Entry deterrence with unobservable investment. Revisiting Limit Pricing.[*]

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Abstract

We study a standard entry game where the incumbent makes a long run investment choice and a pricing decision facing the threat of entry. When the investment decision is not observed by the potential entrant and the incumbent has private information on costs we show that an aggressive pricing strategy restores the commitment value of investment in a separating equilibrium and affects the probability of entry.

1 Introduction

As it is well known, aggressive pricing strategies by an established firm has been conjectured for a long time as a barrier to entry by potential entrants. Milgrom and Roberts (1982a) summarize the idea of limit pricing by an incumbent as "pricing below the monopoly price to make new entry appear unattractive".[1] They show that when the incumbent has private information, limit pricing is an equilibrium strategy of a signalling game and the probability of entry may be affected, but not necessarily reduced. A necessary condition for a limit pricing strategy to deter entry is that it does not perfectly disclose private information - as in an equilibrium where at least some pooling occurs.[2] When - as in a separating equilibrium - the equilibrium strategy discloses precise information about some characteristics of the profitability of entry, the probability of entry is left unaffected by the

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[*]First draft.

[1]Bolton et al. (2004) provide a similar definition for a predatory price, "that is profit maximizing only because of its exclusionary or other anticompetitive effects".

[2]Similar results obtain when the signal is exogenously garbled by random shocks as in Saloner (????) and Matthews and Mirman (1983) or endogenously jammed by the entrant, as in Fudenberg and Tirole (????).

incumbent's pricing choices. The influence of this result on the subsequent analysis of managerial strategies cannot be underscored and limit pricing has been considered as a self defeating strategy for an incumbent subject to threat: by "limit pricing, incumbent sacrifices short-term profits without affecting long term competition" (Besanko, Dranove and Shanley, 2000) in the logic of burning money result related to the interpretation of the separating equilibrium in signaling games.

In this paper we argue that pricing strategies can promptly convey information *and* affect the probability of entry when an *unobservable* long term investment decision is made by the incumbent firm, along with a price decision, in a limit pricing framework. When information is disclosed, as in a separating equilibrium, we show that limit pricing is useful to effectively manipulate the profitability of entry and its equilibrium probability.

Under the two assumptions of no long term commitment to price and non observability of long term investments it may seem-prima facie- that, relying on the standard models of entry deterrence there is no way for the incumbent firm to affect entry decisions. We show that this is not the case and that limit pricing strategies, to the extent they convey information to the potential entrant, restore the commitment value of long term investments and will affect entry decisions.

The reason why limit pricing may restore the commitment value of unobservable long term investments is quite intuitive and rests on a simple argument. The incentives to limit price in the standard entry game with private information interact with the incentives to long term investments. Consider a simple extension of the model analyzed in Milgrom and Roberts (1982), where the incumbent has private information on its costs, extended to the case where an incumbent's strategy space is enlarged to include both an unobservable cost reducing investment decision and a pricing decision taken under the threat of entry. A low cost incumbent limit prices in order not to be confused with a high cost one, should entry occur. As a result larger quantities are produced and the incentives to invest in cost reducing activities are increased, since they depend on the scale of production. On the other hand, by observing price, the potential entrant has to make inferences on the post entry market profitability and will try to asses both his competitor's cost *and* the amount of investment that has been provided before entry. Since a low cost incumbent has larger incentives to invest than a high cost one, limit pricing may be reinforced because lower prices credibly convey the information that larger investment has been performed, reducing the probability of entry. When the incumbent's cost is common knowledge, on the other hand, both types of incumbent firms will price at monopoly level and, due to non observability of the investment choice, will invest without taking the entry deterrence impact of their choices into account. At equilibrium, therefore, one should observe larger investment under limit pricing than under complete information. As a consequence of greater incentives to invest in cost reducing activities the probability of entry is lower under limit pricing.

