

Endogenous Growth and Research Activity under Private Information *

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Abstract

We setup an endogenous growth model where innovators produce ideas and privately know their production cost. Developers react by offering non-linear contracts that affect the mass of innovators, and then the growth of the economy. Two main results are obtained. First, there is an equilibrium contract under asymmetric information that entails more selection of talented workers in R&D activities and higher profit for the developer. Second, there is efficiency – extraction rent tradeoff that increases the dispersion of the technology and reduces the economy rate of growth with respect to the full information case. In addition I provide the characterization of the equilibrium contracts when there is competition across principals. I found countervailing incentives that drive interesting dynamic of the productivity in the long run.

Key Words: Adverse Selection, Innovation and Endogenous Growth.

JEL Codes: 031,033,D82

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1 Introduction

Many industries face agency problems that affect R&D's return on investment. A firm who develops new projects has to design contracts and to monitor talented innovators who are private informed about their ideas, ability, productivity and effort. The Developer aims to optimize the profitability of the project and therefore to align the innovators' incentives and interests with her objectives to reduce the contracting and monitoring costs. The purpose of this paper is to study endogenous technological change in an environment in which the incentives to innovate are private information. In particular, I consider a situation in which productivity in the innovation process is not observable. Indeed this has important consequences on research activity and on the growth rate of the economy.

Recent research on industrial organization and incentive theory address some issues about difficulty of resources allocation to R&D. Specifically, this literature emphasizes on problems of assigning correctly property rights to innovation activity. A considerable research effort has been put toward problem agency in R&D. In this paper I focus on analyzing the impact of allocation of talented innovators with private information about economic growth.

In this setup, innovation activity comes from an interaction between a representative developer and a continuum of innovators. Growth is engine by improvements in quality of the intermediate goods. Innovators create knowledge by producing blueprints and developer uses them as inputs.

I study how R&D is produced when the developer proposes specific contract on the level production of blueprints. The agency problem arises since innovators and developer have different interests. Depending on the expected value of the new idea (or project), innovators want to maximize their utility by minimizing their level of effort. On the other hand developer aims to extract all the possible surplus of the innovators in order to maximize her profits. In the literature it is commonly arguing that patents are the mechanisms that reward innovators. Patents work as a system in which the regulator assigns to the R&D actors' property rights on the use of ideas. However, there is not consensus about how the surplus is allocated between innovators, developer and final consumers. In this paper, the aim is to combine the previous microeconomics literature on incentives to innovation process into the macroeconomics setup. I extend the previous research by using a simple model of adverse selection into a standard endogenous growth framework.

The model answers the following questions: What is the impact on the economic growth of adverse selection in innovation activity on economic growth? How does non-observable

heterogeneity in the innovators' productivity affect the resource allocation to R&D?. One of the main results under asymmetric information is that the dispersion among different qualities of ideas matters. This dispersion determines the likelihood to choose the more efficient technology with higher productivity and economic growth.

The second result emphasizes on the negative effect of asymmetric information on the threshold productivity. This threshold determines the distribution of labor force between final sector and R&D activities. In particular under private information I found this threshold bigger. This result suggests that more selection of talented innovators in R&D sector leads to higher expected profits for developers. The decision-making in selection implies the standard tradeoff between extraction rent and efficiency. As a consequence the rate of economic growth is reduced in contrast to the case of full information.

The third result is the characterization of equilibrium contracts under developers' competition facing innovators' private information. Developers offer compensation schemes in order to attract the most talented innovators. This developers' competition induces that innovators can over or under report their productivity type with respect to their true value. In this sense, the model predicts that for some regions of the productivity there is over production of blueprints. In contrast when the dispersion of the productivity increases, the production of blueprints decreases.

This paper is structured as follows: Section I is dedicated to study the related literature. Section IV presents the benchmark model setup under full information. Section V examines the main implications of the model under asymmetric information. Section VI provides extends the previous setup to consider competition across developers under private information. Section VII concludes.

2 Related Literature

This paper is related to several strands of literature. In first place, there is a large literature about the incentives in R&D and growth. Seminal papers of Romer [1990], Grossmann and Helpman [1991], Aghion and Howitt [1992], focus on incentives to innovation are characterized by monopoly rents (through patents) and investment of resources in R&D. O'Donoghue, and Zweimüller, Josef [1998] study the patent protection for sequential innovation and they identify different structures of the optimal patent and breadth. They find that for leading breadth the patent is finite and broad and the length of the patent life coincides with the statutory life.

All the previous literature described above shows that when the economy has an accumulation of knowledge, there are spillover effects on activities. Property rights which enhance innovators' protection involve some welfare distortions (i.e monopoly and knowledge externalities). In this sense, the policy intervention is justified through the introduction of subsidies in R&D. In order to have in mind a magnitude of the impact of the externality of knowledge in the economy, Jones and Williams [1998] have been estimated that the social return of R&D in OCDE countries is around the average of 27 %. They find that optimal R&D investment can be multiplied approximately by twice or four times the current level of investment. These results are coherent with similar findings such as Griliches and Lichtenberg [1984].

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Regarding to the purpose of this paper, a second branch of literature that is related to the incentives in R&D with the perspective of industrial organization and incentive theory. I refer to the works of Aghion and Tirole [1994,1995], Anton and Yao [2004], Martimort et al [2008]. Aghion and Tirole [1994,1995] analyze the innovation process as an interaction among the financier, creator, owner and user of the innovation. Using incomplete contracts they study the impact of different structures of organization on their research activities. They find that the structure of the organization depends on how the innovation units are financed. This theory explains the role of the joint venture in development of new R&D as well as the role for the government to subsidize R&D. Anton and Yao [2004] explain the failure of the intellectual protection as a problem of disclosure of information related to the size of the innovation. They find that, big innovations are protected by trade secrecy. In the case of medium innovation, property rights are setting through licenses and patents. The costly full protection fails to protect low size of innovation allowing imitation.

Building on this theoretical work, Martimort, et al [2008] analyze the innovation process

in a bilateral relationship between the developers and innovators. Developers face problems of adverse selection since the innovators have private information about the quality of the projects. Similarly, developers learn about the quality of the innovator's idea and have the incentive to incur in moral hazard. They find that the optimal contract is characterized where the innovator holds an important share of equity into the project. This element of the innovator's compensation package gives a signal about the quality of the idea. For instance the innovator will likely accept a higher variable part of his compensation when his perception of the quality of the idea is good enough. The problem of innovator's incentives is the under provision of their effort in innovation activity since there is a potential business stealing effect to developers. This paper is based on the previous work of Martimort et al [2008], analyzing the implication of the adverse selection on R&D incentives in bilateral relationship developers -innovators and in the accumulation of knowledge.

3 The Model

3.1 Environment: Agents, Preferences and Technologies

The economy is composed by three kinds of agents: households, final sector and developer. At any point of time, there is a continuum of individuals who has identical preferences, that constitutes one household. Each individual differs in the level of productivity or ability measured by θ which is a random variable such that $\theta \sim \mathcal{U}[0,1] \equiv \Theta$ with distribution function $F(\theta)$, density function $f(\theta)$, and where 0 (1) is a measure of the lower type (higher type) of productivity.

Individuals can offer labor in both activities, either final good sector or R&D. In the final sector, as a worker (W), they receive a certain wage w in each period of time independently of the level ability. In R&D activity, as an innovator (I), they earn an income τ according to their level of ability θ delivered at time t . Preferences in both cases are represented by the following utility function:

$$U = E_0 \int_0^\infty \ln \left(c_t^j - \frac{e_t^2}{2} \right) \exp(-\rho t) dt \quad (1)$$

where c_t^j is the consumption of the household depends on the occupational choice $j = W, I$. e measures the level of effort and A indicates the total stock of knowledge. An individual's disutility of effort is assumed increasing, convex and twice continuously differentiable. In addition, it is assumed that the cost of effort reduces as the stock of knowledge increases over time. As in Reib and Weinert [2005] did, this assumption allows stationary level of effort

over time . The intuition is based on the fact that as technology advances, the opportunity cost of leisure increases since the labor is more productive. In this setup, if the agent's choice is being a worker and then offering labor in final sector, he will exert one unit of labor and zero effort. On the contrary, the agent as an innovator offers labor in R&D and exerts positive amount of effort e . This preferences are a particular case of GEE preferences in which there is not income effects in the labor supply that means the intertemporal consumption substitution are ruled out of the choice of labor effort.

