0.5
	26.51	12.19	4.33	20.28	s_2	0.1
	-14.33	-10.24	4.33	-12.35	s_3	0.4
Expected rate of return (%)	16.52	14.43	4.33	15.47		
Beta	1.094	0.906	0.00	1.00		
CAPM-based cost of capital (%)	16.52	14.43	4.33			
Arbitrage-based cost of capital (%)	16.52	14.43	4.33			
CAPM-based value	1307.34	445.66	119.80	22,791,792		
Arbitrage-based value	1307.34	445.6	119.80	22,791,792		
CAPM-based NPV	0	0	0			
Arbitrage-based npv	0	0	0			

Table 3. CAPM versus arbitrage: Valuation

Project A

			state	probability
Cost		738.48		
Cash Flow	$\left\{\vphantom{\begin{array}{c}1\\2\\3\end{array}}\right.$	1200	s_1	0.5
		1000	s_2	0.1
		800	s_3	0.4
Expected rate of return (%)		38.12		
Beta		1.094		
CAPM-based cost of capital (%)		16.52		
Arbitrage-based cost of capital (%)		14.43		
CAPM-based value		875.33		
Arbitrage-based value		891.32		
CAPM-based NPV		136.85		
Arbitrage-based npv		152.84		

Table 4. CAPM versus arbitrage : Project selection

	Project			
	B	C		
Cost	926	64		
			state	probability
	935	466	s_1	0.5
Cash Flow $\left\{\right.$	935	338.6	s_2	0.1
	935	−73	s_3	0.4
Expected rate of return (%)	0.97	271.90		
Beta	0	17.21		
CAPM-based cost of capital (%)	4.33	196.08		

Table 5. Nonadditivity (framing effects)

	Project				
	D	E	F		
Cost	930	60	990		
				state	probability
	950	250	1200	s_1	0.5
Cash Flow	950	150	1100	s_2	0.1
	950	20	970	s_3	0.4
Expected rate of return (%)	2.15	146.66	10.90		
Beta	0	7.76	0.47		
CAPM-based cost of capital (%)	4.33	90.86	9.57		
CAPM-based value	910.52	77.54	1002.01		
CAPM-based NPV	-19.47	17.54	12.01		