We formalize the argument above in a setting similar to Milgrom and Robert's (1982a) where the

incumbent has private information on its production costs and is uncertain about the production costs of the potential entrant. We study a setting with a continuum of types and, for the sake of simplicity, the same assumption of learning upon entry is made, so that the post entry competition is expected to yield a Cournot-Nash duopoly outcome[3]. Under these assumptions we compare the level of investment and pricing strategies by the incumbent in four regimes: *a.* blockaded entry, *b.* entry threat under complete information when both the pricing strategy and the investment decision are observable before entry, *c.* entry threat under complete information when pre-entry pricing, investment and entry decisions are simultaneously taken, *d.* entry threat when the investment decision is not observable whereas the pricing strategy is and satisfies conditions for the Pareto efficient separating equilibrium (Milgrom and Roberts, 1982a, Mailath, 1987).

The reason to concentrate on the separating equilibrium is two-fold. On the one hand- as it is well known- it survives most commonly adopted equilibrium selection criteria among the pletora of equilibria arising in signaling games. The second reason we concentrate on the separating equilibrium relates to our specific interest, i.e., establishing whether, at an equilibrium where precise information is disclosed, limit pricing with unobservable investment may affect the probability of entry by restoring the commitment value of non observable investment in entry deterrence strategies.

The results obtained show that our conjecture holds and limit pricing is part of an entry deterrence strategy and allows incumbent to restore the commitment value of non observable long term investments. As for the resilience of our results, notice that in the analysis of strategic impact of long term investment the commitment value relies on two fundamental features: the investment cost has to be sunk and it has to be observable. Whereas sunkness of most investment strategies is almost undisputed in the literature, observability of investment is less clear cut, specially as for cost reducing activities. It may well be the case that many long term investment strategies are not observable by potential entrants. Consider a few examples of extensively studied commitment devices in oligopoly markets. Long term managerial contracts may affect the competitive stance of the firm but are not necessarily observed by potential entrants (Katz, 1991); credibly disclosing information about the detailed impact of R&D investments profitability may be prevented by the incumbent's willingness to keep his technology secret; the amount of effort dedicated to learning by doing at plant level can be difficult to be observed by potential entrants just by definition. Therefore, in the circumstances where long term investment is not observed by potential entrants, limit pricing strategies can be seen as a device in the incumbent's hand to rescue the commitment value of unobservable investments. This has non negligible consequences both for the analysis of managerial strategies and to frame public policy issues related to anti-trust.

The analysis of industries and firms subject to entry, both from the point of view of its anti-

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[3]By disposing of the assumption of learning upon entry the expected outcome of post entry game is a Bayes-Nash equilibrium of the Cournot game with two sided private information.

trust implications and for the analysis of strategic management has relied on either the impact of reputation concerns on pricing strategies (predatory pricing as in Milgrom and Roberts, 1982b) and on long term strategic investment considerations and related commitment arguments (Fudenberg and Tirole, 1984). In order to have an effective entry deterrence strategy, according to this view the external observer- a potential entrant (but also a policy concerned public agency involved in the welfare assessment of such strategies)- has to base her judgement on a set of indicators and histories that may be quite complicated. When reputational concerns are assumed to be relevant she has to make inferences based on a sufficiently long span of observations and keep close scrutiny of the relevant market in different periods or regions. When strategic barriers to entry may be relevant, the entrant's assessment of their commitment value, with special attention to their observability and irreversibility has to be based on a full range of indicators of the incumbent's long term investment strategy, like advertising, R&D, distribution chains, product quality and so forth.

Under this respect the main point of this paper is that limit pricing may be involved in entry deterrence strategies and that it actually signals their existence when long term investments are not observable. Limit pricing in a model with unobservable investment allows the external observer with a simple signal on which inferences about entry eterrence strategies may be based. We do not address welfare analysis here, but it seems to us that this perspective may be relevant both for the analysis of managerial strategy and for its implications on policy since, pricing below monopoly price credibly signals the incumbent's willingness and his actual ability to raise barriers to entry.

The rest of the paper is organized as follows: Section 2 describes the model and derives equilibrium condition for pre-entry pricing and investment and for the probability of entry under complete information and under private information of the incumbent about its costs, given a separating equilibrium. Section 3 concludes.