Final good Y_t is produced by the final sector and is used to aggregate consumption good by the households and it is also as an input for intermediate goods and R&D.

The R&D activities' output is explained as follows. There is a continuum of R&D sectors, each sector is denoted by $i \in [0, 1]$. At each period of time t , each sector i is characterized by a level of technology $A_{i,t}$ and the total stock of knowledge is the aggregation across sectors i , $A_t = \int_0^1 A_{i,t} di$. . Each intermediate good has a level of quality along that can be improved over time. Each R&D activity has the following assumptions:

1. In sector i , R&D activity produces blueprints q_i that is a combination of the effort exerted by the household e and the specific ability θ in the sector i . R&D output is explained according to the following technology¹:

$$q_i = \theta_i + \frac{e(\theta)}{\sqrt{A_t}} \quad (2)$$

Therefore given a level of blueprints production, this assumption implies two possibilities: if the innovator has low ability he could compensate it by exerting high effort. Otherwise with high ability the innovator does not need to exert marginal effort. The total production of R&D in sector i is given by an aggregation across level of ability θ from a cut-off productivity level that will be determined after. Let this aggregate blueprints $q_i = \int_{\hat{\theta}_i}^1 q_i(\theta) dF_i(\theta_i)$.

2. Innovations in each sector arrives with a Poisson rate of λ . Since there is a continuum of R&D independent sector the total arrival rate of innovation is equal to λq_i . Taking in small period that means from now 0, the flow probability is equal to λq_i .

¹This main results of the analysis are mantained under complementarity assumption between ability and effort as in Aghion and Tirole [1994,b]

3. Variation in the stock of knowledge is given by $\Delta A_{i,t} = \sigma A_t$, where $\sigma > 1$ is the frequency of innovation. That means when innovation occurs at time t the induced change in the knowledge in sector i is proportional to the whole disposable knowledge in the economy. Therefore the law motion for the average knowledge in the economy is given by:

$$A_{i,t+\Delta t} \approx (A_{i,t} + \sigma A_t) \lambda q_{i,t} \Delta t + A_{i,t} (1 - \lambda q_{i,t} \Delta t) \quad (3)$$

$$A_{i,t+\Delta t} = A_{i,t} + \sigma \lambda q_{i,t} A \Delta t$$

4. Then the instantaneous changes on the stock of knowledge is equal to :

$$\lim_{\Delta t \rightarrow 0} \frac{A_i(t + \Delta t) - A_i(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\sigma \lambda q_{i,t} A \Delta t}{\Delta t} \quad (4)$$

5. The average of the change in the stock of knowledge in each sector is given by:

$$\mathbf{E} \left(\dot{A}_{i,t} \right) = \sigma A_t \lambda q_{i,t} \quad (5)$$

Once an innovation appers in sector i , the intermediate good $x_{i,t}$ is produced according to linear technology $x_{i,t} = y_{i,t}$, where $y_{i,t}$ is the quantity of final output used to produce $x_{i,t}$. The total final output Y_t is a combination of labor L^Y and a continuum intermediate goods x_i . Final good technology exhibits constant returns to scale in the aggregated intermediate good $X = \int_0^1 x_i di$ and labor used in final sector L^Y :

$$Y_t = (L^Y)^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{i,t}^\alpha di \quad (6)$$

The total output in the economy is given by:

$$Y_t = \hat{\theta} c_t^w + \int_{\hat{\theta}}^1 c^I(\theta) dF(\theta) + \int_0^1 x_{i,t} di \quad (7)$$

In this sense, total final output is allocated between workers' aggregate consumption, innovators' consumption, and total expenditures in intermediate goods.

3.2 Timing

The production process has these following stages:

1. Each innovator learns his type

2. The R&D sector/final sector proposes a contract that specifies the amount of blueprints in each sector i and the wage/income if the household is a worker/innovator
3. The household by choosing to be worker or innovator accepts or rejects the contract.
4. Each innovator makes a decision about his level of production of blueprint q representing by the productivity parameter θ
5. The contract is executed and thus intermediate goods are produced
6. Competitive firms in final sector use them as an input into final output production
7. Each household decides the amount of consumption.

In the first part I characterize the situation under full information about the productivity parameter. I start by characterizing the first best and the equilibrium outcome. In the second part the information frictions are studied. In particular, the constrained efficient allocations and the equilibrium contracts under adverse selection are considered.

4 First- Best

Suppose a central planner who decides about consumption, R&D efforts, intermediate goods, labor in final sector and investment in R&D. Therefore, the central planner solve the following problem :

$$\max_{\{c_t^w, c_t^I, q_t, \theta^{\hat{f}^b}, x_{i,t}, A_{i,t}\}} \int_0^\infty \left[\int_0^{\theta^{\hat{f}^b}} \ln(c_t^w(\theta)) dF(\theta) - \int_{\theta^{\hat{f}^b}}^1 (c_t^I(\theta) - A_t((q_t - \theta)^2/2)) dF(\theta) \right] \exp(-\rho t) dt \quad (8)$$

subject to [6], [5], [7].

The central planner maximizes the social welfare utility across individual with specific ability θ . The social weights are determined by the occupational choices $\theta^{\hat{f}^b}$. The first order conditions are:

$$[c_t^w, c_t^I] : \quad \frac{\exp^{-\rho t}}{c_t^w} = \frac{\exp^{-\rho t}}{c_t^I - A_t e_t^2/2} = \mu \quad (9)$$

$$[q_t] : \quad \frac{(q_t - \theta) \exp^{-\rho t}}{c_t^I - A_t e_t^2/2} = \lambda \sigma \int_0^1 \eta(i) di \quad (10)$$

$$[A_t] : \quad -\frac{e_t^2/2 \exp^{-\rho t}}{c_t^I - A_t e_t^2/2} + \mu \left[(1 - \alpha) (L^Y)^{1-\alpha} \int_0^1 A_{i,t}^{-\alpha} x_{i,t}^\alpha \right] + \lambda \sigma \int_0^1 \eta(i) di \int_{\hat{\theta}}^1 q_t dF(\theta) = -\dot{\eta} \quad (11)$$

$$[\hat{\theta}^{fb}] : \quad \mu \left[(1 - \alpha) \frac{Y_t}{\hat{\theta}^{fb}} \right] = \lambda \sigma \left(e_t + \hat{\theta}^{fb} \right) \int_0^1 \eta(i) A_{i,t} di \quad (12)$$

$$[x_t] : \quad \mu \left[\alpha (L^Y)^{1-\alpha} \int_0^1 A_{i,t}^{1-\alpha} x_{i,t}^{\alpha-1} - 1 \right] \quad (13)$$

From equations 9, μ is the lagrange multiplier associated to the aggregate resources constraint. Then, from the first order condition with respect to consumption in each occupation it has that $c_t^w = c_t^I - A_t e_t^2/2$. That means, the central planner chooses the allocations in such way that the difference between consumption of innovators and workers is the amount of R&D effort.

Second, the first order condition [10] entails that the marginal value of the effort in R&D must be equal to marginal value of the investment in R&D, where $\int_0^1 \eta(i) di$ is the sequence of the Lagrange multipliers associated to the law motion of knowledge. The third first order condition entails that the marginal value of unit of knowledge is equal to the disutility of provide effort in R&D activities plus the marginal productivity of knowledge and the marginal value of the investment in R&D.

The other conditions equations [12] and [13] are the standards demand for labor in final sector and the demand for intermediate goods. Using this equation system, the optimal allocations are characterized. As it is presented in the next preposition the central planner allocations can be characterized to the case of symmetric allocations.

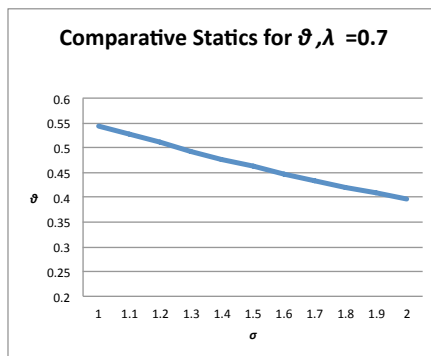
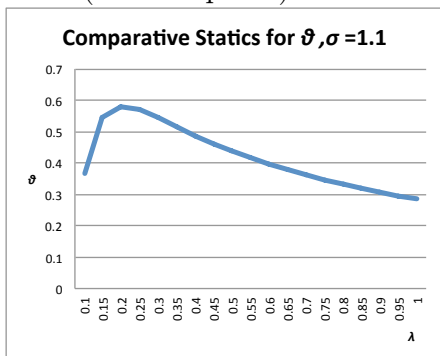
Proposition 1 *At the symmetric steady-state first best, the central planner's allocations are determined by: $Y^{fb} = \hat{\theta}^{fb} A \alpha^{\alpha/(1-\alpha)}$, $x^{fb} = \hat{\theta}^{fb} A \alpha^{1/(1-\alpha)}$, $C^{fb} = \hat{\theta}^{fb} A \alpha^{\alpha/(1-\alpha)} (\alpha^\alpha - 1)$, $q^{fb} = \left(\sqrt{\hat{\theta}^{fb^2} + 4(1-\alpha)\alpha^{\alpha/(1-\alpha)}} - \hat{\theta}^{fb} \right) / 2 + \theta$. The rate of growth in steady-state $g_Y^{fb} = g_x^{fb} = g_C^{fb} = g^{fb} = \sigma \lambda \frac{(1-\hat{\theta}^{fb})}{2} \left[\sqrt{1 + \hat{\theta}^{fb^2} + 4(1-\alpha)\alpha^{\alpha/(1-\alpha)}} \right]$ and for $\hat{\theta}^{fb} \in [0, 1]$ there is a unique productivity threshold $\hat{\theta}^{fb}$.*

Proof see appendix

Some comments about the proposition 1 are presented. Firstly the allocations of output, intermediate goods and aggregate consumption are proportional to the stock of knowlege

and the productivity threshold. Secondly, the optimal growth rate depends positively on the total labor allocated to R&D and the parameters related to the intensity of knowledge spillovers measures by λ and σ . The parameter λ indicates the contribution of the production of blueprints to the increments on the stock of knowledge. The parameter σ accounts the influence of the total stock of knowledge in the quality improvements in each sector i .

Under a set of reasonable parameters for the technology of final output $0 < \alpha < 1$ and $0 < \lambda < 1$ and $\sigma > 1$ there is a unique productivity threshold of labor $\hat{\theta}$ as it is presented in the next graph. Numerical excersises about the sensitivity of the threshold to the productivity parameters. In the first panel, the reaction of the threshold is analized when the production of blueprints becomes more efficient. In this sense, there is less selection of R&D labor when the production of blue print is high productive. Contrary it is the case where there is more influence of the total stock of knowledge on the production of the new qualities (second panel).



5 Decentralized allocations

5.1 Households and Occupational Choices

Households offer labor in both activities, a threshold such that determines the level of activity in which households will work. Then occupational choice for each type θ is determined by:

$$v = \max \{v_W, v_I\}$$

Where v_W, v_I are the value functions either is if the household offer labor in final sector or R&D. If the value function exist then there is a productivity threshold that determines the allocation of labor:

- $\hat{\theta}$ if $v_w(w) = v_I(\theta, e^D(\theta)) \rightarrow$ Individuals are indifferent between both activities
- $\theta < \hat{\theta}$ if $v_w(w) < v_I(\theta, e^D(\theta)) \rightarrow$ Individuals aim to be innovator within R&D sector
- $\theta > \hat{\theta}$ if $v_w(w) > v_I(\theta, e^D(\theta)) \rightarrow$ Individuals prefer to be a worker within final sector

The value function comes from solve the following problem for each household of type θ :

$$\max_{\{c_t^j, b_{t+1}^j\}_{t=0}^{\infty}} \int_0^1 \ln \left(c_t^j - A_t \left(\frac{e_t^2}{2} \right) \right) \exp \rho t \quad (14)$$

subject to

$$c_t^j + \dot{b}^j = \tau_t + r_t b^j$$

with b_0 given.

The objective of household $-\theta$ is to choose a sequence of consumption c_t^j and assets b_t^j for each occupational choice $j = W, I$ in order to maximizes the discounted utility over time net of the labor decisions. Each household $-\theta$ the total revenue (i.e total of income that depend on the occupational choice and the assets-earnings) is allocated in buying consumption goods and assets. The standard Euler condition entails that the marginal rate of substitution is constant over time and equal to the interest rate:

$$g_{\tilde{c}^j} = r_t - \rho \quad (15)$$

where $\tilde{c}^j = c_t^j - A_t \left(\frac{e_t^2}{2} \right)$ and $g_{\tilde{c}^j}$ is the rate of growth of \tilde{c}^j .

Since there aren't income effect on the preferences then the level of effort is determined by the wages in R&D activity $e = \tau$

5.2 Final good firms

Competitive firms purchase intermediate goods and use them to produce final homogeneous good according to the technology previously specified. Price of final good is the numeraire in the economy. Let x the amount of intermediate goods that is bought by the firm and let p the price per unit of intermediate goods. The problem that final good firms solve in each period t is:

$$\max_{x_{i,t}, L_t^Y} B_{i,t} = (L^Y)^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{i,t}^\alpha di - w_t L_t^Y - x_{i,t} p_{i,t} \quad (16)$$

First order condition with respect to the intermediate goods is:

$$x_{i,t} = \left(\frac{\alpha}{p_{i,t}} \right)^{\frac{1}{1-\alpha}} A_{i,t} L^Y \quad (17)$$

The equation of intermediate goods demand depends positively on sectorial productivity and negatively on the price of intermediate goods. The labor demand in final output activity is given by:

$$(1 - \alpha) \frac{Y_{i,t}}{L} = w_t \quad (18)$$

5.3 Development activity

In each sector i there is a monopolistic developer who is in charge of the production of intermediate goods. Once an innovation is made, each developer gets a infinite horizon patent for the new intermediate goods.

Developer gets a monopolistic profit by selling intermediate goods to final sector. Secondly she chooses the level of investment in R&D. Each developer in sector i chooses the price such that maximizes her profit:

$$\pi_{i,t} = p_{i,t} x_{i,t} - y_{i,t} \quad (19)$$

First order condition is:

$$p_i = \frac{1}{\alpha} \text{ for all } i \quad (20)$$

Monopolistic's price includes the markup ($1/\alpha$). As the markup is the same for all good i then the demand for each intermediate good is given by:

$$x_{i,t} = x_t = \alpha^{\frac{2}{1-\alpha}} A_t L^Y \quad (21)$$

Therefore profit for each monopolist in sector i is:

$$\pi_{i,t} = \tilde{\pi} A_{i,t} L^Y \quad (22)$$

Where $\tilde{\pi} = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$ is the market power factor. The profit in each sector i is proportional to the effective labor $A_{i,t} L^Y$ and the market power factor $\tilde{\pi}$. Developer's profit by selling intermediate goods arises because of improvement in the technology, increase on final sector labor or a change in the elasticity of substitution.

5.4 R&D Activity

There is a single entrepreneur in each sector i who invests in R&D and innovates with flow probability $\lambda q_{i,t}$. In the case of success the outcome of an innovation is a new version of the intermediate good and she will obtain a monopolistic rent given by the patent that has a mean duration of $\frac{1}{\beta}$.