2 Limit pricing and unobservable investment in a simple entry game

We consider a simple two periods model. In each of the two periods $i = A, B$ the market is described by a linear inverse demand function $P^i = \alpha - Q^i$. There are two firms in this market, firm 1 an incumbent and firm 2 a potential entrant. In period A the incumbent enjoys a monopoly and chooses quantities q_1^A and the level of cost reducing investment e_1 affecting current and future costs. In period B, depending on the size of fixed entry costs F a potential entrant may decide to challenge the incumbent and enter the market. Should entry occur, the demand for each firm is $P^B = \alpha - q_1^B - q_2^B$. Marginal costs are constant and, for firm 1, are given by $\theta_1 = c_1 - e_1$, where c_1 represents a cost component exogenously given and e_1 represents the amount of cost reducing activity that may be performed inside the firm at a cost ψe_1^2. The entrant's marginal costs are given

by θ_2.

The timing of the game is standard: in the first period the incumbent operates as a monopolist and has to decide a quantity q_1^A and the size of a long term investment e_1^A. After possibly observing first period strategies by the incumbent, the entrant may decide to challenge the incumbent and entering the market.

The incumbents profits are given by

$$\pi_1^A = (P^A - \theta_1)q_1^A - \psi(e_1^A)^2 \tag{1}$$

in the first period and by

$$\pi_1^B = (P^B - \theta_1)q_1^B = (\alpha - q_1^B - q_2^B)q_1^B \tag{2}$$

in the second period.

If the entrant decides to enter his profits will be

$$\pi_2^B = (P^B - c_2)q_2^A - F = (\alpha - q_1^B - q_2^B)q_2^A - F \tag{3}$$

As for the information structure of the game, we follow Milgrom and Roberts (1982) and assume that, unless differently specified, the incumbent has private information on its production costs and c_1 is distributed with a atomless distribution function $H(c_1)$ on a closed interval $[\underline{c}_1, \overline{c}_1]$, with density $h(c_1) > 0$ over the entire support, from the point of view of the entrant. As for the entrant's cost, c_2 is distributed with a atomless distribution function $G(c_2)$ on a closed interval $[\underline{c}_2, \overline{c}_2]$ with density $g(c_2) > 0$ over the entire support, from the point of view of the incumbent with α , ψ and F being common knowledge. In the analysis of the incumbent equilibrium behavior we will assume no discounting of future profits. To simplify the algebra without altering our results we follow Milgrom and Roberts (1982a) and assume learning upon entry so that, the expected outcome of the game in the event of entry is the standard Cournot-Nash outcome. Before studying the entry game we provide some simple characterization of the pricing and investment decision choices for an incumbent firms not facing an entry threat.

2.1 Pricing and Investment under blockaded entry

When the incumbent does not face an entry threat (fixed costs large enough) she will maximize the flow of profits with respect to effort and quantities. With blockaded entry, equilibrium quantity in the second period will be given by

$$q_1^{B,m}(e_1^A) = [\alpha - (c_1 - e_1^A)]/2 \tag{4}$$

yielding monopoly profits given by

$$\pi_1^{B,m} = [\alpha - (c_1 - e_1^A)]^2/4 \tag{5}$$

in the first period, anticipating blockaded entry, the equilibrium strategies of the incumbent firm will satisfy:

$$\underset{(q_1^A, e_1^A)}{Max} \; \pi_1^A + \pi_1^{B,m} - \psi(e_1^A)^2 \tag{6}$$

denoting equilibrium strategies in period A under blockaded entry as $q_1^{A,m}, e_1^{A,m}$, they satisfy the first order conditions:

$$\alpha - 2q_1^{A,m} - (c_1 - e_1^{A,m}) = 0 \tag{7}$$
$$q_1^{A,\beta} + q_1^{B,m}(e_1^{A,m}) - 2\psi e_1^{A,m} = 0$$

solving (7) we get

$$q_1^{A,m} = \frac{\psi}{2\psi - 1}(\alpha - c_1) \tag{8}$$
$$e_1^{A,m} = \frac{1}{2\psi - 1}(\alpha - c_1)$$

With global concavity of (6) warranted by $\psi > 1/2$ implied by the condition on positive marginal costs, $\theta_1 > 0$, warranted by $\psi > \alpha/2c_1$ which we assume to hold.

Investment and pricing by an incumbent firm under blockaded entry will be one of our benchmark to evaluate the effects of limit pricing strategy under private information. Before moving to the study of limit pricing under private information, however, let's study entry deterrence under complete information. We will consider two alternative regimes for our comparison of the effects of private information and commitment observability in the presence of an entry threat. In the first regime we analyze the incentives for an established firm to use investment to deter entry, when pricing strategies and investments by the incumbent are made simultaneously with the entry decision by the entrant. In the second regime we analyze sequential entry game with observable investment and pricing strategy.