The investment in R&D constitutes the aggregate payment to individual innovators for the production of q_i units of R&D in sector i . At each time period of time t , a contract specifies the R&D outcome and payoff for each productivity- type $\{q_i(\theta), \tau_i(\theta)\}$ for all productivity parameter θ such that $\hat{\theta} \leq \theta \leq 1$ in each sector i . After the innovation is realized, the innovator exclusively contracts with one sector. I consider the case of short contracts, where the R&D firm offers contracts that are at least as good as the contracts offers by the manufacturing sector.

Then the problem that solves R&D entrepreneur is:

$$\max_{q_{i,t}(\theta), \tau_t(\theta)} \lambda q_{i,t}(\theta) V_{i,t} - \tau_t(\theta) \quad (23)$$

Subject to [2] and

$$U^I(\theta) = \tau_t(\theta) - A_t e_t^2(\theta) / 2 \geq w_t \quad (24)$$

Where $A_{i,t} \tau_t(\theta)$ is the total payment to the innovator in sector i . Here we assume that the remuneration to the innovator is proportional to the specific stock of knowledge in sector i .

The objective of the R&D entrepreneur is to maximize their expected benefit that in the case of success is the monopolistic rent of selling intermediate goods to final sector subtracting the productivity adjusted value for each innovation, equation [23]. Equation [24] is the participation constraint of the innovator in each sector i . Notice that given the technological constraint equation [2] each innovator submits different R&D outcomes depending of the specific level of knowledge in the sector i , the ability and effort.

Replacing equation [2] in [24] the participation constraint is binding at w_t , therefore the first order condition entails:

$$V_{i,t} = \frac{(q_{i,t} - \theta_{i,t})}{\lambda} A_{i,t} \quad (25)$$

In this sense the benefit for the R&D firm of invest $q_{i,t}$ of final output is equal equal to the level of effort in R&D.

5.5 Capital Markets

R&D is financed through the issues of equity claims on the flow of profits generated by the innovation that are given by:

$$V_{i,t} = \int_t^\infty \pi_{i,s} \exp^{-\int_t^s (r(u)+\beta)ds} ds \quad (26)$$

Thus, at interval of time dt the developer of sector i receives a flow of profits equivalent to $\pi_i dt$ and therefore the value of the firm in sector i increases by $\dot{V}_i dt$ in each industry i . Because there are improvements in the level of quality, shareholders will be impacted by a lost if a new innovation arrives equivalent to V_i and this happens with probability β . As a consequence the existing developer receives zero profits and is replaced by other with more efficient quality technology.

Since the capital markets are efficient, the expected rate of return of holding stock in R&D activity must be equal to the free risk rate that is achieved under complete markets $rd(t)$. Therefore the non arbitrage condition in the capital markets yields to:

$$r(t) + \beta = g_v + \frac{\pi_{i,t}}{V_{i,t}} \quad (27)$$

Since the interest rate is endogenous the non arbitrage condition for the asset markets is $g_c + \rho + \beta = g_v + \frac{\pi_{i,t}}{V_{i,t}}$. Where g_v measures the capital gains for the shareholders. If the agents have bonds can smmoth consumption over time

5.6 Definition of equilibrium

An equilibrium for this economy is a collection of time paths of $\{c_t^w(\theta), c_t^I(\theta), e_t(\theta), x_t(\theta), L_t^Y(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$, a path of state variable $\{A_t\}_{t=0}^\infty$ and a sequences of prices $\{w_t, p_t\}_{t=0}^\infty \{\tau_t(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$ such that:

Given $\{w_t\}_{t=0}^\infty, \{c_t^w(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$, solves the household's problem as being a worker.

Given $\{\tau_t(\theta)\}_{t=0}^\infty, \{c_t^I(\theta), e_t(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$ solves the household's problem as being an innovator.

Given $\{p_t, w_t\}_{t=0}^\infty, \{x_t(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$ solves the final sector's problem.

Given $\{A_t\}_{t=0}^\infty, \{p_t\}_{t=0}^\infty \{\tau(\theta)\}_{t=0}^\infty, \{A_{t+1}\}_{t=0}^\infty, \{e_t(\theta)\}_{t=0}^\infty$ for all $\theta \in \Theta$ solves the developer problem.

Market clears for every time t

$$\begin{aligned} L^Y &= F(\hat{\theta}) \\ 1 - F(\hat{\theta}) &= e(\hat{\theta}) \end{aligned}$$

$$Y_t = \hat{\theta} c_t^w + \int_{\hat{\theta}}^1 c^I(\theta) dF(\theta) + \int_0^1 x_{i,t} di \quad (28)$$

5.7 Characterization of the steady-state symmetric equilibrium

In this subsection the steady-state allocations are characterized in a symmetric case. Next proposition study the balance growth path of the model and the threshold productivity that arise in equilibrium.

Proposition 2 *Under full information, with a uniform distribution of types θ , there is a unique balanced growth path characterized by a symmetric equilibrium in the terms of prices and quantities therefore quantities: $Y_t^* = \alpha^{\frac{2\alpha}{1-\alpha}} \hat{\theta} A_t$, $q^* = \frac{\tilde{\pi}\lambda\hat{\theta}}{(\rho+\beta)} + \theta$, $x_t^* = \alpha^{\frac{2}{1-\alpha}} A_t \hat{\theta}$ prices $r_t = \rho + g^*$, $w_t = (1 - \alpha) \alpha^{(2\alpha/(1-\alpha))} A_t$, $p = \frac{1}{\alpha}$. rate of growth $g^* = (1 - \hat{\theta}) \left[\hat{\theta} \left(\frac{\sigma\lambda^2\tilde{\pi} + (\rho+\beta)}{2(\rho+\beta)} + \frac{1}{2} \right) \right]$ and productivity cut-off $\hat{\theta} = \sqrt{\frac{2(\rho+\beta)}{\lambda\alpha[\tilde{\pi}\lambda+1]}} > 0$.*

Proof see the appendix

The rate of growth of this economy is determined by the proportion of the monopoly rents of the developer adjusted by the rate of creative destruction and the mean value of the skill of the workers. As standard Schumpeterian models there is an scale effect due to labor in R&D.

In this sense, increments in R&D investment have two effects: Firstly, there is a positive effect on growth since the production of blue prints increases, that means more monopolistic rents and therefore more revenue. Secondly, more investment in R&D implies that more selection of labor in the economy that reduces the total production in R&D. These two forces determine the optimal amount of R&D effort in the economy. Developers offer no linear transfers to the agents such that are competitive with respect to final sector and determines the level of productivity in equilibrium.

The productivity cut-off is determined in equilibrium, where the worker is indifferent between offer labor in one of the both activities. Thus, this is the situation where the

developers compete for labor with respect to the final sector. In this sense, developer's offers a menu of contract that are type contingent and are at least as good as the wage offer by the final sector.

In equilibrium, productivity threshold is positive with respect to the markup and the rate of the creative destruction. In both cases the effect is positive but decreasing. In the first place as the economy advances there are more demand for the intermediate good increases but a minor rate then the profits of the developers doesn't increase at all therefore more labor is allocated to final sector. Secondly as the probability of the replacement one technology but another with higher quality increases more mass of the workers goes to final sector.

6 Asymmetric information

In this section, I will describe the constrained efficient allocations when a central planner faces informational constraint about the skill level in the blueprints' technology. Secondly I will study the problem in a decentralized way. The optimal contract between developer and innovators is characterized and I will explain the main distortions that arise with respect to the constrained efficient outcome.

6.1 Constrained Efficient Allocation

From now assume that innovator is private informed about his productivity. Let's define σ as a reporting strategy with $\sigma : \Theta \rightarrow \Theta$. Let's define an allocation contingent on the productivity report as the $Z(\theta) \equiv \{c_t(\theta), c_t(\theta), q_t(\theta), e_t(\theta), A_t(\theta)\}_{t=1}^{\infty}$ for all $\theta \in \Theta$ for each t .

I define $Z(\sigma(\theta^*))$ as the truthfully reporting strategy where for each t $Z(\sigma(\theta^*)) = Z(\theta)$. Assume that the central planner implements a direct mechanism. That means the central planner ask for a productivity report $\tilde{\theta}$ and in exchange delivers an allocation $Z(\tilde{\theta})$. In order to induce truthfull strategies $Z(\tilde{\theta}) = Z(\theta)$ the optimal allocation must be satisfy the following incentive compatibility constraint:

$$\int_0^{\infty} \ln \left(c_t^I(\theta) - A_t \left((q_t(\theta) - \theta)^2 / 2 \right) \right) \exp(-\rho t) dt \geq \int_0^{\infty} \ln \left(c_t^I(\theta') - A_t \left((q_t(\theta') - \theta)^2 / 2 \right) \right) \exp(-\rho t) dt \quad (29)$$

The incentive compatibility constraint [29] shows lifetime discounted utility of an agent that report the true type θ and work for R&D sector is higher than any other possible deviation θ' . I work with the following transformation $\tilde{u}_t^I(\theta) = c_t^I(\theta) - A_t((q_t(\theta) - \theta)^2/2)$ and where $\varphi(\theta) = \int_t^\infty \tilde{u}_t^I(\theta) \exp(-\rho t) dt$. This transformation allows preserve the type ranking.

The incentive compatibility can be expressed in two parts: The information rent and monotonicity constraint. Information rent says that the central planner will deliver to the agent at least a level of utility equivalent to the discounted reservation utility (that is the discounted utility of the worker in final sector) and a reward that is characterized lifetime discounted utility of the innovator of type (θ) is equal to discounted utility when the agent is indifferent between to be worker in final sector or in R&D sector and an additional surplus that is proportional to the total of effort that make the agent when work in R&D activities.

$$\varphi(\theta) = \varphi(\theta, \hat{\theta}) + \underbrace{\int_t^\infty \int_{\hat{\theta}}^\theta (A_t(q(x) - x)) dx}_{\text{Information Rent}} \exp(-\rho t) dt \quad (30)$$

Second the central planner will reward schemes that are based on the increasing effort. In that sense, the production of blueprints is decreasing with respect to θ .

$$\int_t^\infty \left(\frac{dq_t}{d\theta}(\theta) \right) \exp(-\rho t) \leq 0 \quad (31)$$

To solve the following problem I will solve the relaxed problem that consists in solve the problem taking into account the information rents [30] and after the monotonicity condition is verified. It turn now that the central planner solves the following problem :

$$\max_{\{c_t^w, c_t^I, q_t, \theta^{\hat{b}}, x_{i,t}, A_{i,t}\}} \int_0^\infty \left[\int_0^{\theta^{\hat{b}}} (c_t^w(\theta)) dF(\theta) - \int_{\theta^{\hat{b}}}^1 (c_t^I(\theta) - A_t((q_t - \theta)^2/2)) (1 + \Delta(\theta)) dF(\theta) \right] \exp(-\rho t) dt \quad (32)$$

Where $\Delta(\theta) = \frac{1-F(\theta)}{f(\theta)}$ and subject to [6], [5], [7].

The first order condition of the problem entails:

$$[c_t^w, c_t^I] : \quad \exp^{-\rho t} = \mu \quad (33)$$

$$[q_t] : \quad (q_t - \theta) (1 + \Delta(\theta)) \exp^{-\rho t} = \lambda \sigma \int_0^1 \eta(i) di \quad (34)$$

$$[A_t] : \quad -((q - \theta)^2 / 2) (1 + \Delta(\theta)) \mu + \mu \left[(1 - \alpha) (L^Y)^{1-\alpha} \int_0^1 A_{i,t}^{-\alpha} x_{i,t}^\alpha \right] + \lambda \sigma \int_0^1 \eta(i) di \int_{\hat{\theta}}^1 q_t dF(\theta) = -\dot{\eta} \quad (35)$$

$$[\hat{\theta}^{fb}] : \quad \left[(1 - \alpha) \frac{Y}{\hat{\theta}} \right] = \lambda \sigma \left(q(\hat{\theta}) \right) \frac{\eta}{\mu} A + A \frac{(q(\hat{\theta}) - \hat{\theta})^2}{2} (1 + \Delta(\theta)) \quad (36)$$

$$[x_t] : \quad \mu \left[\alpha (L^Y)^{1-\alpha} \int_0^1 A_{i,t}^{1-\alpha} x_{i,t}^{\alpha-1} - 1 \right] \quad (37)$$

The new first order equations introduce two distortions: The first one is on the optimal amount of blueprints [34]. The LHS of equation shows that the marginal cost of provide effort increase in $(q_t - \theta) \Delta(\theta)$ due to the information friction. This distortion in the production of blueprints will affect also the invesment on R&D, equation [35] as well as the optimal occupational choice, equation [36].

6.2 Bilateral Asymmetric Information: Developers Innovators

Under this new assumption the principal offer contracts $\{q_t(\theta), \tau_t(\theta)\}_{\theta \in \Theta}$ that constitutes a payment $\tau_t(\theta)$ to a type θ agents, in exchange of certain amount of blueprints $q_t(\theta)$. The contractual problem is symmetric for all i sector, therefore we skip the index i . Under this informational constraint, the principal must be sure that for each type θ the agent is motivated to choose the contract $\{q_t(\theta), \tau_t(\theta)\}$ rather than $\{q(\theta'), \tau(\theta')\}$.

In this setting, I assume that the private information of the productivity affect the distribution of the output produced by innovator at each period t . An important assumption is the commitment for both parties. That means that at period 0 developer decides the intertemporal plan of amount of payments and the demand of effort based on her belief about the productivity of the agent. Alternatively, innovators need to decide the level of effort at each period that is according to the following incentive compatibility constraint:

$$\theta \in \arg \max_{\hat{\theta}} \left(u(\hat{\theta}) - A_t \frac{e(\theta)^2}{2} \right) \text{ for all } \theta \in \Theta \quad (38)$$

Therefore I will restrict to a set of announcements of innovator's productivity θ that satisfies truthtelling strategies. Innovator's preferences satisfy the Spence- Mirless condition:

$$\frac{\partial}{\partial \theta} \left(\frac{\partial U / \partial e}{\partial U / \partial \tau(\theta)} \right) = -e(\theta) < 0 \quad (39)$$

The condition [39] expresses the marginal rate of substitution between effort and the innovator's payment. Therefore the condition entails as lower is the type (more efficient agent) lower is the wage required to induce a determined level of effort, therefore the analysis is restricted to effort functions that are increasing in the agents efficiency.

The participation constraint can be re-written expressing in term of value function:

$$U^I(\theta) = \max_{\tilde{\theta}} \left(c_t^I(\tilde{\theta}) - A_t \frac{(q(\tilde{\theta}) - \theta)^2}{2} \right) = c^I(\theta) - A_t \frac{(q(\tilde{\theta}) - \theta)^2}{2}$$

Thus, the first order condition for type θ^* is :

$$\frac{dc^I(\tilde{\theta})}{d\tilde{\theta}} - A_t (q(\tilde{\theta}) - \theta) \frac{dq(\tilde{\theta})}{d\tilde{\theta}} = 0$$

Under truthfully strategies must be satisfy:

$$\frac{dc^I(\theta)}{d\theta} - A (q(\theta) - \theta) \frac{dq(\theta)}{d\theta} = 0 \text{ for all } \theta \in \Theta \quad (40)$$

It's also necessary that satisfy the second order conditions:

$$\frac{d^2c^I(\theta)}{d\theta^2} - \frac{d\tilde{q}^2(\theta)}{d\theta^2} [1 - (\tilde{q}(\theta) - \theta)] + \frac{d\tilde{q}(\theta)}{d\theta} \leq 0 \quad (41)$$