2.2 Threat of entry under complete information and observable investment (commitment)

In this subsection we study the entry game under complete information on c_1 and observable e_1^A. The entrant has private information on c_2 before entry occurs, i.e., at the time the incumbent makes its first period choices. Due to the learning upon entry assumption, the second period outcome in the event of entry will be the standard Cournot-Nash outcome

$$\pi_1^{B,d}(e_1^A) = [\alpha + c_2 - 2(c_1 - e_1^A)]^2/9 \tag{9}$$

If the entrant decides to enter his profits will be

$$\pi_2^{B,d}(e_1^A) = [\alpha + (c_1 - e_1^A) - 2c_2]^2/9 - F \tag{10}$$

As for the first period the incumbent rationally expects the entrant to enter if and only if $\pi_2^B \geq 0$ that is, if and only if it holds

$$c_2 \leq \tilde{c}_2(e_1^A) = [\alpha + (c_1 - e_1^A) - 3\sqrt{F}]/2 \tag{11}$$

Therefore the equilibrium strategies of the incumbent can be obtained by solving

$$\underset{(q_1^A, e_1^A)}{Max} \ \pi_1^A - \psi(e_1)^2 + \int_0^{\tilde{c}_2(.)} \pi_1^{B,d}(.)dG(c_2) + \int_{\tilde{c}_2(.)}^{\bar{c}_2} \pi_1^{B,m}(.)dG(c_2) \tag{12}$$

Denoting with $q_1^{A,\varkappa}$, $e_1^{A,\varkappa}$ the equilibrium quantities and investment under the commitment regime, equilibrium strategies satisfy the following first order conditions:

$$\alpha - 2q_1^{A,\varkappa} - (c_1 - e_1^{A,\varkappa}) = 0 \tag{13}$$

$$q_1^{A,\varkappa} + \pi_1^{B,d}(\tilde{c}_2)g(\tilde{c}_2)\frac{\partial \tilde{c}_2}{\partial e_1^A} + \int_0^{\tilde{c}_2(.)} \frac{\partial \pi_1^{B,d}(.)}{\partial e_1^A}dG(c_2)$$

$$-\pi_1^{B,m}(\tilde{c}_2)g(\tilde{c}_2)\frac{\partial \tilde{c}_2}{\partial e_1^A} + \int_{\tilde{c}_2(.)}^{\bar{c}_2} \frac{\partial \pi_1^{B,m}(.)}{\partial e_1^A}dG(c_2) - 2\psi e_1^{A,\varkappa} = 0$$

The first equation in (13) shows that monopoly pricing rule still obtains in the first period whereas, from inspecting the second equation in (13), incentives to investment are modified because

of the commitment effect. To provide some intuition and to analyze the second equation in (13) it may be instructive to interpret its different terms driving the investment effects. Given monopoly pricing, the equilibrium level of investment due to the commitment effect is balanced by different forces, as in the analysis of strategies introduced in Fudenberg and Tirole (1984) and commonly adopted to classify business strategies.

$$\underbrace{q_1^{A,\varkappa}}_{\text{Direct MB } e_1^A} + \underbrace{[\pi_1^{B,d}(\tilde{c}_2(.)) - \pi_1^{B,m}(\tilde{c}_2(.))]g(\tilde{c}_2(.))\frac{\partial \tilde{c}_2}{\partial e_1^A}}_{\text{MB of } e_1^A \text{ from entry deterrence}} +$$

$$\underbrace{\int_0^{\tilde{c}_2(.)} \frac{\partial \pi_1^{B,d}(.)}{\partial e_1^A}dG(c_2)}_{\text{MB of } e_1^A \text{ to accommodate entry}} + \underbrace{\int_{\tilde{c}_2(.)}^{\bar{c}_2} \frac{\partial \pi_1^{B,m}(.)}{\partial e_1^A}dG(c_2)}_{\text{MB of } e_1^A \text{ if blockaded entry}} - \underbrace{2\psi e_1^{A,\varkappa}}_{\text{Marginal Cost of } e_1^A} = 0$$

given our assumptions on the demand and cost functions these different bits may be computed given (9), (5) and (11) and evaluated at e_1^m from equation (8) in order to verify whether overinvestment due to entry deterrence is supported by commitment considerations for specific $G(c_2)$.