Differentiating [40] with respect to $\tilde{\theta}$ we obtain:

$$\frac{d^2c^I(\tilde{\theta})}{d\tilde{\theta}^2} - \frac{dq^2(\tilde{\theta})}{d\tilde{\theta}^2} [1 - (q(\tilde{\theta}) - \theta)] \leq 0 \quad (42)$$

under truthtelling strategies $\theta = \tilde{\theta}$, replacing [42] in [41] implies:

$$\frac{dq}{d\theta}(\theta) \leq 0 \quad (43)$$

This is the monotonicity constraints for the blueprints. Using the envelope theorem must be satisfy:

$$\frac{dU^I}{d\theta}(\theta) = A (q(\theta) - \theta) \quad (44)$$

Therefore integrating [44] from 0 to 1-type we can rewritten the innovator's indirect utility as follow: $U^I(\theta) = U^I(\underline{\theta}) + \int_0^1 (q(x) - x) dx$. Nevertheless $U^I(\underline{\theta}) = \bar{\omega} (1 - \hat{\theta})$ the outside

option of the agent that represents wages that is offered in final goods sector. The indirect utility function of the contract is $U^I(\theta) = c^I(\theta) - \frac{(q(\theta) - \theta)^2}{2} = \bar{\omega} (1 - \hat{\theta}) + \int_{\theta}^1 (q(x) - x) dx$. In this sense, the scheme of payments to innovator is according to:

$$c^I(\theta) = A \left(\frac{(q(\theta) - \theta)}{2} + \bar{\omega} (1 - \hat{\theta}) + \underbrace{\int_{\theta}^1 (\tilde{q}(x) - x) dx}_{\text{Information rent}} \right) \quad (45)$$

The participation constraint under asymmetric information is the similar to the symmetric information case but in addition, there is an information rent that the developer gives to the agent to reveal the private information. The total payment for the innovator must be at least equal to the reservation utility plus the disutility of effort. It means that the payments are increasing with the level of effort.

The problem for the developer under asymmetric information is:

$$\max_{q_t(\theta), \tau_t(\theta)} \int_{\hat{\theta}_i}^1 [\lambda q_t(\theta) V_t - \tau(\theta)] dF(\theta)$$

subject to the participation constraint [45] and the monotonicity constraint :

$$\frac{d\tilde{q}_t}{d\theta}(\theta) \leq 0 \quad (46)$$

This condition implies that the innovators' type θ don't lie locally. Since the Spence- Mirless condition is satisfied then local incentives constraint implies lead to global constraints. In order to solve the previous problem I characterized the *relaxed problem* in which the monotonicity constraint on effort is ignored and after the solution of the relaxed problem is verified under the monotonicity constraint. Integrating by parts the problem of the developer is written as:

$$\max_{q_t(\theta), \tau_t(\theta)} \int_{\hat{\theta}_i}^1 \left[\lambda q_t(\theta) V_t - A_t \left(\frac{(q_t - \theta)^2}{2} + \frac{(1 - F(\theta))}{f(\theta)} (q_t - \theta) \right) \right] dF(\theta) - \bar{\omega} (1 - \hat{\theta}) \quad (47)$$

subject to [46] .

The first order condition entails:

$$[q_t] : V_t = \frac{A_t}{\lambda} \left[(q_t - \theta) + \frac{(1 - F(\theta))}{f(\theta)} \right] \quad (48)$$

$$(49)$$

The equation [48] establishes the value of an innovation. This value is equal to the level of effort under asymmetric information and it that includes the trade-off between efficiency versus *current extraction-rent friction*. The friction introduced by asymmetric information is captured by the inverse of the hazard rate defined by $\Delta(\theta) = \frac{1-F(\theta)}{f(\theta)}$. In the margin, the cost of the effort is distorted by the conditional probability of θ - *type* that lies on the interval $[\theta, 1]$ can change in a small neighborhood $[\theta, \theta + d\theta]$.

The next proposition analyzes the impact of the information friction into the equilibrium amount of blueprints and rate of economic growth:

Proposition 3 *Growth and Selection, under asymmetric information, the equilibrium menu of contracts entails:*

- *Distortion in the equilibrium quantity of blueprints is: $q^{AI}(\theta) = \frac{\lambda\bar{\pi}\hat{\theta}^{AI}}{(\rho+\beta)} + \left(\theta - \frac{(1-F(\theta))}{f(\theta)}\right) < q^*(\theta)$.*
- *Reduction on the rate of growth of the economy with respect to the full information is $g^{AI} = \left(1 - \hat{\theta}^{AI}\right) \left[\hat{\theta}^{AI} \left(\frac{\sigma\lambda^2\bar{\pi}}{(\rho+\beta)} + \frac{\lambda}{2}\right)\right] < g^*$.*
- *Distortion in the cut-off level of productivity such that $\hat{\theta}^{AI} > \hat{\theta}^*$*

Proof see in the appendix

Under asymmetric information the value of one unit of production of blueprints is given by the level of effort in R&D plus the information rent that the developer transfers to the innovator as incentive to exert higher level of effort. Adverse selection problem leads to a distortion of the amount of blueprints in equilibrium. In particular, the amount of blueprints chosen by the developer affects the mass for workers in each activity. This is the traditional extraction -rent tradeoff that generates more dispersion in the productivity types. Nevertheless there is also a selection effect. As the mean value of the productivity increases there is a scale effect that affect positively the rate of growth of the economy.

6.3 Multiple principals (To be Completed).

In the previous section, I characterized the case when a single developer in sector i offers a set of constructs that are incentive compatible with respect to the outside option given by the wages offered in final sector. In this section I go more deeply in to understand

how competition inside R&D activities can affect the incentives to innovate under private information. In this setting each firm in sector i competes against other R&D firms in order to attract talented innovators.

In this case, the participation constraint is type dependent on the level of productivity θ . In this section I will show that given a level of productivity threshold $\hat{\theta}$ there is some incentives for the developers to propose alternative contracts that can be attractive for the innovators. I argue at least for two reason the developers have incentives to competition: Firstly the developers want to maintain the monopolistic position and therefore escape from competition. Then developers wants to hired talented innovators, invest in R&D to maintain the market power.

Second, there is an important contractual externality that emerges from the competition. This is that reveal information about the developers' willingness to for a new innovation. This point is important because in our previous setup the demand for new innovation is passive since the demand for new intermediate goods depends on the markup.

The technique applies in this subsection follows that are used by Biglaiser and Mezzetti [1993], Champasaur - Rochet [1989] , and Jullien [2000] . In general the problem to consider is :

$$\max_{q_t(\theta)U^I} \int_{\hat{\theta}_i}^1 \{ \lambda q_t(\theta) V_t - A_t [(q(\theta) - \theta)^2 / 2 - U^I(\theta)] \} dF(\theta) \quad (50)$$

Subject to [2] and

$$\frac{dU^I}{d\theta}(\theta) = (q(\theta) - \theta) \quad (51)$$

$$U^I(\theta) \geq U^o(\theta) \quad (52)$$

The objective function considers as before the innovation flow and the cost of effort taking into account the monotocity constraint and the outside option given by other firm o in sector i . The key aspect is that the informational rent is not monotonic and lies on the interior of the participation constraint. When there is competition among developers there is not trivial characterization for R&D contracts. In fact there is place for endogenous exclusion and bunching regions see Jullien [2000].

Why it is matter for growth?. Competition leads to change in R&D investments, as I will present, more specific, the shape of the investment on R&D is not monotonic with the

type of the agent and induces more selection from the point of view of the developers implies more rent extraction. The next proposition studies the configuration of cream skimming contracts in R&D in the model:

Proposition 4 (*Cream skimming contracts*), *Suppose that firm offers a contract $o \in i \{\tau^o, q^o\}$ then the cream skimming contract is characterized by a two productivity thresholds $\{\theta_1, \theta_2\} \in [\hat{\theta}, 1]$ such that:*

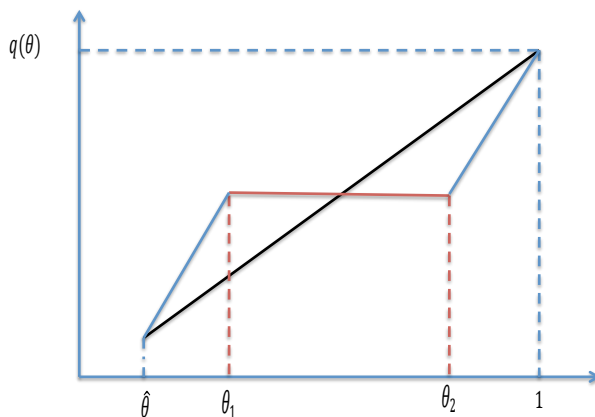
- *On $[\hat{\theta}, \theta_1]$ there is upward distortion on which the amount of blue print is $\bar{q}(\theta) = \frac{1}{\eta}V + \theta + \frac{1}{\eta}(1 - \hat{\theta})$*
- *On $[\theta_1, 1]$ there is a bunching region and the blue prints quantities are given by $\bar{q}(\theta) = q^o = \frac{1}{\eta}V + \theta - \frac{1}{\eta}(\psi(\theta) - (1 - \hat{\theta}))$ where $\psi(\theta) = \int_{\hat{\theta}}^1 d\psi(x)$ is a measure of Lagrange multipliers. The payment for the innovator is given by $\bar{u}(\theta) = u^o(\theta)$.*
- *On $[\theta_1, 1]$ there is downward distortion where $\bar{q}(\theta) = \frac{1}{\eta}V + (\theta - \frac{1}{\eta}\hat{\theta})$ and $\bar{u}(\theta) = u^o(1) - \int_{\theta}^1 \bar{q}(s) ds$.*

The proposition establishes that on the region between $[\hat{\theta}, \theta_1]$ innovators respond more to change in the outside option. In particular, innovators have incentives to over-report his type. The result is over-production of blueprints with respect to the full information case. The first part of the proposition shows as the proportion of innovator's type allocated to R&D increases, the reservation utility is larger for higher levels of θ and becomes more attractive for the lower types.

As the participation constraint is binding, there is a region for which the quantity that maximize the profit of the developers is not monotonic. For intermediate type, the principals face conflicts between the incentive compatibility constraints and the minimization of the informational rents.

In this case the developer to restore the incentive compatibility constraints, propose the same transfer scheme for the types between $[\theta_1, \theta_2]$ independent of the level of blueprint that the innovators produces in this range. In fact, under this region there are a sort of countervailing incentives playing with the incentives for the agent to over/ under report their type.

In the region that lies on the interval $[\theta_2, 1]$ there is under-production of blueprints with respect to full information case. In this case, as increase the type, for the innovators are more costly in terms of effort and tends to under report their types. The shape of the optimal contract is given in next figure:



This kind of compensation can be found in high tech industries and financial services. For instance software industry is characterized because there are high variance in the returns to innovations and are more likely to pay for star workers.

As is documented by Andersson et al [2009] talented innovators' compensation in software industry is in average twice than innovators' salary in the other industries. In addition, they show that these industries pay more workers with highly loyalty, in particular stay with a firm for five years implies higher earnings that can represented in stock options or other benefits. This trend is more persistent in firms in which with high variance market payoffs.

Our second example is in the financial industry in which CEO's compensation plays an important role. Gabaix and Landier [2007] have been showed that how CEO's compensation increases as the size of the firm increases. Celerier [2010] shows for France that important premium in the financial sectors are associated to skeness on the wages and the returns on the seniority.

7 Concluding Remarks

In this paper, I construct an endogenous growth with non-observable heterogeneity under adverse selection. The main message of the paper is that heterogeneity introduces a new scale effect that is important in the determination of the rate of growth. In addition adverse selection introduces a negative impact over the economic growth because it increases the

dispersion between the innovators productivity. In conclusion, the equilibrium contracts entail more selection of talented workers in R&D activities and higher profit rate for the developer with respect to the case of full information.

This paper also provides an analysis when there are several principals with adverse selection. The main results establish countervailing incentives for the innovators that affect the total production of blueprints in the economy and therefore the probability of the arrival new innovation. In addition, competition can be welfare enhancing and reduces the rents for the developer on the production of intermediate goods. Nevertheless, our result doesn't take into account possible scheme of communication and information sharing between principals. This is a possible extension in future research.

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9 Appendix

Proof of the proposition 1

From the first order condition [13] in the symmetric case, solving for $x_i^{fb} = x^{fb} = \hat{\theta}^{fb} A \alpha^{\alpha/(1-\alpha)}$, replacing in the production function for the symmetric case it yields $Y^{fb} = \hat{\theta}^{fb} A \alpha^{(\alpha/(1-\alpha))}$. So using the aggregate resources constraint the aggregate consumption is characterized $C^{fb} = \hat{\theta}^{fb} A \alpha^{\alpha/(1-\alpha)} (\alpha^\alpha - 1)$, then the first set of allocations is obtained. Now, to obtain the total amount of blueprints, from the first order condition [9] $c_t^w = c_t^I - A_t e_t^2 / 2$, replacing in the optimality condition for blueprints [10] it yields:

$$q^{fb} - \theta = \lambda \sigma \frac{\eta}{\mu} \quad (53)$$

Since $\frac{\exp^{-\rho t}}{c_t^w} = \mu$. Solving for $\frac{\eta}{\mu}$ from [12] it gets

$$\frac{\eta}{\mu} = \frac{(1 - \alpha) \alpha^{(\alpha/(1-\alpha))}}{\lambda \sigma (q^{fb} + \hat{\theta}^{fb} - \theta)}. \quad (54)$$

Substituting expression [54] in [53] the expression for q^{fb} is obtained. The second part is related to the steady-state rate of growth. Notice that optimal final output, intermediate goods and aggregate consumption are proportional to the aggregate stock of knowledge, then the rate of growth are equalized among them to the rate of growth of productivity. Using the optimal amount of blueprints q^{fb} the rate of growth of the productivity is given by :

$$g^{fb} = \sigma \lambda \frac{(1 - \hat{\theta}^{fb})}{2} \left[\sqrt{1 + \hat{\theta}^{fb^2} + 4(1 - \alpha) \alpha^{\alpha/(1-\alpha)}} \right] \quad (55)$$

The last part of the proposition is concerning about the characterization of the productivity threshold $\hat{\theta}^{fb}$. Using the first order condition [11] and using the fact that $\frac{e \exp(-\rho t)}{c_t^w} = \lambda \sigma \eta$, dividing all the expression by η and replacing the optimal amount for intermediate goods yields:

$$-\lambda\sigma e/2 + \frac{\mu}{\eta} \left[(1 - \alpha) \theta^{\hat{f}^b} \alpha^{\alpha/(1-\alpha)} \right] + \lambda\sigma \int_{\theta^{\hat{f}^b}}^1 q(\theta) dF(\theta) = -g_\eta \quad (56)$$

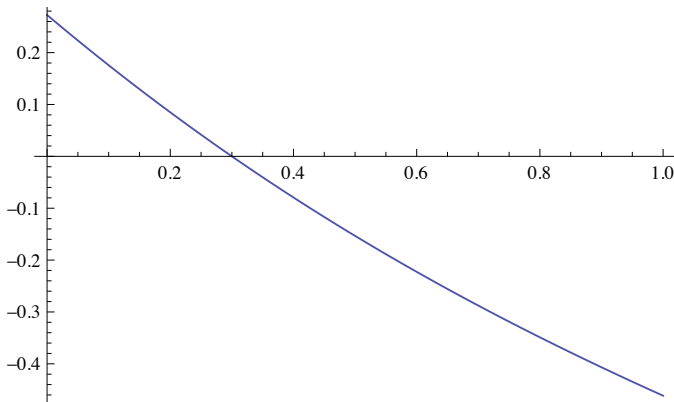
where $g_n = \frac{\dot{n}}{\eta}$. As $q^{\hat{f}^b}$ is a stationary variable and using the expression [54] implies that $g_\eta = g_\mu = -g_{c^w} - \rho$ then as g_{c^w} in steady-state grows at the rate of the technology g_A then $-g_\eta = g_A + \rho$. Similarly $g_A = \lambda\sigma \int_{\theta^{\hat{f}^b}}^1 q(\theta) dF(\theta)$ therefore can be simplified as :

$$-\lambda\sigma e/2 + \frac{\mu}{\eta} \left[(1 - \alpha) \theta^{\hat{f}^b} \alpha^{\alpha/(1-\alpha)} \right] = \rho \quad (57)$$