In general, define $ED = (1/2)[\pi_1^{B,d}(\tilde{c}_2(.)) - \pi_1^{B,m}(\tilde{c}_2(.))]g(\tilde{c}_2(.))$ evaluated at $e_1^{A,m}$, the marginal benefit from increasing e_1^A, due to the expected entry deterrence effect; it measures the extra incentives to aggressive investment generated by the threat of entry. Notice that $ED > 0$. Define $EA = \int_0^{\tilde{c}_2(.)} \left[\frac{\partial \pi_1^{B,d}(.)}{\partial e_1^A} - \frac{\partial \pi_1^{B,m}(.)}{\partial e_1^A}\right] dG(c_2)$, evaluated at $e_1^{A,m}$ the extra-incentives to invest due to the reduction in the market shares provided entry occurs, a scale effect; it measures the possibly lower incentives to investment by a monopolist when her market shares are threatened. This effect has been extensively studied in the Shumpeterian tradition on the relationship between market structures and investment incentives. Notice that $EA \gtreqless 0$, therefore the commitment value of long term investment e_1^A may or may not lead to over investment by an incumbent firm under entry threat compared to blockaded entry; $e_1^{A,\varkappa} > e_1^{A,m}$ if and only if $ED + EA > 0$.

In this subsection, reformulating the standard analysis in Fudenberg and Tirole (1984) and Tirole (1988) on investment choices, we have shown that effective entry deterrence strategies may indeed be supported by the commitment value. In order to study the effect of commitment on the probability of entry we move now to the analysis of long term investment strategy with non observability of the incumbent's strategies.

8

2.3 Threat of entry under complete information: investment for deterrence with simultaneous entry decision

In this subsection we study the simple entry game when the pricing and investment strategies by the incumbent are taken simultaneously with the entry decision. The interest of this case to our aim is in that it drastically limits the possibility for the incumbent to convey credible information to the entrant about post entry competition[4]. It will provide us with a benchmark to assess the commitment value of long term investment in the case of observability of investment choices and the scope for limit pricing to restore the commitment value under non observability of long term investment.

In the occurrence of entry, second period profits are given by (9) and by (10) to the incumbent and the entrant respectively. Given the post entry outcome first period equilibrium strategies are given by the equilibrium pricing and investment strategy by the incumbent and the entry choice by the entrant. The cut off level of c_2 below which entry occurs is given by (11), whereas the first period strategies by the incumbent are the solution to

$$\underset{(q_1^A, e_1)}{Max} \pi_1^A - \psi(e_1^A)^2 + \int_0^{\tilde{c}_2} \pi_1^{B,d}(.)dG(c_2) + \int_{\tilde{c}_2}^{\bar{c}_2} \pi_1^{B,m}(.)dG(c_2) \qquad (14)$$

where, due to simultaneity, $\tilde{c}_2$ is taken as given with respect to e_1^A .

Denoting with $q_1^{A,s}$, $e_1^{A,s}$ the equilibrium quantities and investment under no observability, they satisfy the following first order conditions:

$$\alpha - 2q_1^{A,s} - (c_1 - e_1^{A,s}) = 0 \qquad (15)$$

$$q_1^{A,s} + \int_0^{\tilde{c}_2} \frac{\partial \pi_1^{B,d}(.)}{\partial e_1^A} dG(c_2) + \int_{\tilde{c}_2}^{\bar{c}_2} \frac{\partial \pi_1^{B,m}(.)}{\partial e_1^A} dG(c_2) - 2\psi e_1^{A,s} = 0$$

The first equation in (15) shows that monopoly pricing rule still obtains in the first period whereas, from inspecting the second equation in (15) incentives to investment are modified in the absence of the commitment effect. Simple manipulation allow us to write the LHS of the second

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[4]There are of course situtations in which this simple setting is also empirically relevant as in markets where secret price discounting practices are adopted.