Now there is expression of the level of effort and for μ/η then equation [57] collapses to a polynomial of parameters that is denoted by $\Psi(\hat{\theta})$:

$$\Psi(\hat{\theta}^{\hat{f}^b}) = \sqrt{\frac{\hat{\theta}^{\hat{f}^b 2} + 4(1 - \alpha) \alpha^{\alpha/(1-\alpha)}}{2}} - \sqrt{\frac{\rho^2 + 2\hat{\theta}^{\hat{f}^b} (1 - \alpha) \alpha^{\alpha/(1-\alpha)} (\sigma\lambda)^2}{(\lambda\sigma)}} - \left(\frac{\theta}{2} + \frac{\rho}{\lambda\sigma} \right) \quad (58)$$

So, the polynomial $\Psi(\hat{\theta}^{\hat{f}^b})$ is studied in the interval of parameters $\theta \in [0, 1]$. In this sense, it's to shown that there is a number M such that $\Psi(0) < M < \Psi(1)$ or the way around. Characterizing $\Psi(0) = \sqrt{2(1 - \alpha) \alpha^{\alpha/(1-\alpha)}} - \frac{\rho}{\sqrt{\lambda\rho}} \left[1 + \frac{1}{\sqrt{\lambda\rho}} \right]$ and for $\Psi(1) = \sqrt{\frac{1+4(1-\alpha)\alpha^{\alpha/(1-\alpha)}}{2}} - \sqrt{\frac{\rho^2+2(1-\alpha)\alpha^{\alpha/(1-\alpha)}(\sigma\lambda)^2}{(\lambda\sigma)}} - \left(\frac{1}{2} + \frac{\rho}{\lambda\sigma} \right)$. Then for standard values of $0 < \rho < 1$, $0 < \lambda < 1$, $\sigma > 1$ it has $\frac{\partial\Psi(0)}{\partial\alpha} > 0$ and $\frac{\partial\Psi(1)}{\partial\alpha} < 0$. As the $\Psi(\hat{\theta}^{\hat{f}^b})$ is continuous for all $\hat{\theta}^{\hat{f}^b} \in [0, 1]$ and in particular for $M = 0$ the intermediate value theorem applies and therefore the polynomial does have a root between 0 and 1. The next graph shows the root that it was just proved existed.



Proof of the proposition 2

Replacing equation 25 in the non arbitrage condition in the asset market (equation 27) we obtain the equilibrium value of blue prints:

$$q^*(\theta) = \frac{\lambda \tilde{\pi} \hat{\theta}}{\rho + \beta} + \theta \quad (59)$$

Replacing the markup in the demand of intermediate good we obtain $x_t^* = \alpha^{\frac{2}{1-\alpha}} A_t \hat{\theta}$ and entails that the total output in the economy is given by $Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \hat{\theta} A_t$ and the total profits for the developer are $\pi_t = \tilde{\pi} A_t \hat{\theta}$. In the balanced growth path for a symmetric industries we have that $g = g_c^I = g_c^W = g_y = g_A$. Replacing this value in 5 we obtain the rate of growth of the productivity and therefore the rate of growth of the economy is :

$$g = g^* = (1 - \hat{\theta}) \left[\hat{\theta} \left(\frac{\sigma \lambda^2 \tilde{\pi} + (\rho + \beta)}{2(\rho + \beta)} + \frac{1}{2} \right) \right] \quad (60)$$

Since the preferences follows a logarithmic form, and in the setady state aggregate consumption grows at the rate of the technology then interest rate also. In the case of wages as are proportional to the aggregate stock of knowledge grows at the rate of the technology.

$$r_t = \rho + g^*, \quad w_t = (1 - \alpha) \alpha^{(2\alpha/(1-\alpha))} A_t, \quad p = \frac{1}{\alpha}. \quad (61)$$

The determination of the cutoff productivity is given by the participation constraint. As there is free entry in the R&D activity then we can obtain the payments for the innovator in equilibrium according with the level of productivity θ and the total amount of blue-prints in equilibrium:

$$\int_{\hat{\theta}}^1 \tau(\theta) dF(\theta) = \bar{\tau} = Ae^*(e^* + E_{\theta}(\theta)) \quad (62)$$

Replacing the total of payment of innovation for all type selected in R&D $\theta \in [\hat{\theta}, 1]$ in the participation constraint 24 we will obtain that $\frac{(e^*)^2}{2} + e^* \theta = (1 - \alpha) \alpha^{(2\alpha/(1-\alpha))}$. For $\theta = \hat{\theta}$ replacing 59 and solving for a uniform distribution we obtain that the cutoff of the productivity is determined by: $\hat{\theta} = \sqrt{\frac{2(\rho+\beta)}{\lambda\alpha[\tilde{\pi}\lambda+1]}} > 0$

Proof of the proposition 3

The proof is similar to the case of full information, but now with asymmetric information add the virtual surplus capture by the inverse of the hazard rate. Then replacing equation

48 in the non arbitrage condition for asset markets, equation 27 we obtain that $q^{AI}(\theta) = \frac{\lambda \tilde{\pi} \hat{\theta}^{AI}}{(\rho + \beta)} + \left(\theta - \frac{(1-F(\theta))}{f(\theta)} \right) < q^*(\theta)$ as $\Delta(\theta)$ is increasing on θ then $q^{AI}(\theta) < q^*(\theta)$. Replacing this expresion in the rate of growth of the economy we obtain that :

$$g^{AI} = \left(1 - \hat{\theta}^{AI} \right) \left[\hat{\theta}^{AI} \left(\frac{\sigma \lambda^2 \tilde{\pi}}{(\rho + \beta)} + \frac{\lambda}{2} \right) \right] < g^*$$

To find the cut-off the participation constraint, is used, replacing the value for $q^{AI}(\theta)$, the equilibrium cut-off is given by:

$$\hat{\theta}^{AI} = \left(\sqrt{\delta^2 + 4(\delta^2/2\omega)} + \delta \right) / (\delta^2 - 2) > \theta^*. \quad (63)$$

Where $\omega = (1 - \alpha) \alpha^{(2\alpha/(1-\alpha))}$ and $\delta = \frac{\tilde{\pi} \lambda}{(\rho + \beta)}$

Proof of the proposition 4

The Lagrangean for this problem is :

$$L = \max \int_{\hat{\theta}}^1 \left[\lambda q(\theta) V_t - A_t \left(\frac{q_t - \theta}{2} \right)^2 - U^I(\theta) \right] dF(\theta) + \mu q(\theta) + \int_{\hat{\theta}}^1 (U^I(\theta) - U^o(\theta)) d\psi(\theta)$$

$$[q(\theta)] : [\lambda V_t - A_t (q_t - \theta)] f(\theta) + \mu \quad (64)$$

$$[U^I(\theta)] : -f(\theta) + \psi(\theta) = -\dot{\mu}(\theta) \quad (65)$$

Solving 65 we get that $\mu(\theta) = F(\theta) - \psi(\theta)$

Then the first order condition entails :

$$\lambda V_t = \left(\frac{\psi(\theta) - F(\theta)}{f(\theta)} \right) + A_t (q_t - \theta) \quad (66)$$

with $\psi(\theta) = \int_{\hat{\theta}}^1 d\psi(x)$ is a random measure of the Lagrange multipliers. Evaluating when the participation constraint is binding $\psi(\theta) = 1$ or when $\psi(\theta) = 0$ the results yields.