equation in (15) as

$$q_1^{A,s} + q_1^{B,m} + \int\limits_0^{\tilde{c}_2} \left[\frac{\partial \pi_1^{B,d}(.)}{\partial e_1} - \frac{\partial \pi_1^{B,m}(.)}{\partial e_1} \right] dG(c_2) - 2\psi e_1^s \qquad (16)$$

by evaluating (15) at e_1^m defined in (7) we obtain the following

Lemma 1 *In the absence of commitment value, the equilibrium level of long term investment under entry threat $e_1^{A,s}$ may be above or below the level of investment under blockaded entry $e_1^{A,m}$. A sufficient condition for $e_1^{A,s} > e_1^{A,m}$ is that entry is expected to occur at a sufficiently low scale.*

Proof. by evaluating (15) at $e_1^{A,m}$ defined in (7) we obtain that $q_1^{A,s} = q_1^{B,m}$ and therefore (16) can be rewritten, after some simple algebra as $\int\limits_0^{\tilde{c}_2} \left[4/3 q_1^{B,d} - q_1^{B,m} \right] dG(c_2)$. Therefore, a sufficient condition for $e_1^{A,s} > e_1^{A,m}$ is $(4/3)q_1^{B,d} - q_1^{B,m} > 0$. ∎

That there may be overinvestment in the case of non observable strategies compared to blockaded entry by the incumbent may seem surprising at a first look. However, this is the point where the hypothesis of "learning upon entry" bites: the incumbent knows that he can not directly manipulate entrant's expectations at the entry stage. Remember, though, that it is common knowledge that once entry has occurred costs become revealed, therefore the incumbent may still be willing to accommodate entry on his preferred terms, manipulating the reaction functions in the post entry game.[5]

More interestingly for our aim is the following

Lemma 2 *Equilibrium investment with commitment is larger than in the case of non observability of the entrant's strategy. It holds: i. $e_1^{A,\varkappa} > e_1^{A,s}$ and ii. $\tilde{c}_2(e_1^{A,\varkappa}) < \tilde{c}_2(e_1^{A,s})$*

Proof. Evaluate (13) at $e_1^A = e_1^{A,s}$ and get result i.; to get ii. evaluate $\tilde{c}_2(.)$ at the two investment levels. ∎

Therefore we have seen that observability of commitment improves the incentives to invest in long term cost reducing activities and negatively affects the probability of entry compared to non observability of the incumbent's strategies. We are ready now for studying the case of private information of the incumbent on its costs.

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[5]In this case rather than raising entry barriers the incumbent is escavating entry traps, invisible from outside the market, indeed not unexpected in a Nash equilibrium, and definitely observed after entry because of our simplifying hypothesis of learning upon entry.

2.4 Observable pricing strategies restore the commitment value of long term investments: a limit pricing mechanism

In this section we show that the observability of pricing strategy restores the commitment value of non observable investments when the incumbent faces an entry threat. Define $\theta_1 = c_1 - e_1^A$ and its expectation conditional on the observed first period quantity strategy as $E\left[\theta_1 \mid q_1^A\right] = \hat{\theta}_1$. Given the assumption of learning upon entry, at the time the entry decision has to be made, the entrant has an expected profit from entry defined by

$$E\left[\pi_2^{B,d}(e_1) \mid q_1^A\right] = E\left[(\alpha + \theta_1 - 2c_2)^2/9 - F \mid q_1^A\right] \tag{17}$$

In a separating equilibrium $q_1^A = \phi(\theta_1)$ with ϕ continuous and strictly monotone (Milgrom and Roberts, 1982 and Mailath, 1987). Therefore equilibrium beliefs satisfy

$$\hat{\theta}_1 = \theta_1 = \phi^{-1}(q_1^A). \tag{18}$$

The equilibrium cut off level of c_2 below which the entrant will enter is

$$\tilde{c}_2 = [\alpha + \phi^{-1}(q_1^A) - 3\sqrt{F}]/2 \tag{19}$$

the equilibrium strategies by the incumbent can therefore be obtained by studying

$$\underset{(q_1^A, e_1)}{Max} \pi_1^A - \psi(e_1^A)^2 + \int_0^{\tilde{c}_2(q_1^A)} \pi_1^{B,d}(.)dG(c_2) + \int_{\tilde{c}_2(q_1^A)}^{\bar{c}_2} \pi_1^{B,m}(.)dG(c_2)$$

Define $q_1^{A,lp}$ and e_1^{lp} the equilibrium quantity level and cost reducing investment respectively. First order conditions satisfy:

$$\alpha - 2q_1^{A,lp} - \theta_1 + \left\{\pi_1^{B,d}[\tilde{c}_2(q_1^{A,lp})] - \pi_1^{B,m}(.)\right\} g(\tilde{c}_2)\frac{d\tilde{c}_2}{dq_1^A} = 0 \tag{20}$$

$$q_1^{A,lp} - 2\psi e_1^{lp} \int_0^{\tilde{c}_2(q_1^A)} \frac{\partial \pi_1^{B,D}(.)}{\partial e_1} + \int_{\tilde{c}_2(q_1^A)}^{\bar{c}_2} \frac{\partial \pi_1^{B,m}(.)}{\partial e_1} dG(c_2) = 0 \tag{21}$$

Using (19), (18), (9) and (17), simple algebraic manipulations allow us to write (20) as

11

$$\frac{dq_1^{A,lp}}{d\theta_1} = -\frac{[2(\alpha - \theta_1)\sqrt{F} - F]g(\tilde{c}_2)}{16[q_1^{A,lp} - \frac{\alpha - \theta_1}{2}]} \tag{22}$$

Which defines the differential equation to be satisfied at $\hat{\theta}_1 = \theta_1$ with the initial value condition for the Pareto efficient separating equilibrium (Milgrom and Roberts,1982, Mailath, 1987) given by

$$q_1^{A,lp}[\bar{c}_1 - e_1^{lp}(\bar{c}_1)] = q_1^{A,m} = \frac{\alpha - [\bar{c}_1 - e_1^{lp}(\bar{c}_1)]}{2} \tag{23}$$

where $e_1^{lp}(\bar{c}_1)$ satisfies (21).

Simple algebra also allow us to write (21) as

$$q_1^{A,lp} + q_1^{B,m} - 2\psi e_1^{lp} + \int_0^{\tilde{c}_2(q_1^A)} \left[\frac{\partial \pi_1^{B,D}(.)}{\partial e_1^A} - \frac{\partial \pi_1^{B,m}(.)}{\partial e_1^A} \right] dG(c_2) = 0 \tag{24}$$

and as

$$q_1^{A,lp} + q_1^{B,m} - 2\psi e_1^{lp} + \int_0^{\tilde{c}_2(q_1^A)} \left[4/3 q_1^{B,D} - q_1^{B,m} \right] dG(c_2) = 0 \tag{25}$$

The equilibrium strategies for the entry game under private information of the incumbent on c_1 is therefore described by (25), (20), (23) as for the incumbent strategies and (19) as for the entrant. For a given distribution function of the entrant's costs satisfying the assumptions of atomless $G(c_2)$ and $g(c_2) > 0$ we would be able to solve (25) for $e_1^{lp}(c_1)$ to substitute it into (20) and to solve the resulting differential equation for $q_1^{A,lp}(c_1)$ since $\frac{dq_1^{A,lp}}{d\theta_1} = \frac{dq_1^{A,lp}}{dc_1}$.[6] We do not pursue the study of the differential equation here and we limit our selves to notice that, as in the case of Milgrom and Roberts (1982), it holds

Proposition 3 *In a separating equilibrium of the entry game with unobservable investment it holds $\frac{dq_1^{A,lp}}{dc_1} < 0$ and $q_1^{A,lp}(c_1) \geq \frac{\alpha - [c_1 - e_1^{lp}(c_1)]}{2}$, with equality at $c_1 = \bar{c}_1$.*
Proof. *$\frac{dq_1^{A,lp}}{dc_1} < 0$ is implied by type monotonicity (Mailath, 1987), that is $\partial^2 \pi_1 / \partial q_1^A \partial c_1 < 0$ holding in this model. Consider next that the numerator of (22) is given by $\left\{ 2[\alpha - (c_1 - e_1^{lp}(c_1))]\sqrt{F} - F \right\} g(\tilde{c}_2)$. $g(\tilde{c}_2) > 0$ by assumption for $c_2 \in [\underline{c}_2, \bar{c}_2]$, therefore it is positive iff $[2(\alpha - \theta_1)\sqrt{F} - F] > 0$, which is satisfied under the necessary condition for entry to be viable, i.e. that fixed entry costs*

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[6]As an example we computed, but not reported here, the solution for the case of a uniform distribution with $g(c_2) = 1/[\bar{c}_2 - \underline{c}_2]$. The structure of (22) allows the separation of variables method to be solved yielding an implicit function describing the equilibrium relationship between $q_1^{A,LP}$ and c_1.

are low enough to be covered by monopoly profits, i.e. $F < (\alpha - \theta_1)^2/4$. Therefore it must be $-[q_1^{A,lp} - \frac{\alpha - [c_1 - e_1^{lp}(c_1)]}{2}] < 0$ *that is* $q_1^{A,lp} > \frac{\alpha - [c_1 - e_1^{lp}(c_1)]}{2}$. *Finally the initial value condition requires* $q_1^{A,lp} = \frac{\alpha - [c_1 - e_1^{lp}(c_1)]}{2}$ ∎

In other words upward distortion of first period quantity levels by the incumbent firm obtains, i.e. limit pricing must part of the equilibrium entry deterrence strategy in a separating equilibrium. Limit pricing requires a larger scale of production in the first period by the incumbent firm than under complete information. To evaluate whether the scale effect associated to limit pricing in the first period enhances incentives to investment and the probability of entry evaluate the set of first order conditions under private information with the set of first order condition under non observability, i.e. (25) at (16) and get the following

Corollary 4 *Investment (probability of entry) under limit pricing is larger (lower) than under first period simultaneous choice of price investment entry decision equilibrium; i.e., it holds i.* $e_1^{lp} \geq e_1^s$ *and ii.* $\tilde{c}_2(e_1^{A,lp}) > \tilde{c}_2(e_1^s)$.
Proof. *notice that (25) is positive at* e_1^s *as long as* $q_1^{A,lp} > q_1^{A,s}$ *which is true since* $q_1^{A,s} = q_1^{B,m} = \frac{\alpha - [c_1 - e_1^s(c_1)]}{2}$. *By using the definition of* $\tilde{c}_2(.)$,*notice that i. immediately implies ii.* ∎

Therefore, we have shown that in a separating equilibrium the scale effect associated with the limit pricing strategy involves a scale effect that drives overinvestment compared to the case of simultaneous choice of entry deterrence strategies and entry decision by the entrant. This effect drives the probability of entry down restoring some commitment value of non observable investment with long term effect on the incumbent's costs.[7]

3 Conclusions

In this paper we have shown that limit pricing strategy are part of an effective entry deterrence strategy when the incumbent cost reducing investments are not observable by the entrant. In this case the neutrality of limit pricing separating strategies on the probability of entry does not hold anymore. In many industries the assumption of observability of long term investments by agents external to the firm does not necessarily hold, as in the case where detailed information on R&D

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[7]The relationship between $e_1^{A,LP}$, $e_1^{A,M}$ and $e_1^{A,\varkappa}$ can be studied under specific assumptions on $G(c_2)$ but we do not pursue this here. It may be worth noticed that the comparison between $e_1^{A,LP}$ and $e_1^{A,M}$ depend on the expected impact of entry on the market shares. Limit pricing involve an increase in the scale of production in period A but anticipate the possibility of scale reduction due to entry whereas, under blockaded entry, the monopolist discount a larger market share in period two. A similar mechanism is at work in the comparison between $e_1^{A,LP}$ and $e_1^{A,\varkappa}$. Also notice that, since the scale effect under limit pricing depends on c_1 it may well be the case that, under private information and obsevable prices low cost incumbent over-invest compared to the case of observable commitment, whereas high cost types will under invest.

expenditure cannot be released because of secrecy concerns by the incumbent, in the case where learning by doing occurs at the plant level or firms costs depend on managerial effort which is not observed by the entrant. Our results suggest that, specially in these industries, limit pricing is still a valuable framework both for the analysis of managerial strategies and for the assessment of the incumbent's behavior from the point of view of public agencies in charge of controlling anti competitive behavior by established firms. It goes almost without saying that, by increasing incentives to investment for incumbent firms limit pricing may be welfare improving. The precise assessment of the impact of limit pricing strategies on social welfare is, however, outside the scope of the paper and left for future work.

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